NUMERICAL PHYSICS

VOL. II

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Based on the latest syllabus prescribed by the Central Board of Secondary Education, New Delhi for Class XII for the Delhi & All India Senior School Certificate Examinations.

NUMERICAL PHYSICS

VOL. II

(FOR CLASS XII)

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PREFACE

The syllabus in Physics for the students preparing for class XII for Delhi and All India Senior School Certificate Examinations of the Central Board of Secondary Education, New Delhi has been changed under the National Policy on Education, 1986. The present syllabus has given a new approach to the study of Physics at school level. This approach is essentially based on the active participation of the students in the learning process through experimentation, supplemented by demonstration by teachers, practical activity by the students and discussion leading to the understanding of basic concepts in Physics without loss of mathematical rigour.

Each topic in the new syllabus has been tried to be made problem based by the team of the experts in writing the physics book for XII published by NCERT. The idea is to build up the ability of the students to think about a phenomenon and then to solve it scientifically and to stimulate their interest. So a good book of Physics Numerical is essentially needed to meet the challenges of the new approach of syllabus in Physics.

I have followed the same approach in writing this book. It is according to the prescribed syllabus of C.B.S.E., New Delhi for +2 (Senior Secondary Certificate) Examination, 1991 and onwards and fullfils the needs of students. The material included in the book is in conformity with the trend of new syllabus which makes the study of Physics more comprehensive and illustrative. I have been conscious that every student has his own pace of learning and some students can progress at a rapid pace. It has been my attempt to bring basic ideas with care so that every student may enjoy the learning of Physics and can make its applications in daily life. In order to facilitate easy comprehension, a number of problems have been solved with diagrams.

I hope that the book will be found useful by the students and teachers of various institutions, which have adopted the 10+2 pattern. It will also equally help the students preparing for admission in engineering and medical courses. It is for this reason that the multiple choice type and other questions from various competitive examinations like I.I.T., J.E.E., Roorkee, C.P.M.T., D.P.M.T., etc. have been inclued in this book in large number at the end of the exercises.

I will appreciate suggestions from the readers for the improvement of the book. In the end I sincerely thank Shri Ved Bhushan, of M/S. Pitambar Publishing Company for his painstaking efforts in publishing this book well in time.

V.K. AGRAWAL

CONTENTS

Electrostatic Force, Field and Potential

IMPORTANT FORMULAE

1. **Coulomb's Law.** The force of attraction or repulsion between two charges q_1 and q_2 separated by a distance r is given by

$$F=k\frac{q_1 q_2}{r^2}$$

where k=Constant of proportionality $=9\times10^9$ for air

2. Electric Field Intensity,

$$E=\frac{kq}{r^2}$$

Also $k=\frac{1}{4\pi\epsilon_0}$ where ϵ_0 is known as permittivity of vacuum and has the value $8{\cdot}854\times10^{-12}\ C^2/Nm^2$.

3. Force on a charged particle (q) due to an electric field (E) :

$$F=qE$$

4. Electric potential, $V=k\frac{q}{r}$

5. Work done on a charge in displacing it from one point to the other in an electric field,

$$W=q\times V$$

where V=Potential difference between the two points.

6. Charge density,

$$\sigma=k\frac{q}{A} \qquad \text{where } A=\text{Area}$$

7. The dipole moment of two point charges q and $-q$ separated by a distance 'a' is given by

$$p=q\times a$$

8. The torque on a dipole in a uniform electric field E,

$$\vec{\tau}=\vec{q}\times\vec{E}$$

9. The potential energy,

$$E=k\frac{q_1 q_2}{r}$$

SOLVED EXAMPLES

Example 1. *An ebonite rod is rubbed with fur, the later is found to have a negative charge of 4·8 μc.*

(a) Calculate the number of electrons transferred from ebonite to fur.

(b) Is there any transfer of mass from ebonite to fur ?

Solution. (*a*) $q=4{\cdot}8\ \mu c=4{\cdot}8\times10^{-6}$ coulomb

and $e=1{\cdot}6\times10^{-19}$ coulomb

Now we know $q=ne$

$\therefore$ $n=\frac{q}{e}$

$$=\frac{4{\cdot}8\times10^{-6}}{1{\cdot}6\times10^{-19}}$$

$$=3\times10^{13} \text{ electrons}$$

(*b*) Mass transferred to fur from ebonite

$$=nm_e=3\times10^{13}\times9{\cdot}1\times10^{-31}$$

$$=2{\cdot}73\times10^{-17}\text{ kg}$$

Example 2. *Three charges 8μC, +5μC and −4μC are placed at the vertices A, B, C respectively of a equilateral triangle whose side is 5 cm. Find the resultant force on the charge at 'A' due to the two other charges.*

Solution. The force on the charge at A due to that at B, by

$$F=k\frac{q_1 q_2}{r^2},$$

$$F_1=\frac{(9\times10^9)\times(8\times10^{-6})\times(5\times10^{-6})}{({\cdot}05)^2}$$

$$=144\text{ N along }\overrightarrow{BA}$$

Similarly force on the charge at A due to that at C,

$$F_2=\frac{(9\times10^9)(8\times10^{-6})(4\times10^{-6})}{({\cdot}05)^2}$$

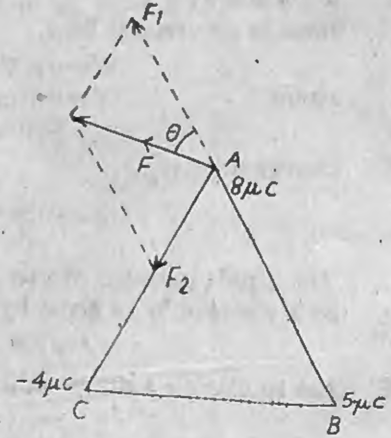


Fig. 1.1.

$F_2 = 119{\cdot}2$ N along $\overrightarrow{AC}$

$\therefore$ Resultant force at A,

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos\theta}$$
$$= \sqrt{144^2 + (119{\cdot}2)^2 + 2\times 144 \times 119{\cdot}2 \cos 120}$$
$$= \sqrt{20736 + 14208{\cdot}64 + 2\times 17164{\cdot}8\ (-\tfrac{1}{2})}$$
$$= \sqrt{17779\ 84} = 133{\cdot}34 \text{ N}$$

$$\tan\theta = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$
$$= \frac{119{\cdot}2 \sin 120}{144 + 119{\cdot}2 \cos 120}$$
$$\tan\theta = \frac{119{\cdot}2\ (\sqrt{\tfrac{3}{2}})}{1444 + 119{\cdot}2\ (-\tfrac{1}{2})} = 1{\cdot}223$$
$$\therefore \quad \theta = 54^\circ\ 44'$$

$\therefore$ The resultant force on the charge at A is $133{\cdot}34$ N and this force acts in a direction making an angle $54^\circ\ 44'$ with the direction of F_1.

Example 3. *A proton falls vertically downward through a distance of 2 cm in a uniform electric field of magnitude $3{\cdot}34\times 10^3$ Nc^{-1}. Calculate (a) the acceleration of proton falling down, (b) the time taken by proton to fall a distance of 2 cm & (c) the direction of electric field.*

Solution. (*a*) $F = eE$

$$\therefore \quad a = \frac{F}{m_p} = \frac{eE}{m_p}$$

Now $e = 1{\cdot}6\times 10^{-19}$ C, $E = 3{\cdot}34\times 10^3$ NC^{-1}

and $m_p = 1{\cdot}67\times 10^{-27}$ kg.

$$\therefore \quad a = \frac{1{\cdot}6\times 10^{-19}\times 3{\cdot}34\times 10^3}{1{\cdot}67\times 10^{-27}}$$
$$a = 3{\cdot}2\times 10^{11}\ \text{ms}^{-2}$$

(*b*) Neglecting acceleration due to gravity $9{\cdot}8$ ms^{-2} in comparison to $a = 3{\cdot}2\times 10^{11}$ ms^{-2},

$$a = 3{\cdot}2\times 10^{11}\ \text{ms}^{-2}, \quad V_i = 0, \quad s = {\cdot}02 \text{ m}$$

$\because$ By $s = V_it + t + \frac{1}{2}at^2$,

$$t = \sqrt{\frac{2s}{a}}$$
$$= \sqrt{\frac{2\times{\cdot}02}{3{\cdot}2\times 10^{11}}}$$
$$= 3{\cdot}535\times 10^{-7}\ \text{S}$$

(*c*) The direction of electric field is vertically downward.

Example 4. *The voltage across the electrodes of a cathode ray gun is 400 volts. Calculate the energy gained by electrons, (b) the speed of electrons & (c) the momentum of the electrans. (mass of an electron=9×10^{-31} Kg and charge on electron=$1{\cdot}6\times10^{-19}$ C).*

Solution. (*a*) $e=1{\cdot}6\times10^{-19}$ C ; V=400 volts

$$m=9\times10^{-31}\text{ kg.}$$

$$E=eV$$

$$=1{\cdot}6\times10^{-19}\times400\text{ J}$$

But $1{\cdot}6\times10^{-19}$ J=1 *e*V

$\therefore$ $E=400\ eV$

(*b*) $\frac{1}{2}mv^2=E$

$$\therefore\quad v=\sqrt{\frac{2E}{m}}$$

$$=\sqrt{\frac{2\times1{\cdot}6\times10^{-19}\times400}{9\times10^{-31}}}$$

$$=1{\cdot}19\times10^{7}\text{ m s}^{-1}$$

(*c*) The momentum,

$$p=mv$$

$$=9\times10^{-31}\times1{\cdot}19\times10^{7}$$

$$=1{\cdot}071\times10^{-23}\text{ kg ms}^{-1}.$$

Example 5. *An electron beam with velocity 4×10^{7} ms^{-1} passes through a parallel plate capacitor. The distance and electric intensity between the plates are 5 cm and 10 volt cm^{-1} respectively. If the length of each plate is 20 cm, calculate the deflection angle of the beam.*

Solution. $u=0$; $v=4\times10^{7}$ ms^{-1} ; $l=20$ cm$=0{\cdot}20$ m.

$$E=10\text{ V cm}^{-1}=10^{3}\text{ V m}^{-1}$$

From $s=ut+\frac{1}{2}at^2$

$$s=0+\frac{1}{2}a\left(\frac{l}{v}\right)^2$$

Now $a=\dfrac{F}{m_e}=\dfrac{eE}{m_e}$

$$\therefore\quad s=\frac{1}{2}\,\frac{eE}{m_e}\,\frac{l^2}{v^2}$$

$$=\frac{1}{2}\,\frac{1{\cdot}6\times10^{-19}\times10^{3}\times{\cdot}20^2}{9\times10^{-31}\times(4\times10^{7})^2}$$

$$=\frac{2}{9}\times10^{-2}\text{ m}=\frac{2}{9}\text{ cm.}$$

Now $\tan\theta=\dfrac{s}{l}$

$$=\frac{\frac{2}{9}}{20}$$
$$=0{\cdot}0111$$
$$\theta=36'$$

Example 6. *A dipole consists of two charges* $+10\mu C$ *and* $-10\mu C$ *separated by a certain distance. Let they be located at* $x=6{\cdot}0$ *cm,* $y=0$ *and* $x=-6{\cdot}0$ *cm,* $y=0$ *respectively. Calculate the field strength at a point* $x=0$, $y=8$ *cm.*

Solution. OA=OB=6 cm. OP=8 cm.

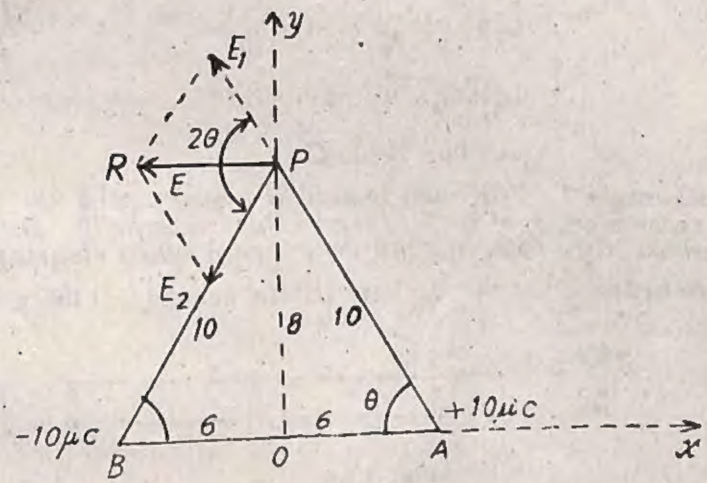


Fig. 1.2.

$$AP=\sqrt{OA^2+OP^2}$$
$$=\sqrt{6^2+8^2}$$
$$=10 \text{ cm.}$$

Intensity of electric field at P due to charge at A,

$$E_1=\frac{kq}{r^2}$$
$$=\frac{(9\times10^9)\,(10\times10^{-6})}{({\cdot}10)^2}$$
$$=9\times10^6 \text{ N/C along } \overrightarrow{AP}$$

Intensity of electrical field at P due to charge at B,

$$E_2=\frac{(9\times10^9)\,(10\times10^{-6})}{({\cdot}10)^2}$$
$$=9\times10^6 \text{ along } \overrightarrow{PB}$$

$$\cos\theta=\frac{6}{10}=\frac{3}{5}$$

$$\cos 2\theta=2\cos^2\theta-1=2\left(\frac{3}{5}\right)^2-1=-\frac{7}{25}$$

$\therefore$ Resultant electric field at P,

$$E=\sqrt{E_1^2+E_2^2+2E_1E_2\cos 2\theta}$$

$$=\sqrt{(9\times10^6)^2+(9\times10^6)^2+2(9\times10^6)^2\times\left(-\frac{7}{25}\right)}$$

$$=9\times10^6\sqrt{1+1-\frac{14}{25}}$$

$$=\frac{9\times10^6}{5}\sqrt{36}$$

$$=1{\cdot}08\times10^5 \text{ N/C.}$$

Example 7. *Two small spheres of equal size 8 cm. apart in air carry charges of +16 μc and −9μc respectively. Determine the position of the point at which there is no resultant electric field.*

Solution. Let the resultant electric field be 0 at the point P,

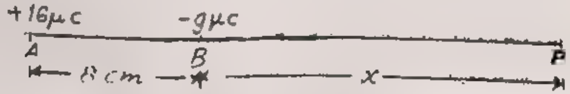


Fig. 1.3.

x cm. away from B. Then

$$k\frac{q_1}{r_1^2}=k\frac{q_2}{r_2^2}$$

$$\frac{16}{(x+8)^2}=\frac{9}{x^2}$$

or
$$\frac{4}{(x+8)}=\frac{3}{x}$$

$$x=24 \text{ cm.}$$

Example 8. *What charge is required to electrify a sphere of 15 cm. radius, so that surface density is* $\frac{7}{11}$ μc/m².

Solution.

$$r=15 \text{ cm.}=0{\cdot}15 \text{ m},\quad \sigma=\frac{7}{11}\ \mu\text{c/m}^2=\frac{7}{11}\times10^{-6}\text{ c/m}^2$$

$\therefore$ By surface density formula,

$$\sigma = \frac{q}{4\pi r^2},$$

$$q = 4\pi\sigma . r^2$$

$$= 4 \times \frac{22}{7} \times \frac{7}{11} \times 10^{-6} \times (0{\cdot}15)^2$$

$$= 1{\cdot}8 \times 10^{-7} \text{ C.}$$

Example 9. *An oil drops of 15 excess electrons is held stationary under a constant electric field of $3{\cdot}75 \times 10^5$ Vm^{-1} in Millikan's oil drop experiment. The density of oil is $\frac{49}{22}$ g cm^{-3}. Calculate the radius of the drop.*

Solution. For the drop to be stationary,

$$mg = qE$$

$$\left(\frac{4}{3}\pi r^3\right)\rho g = neE$$

$$\therefore \quad r^3 = \frac{3neE}{4\pi\rho g}$$

$$r^3 = \frac{3 \times 15 \times 1{\cdot}6 \times 10^{-19} \times 3{\cdot}75 \times 10^5}{4 \times \frac{22}{7} \times \left(\frac{49}{22} \times 10^3\right) \times 9{\cdot}8}$$

$$r = \left(\frac{27000 \times 10^{-18}}{2^3 \times 7^3}\right)^{\frac{1}{3}}$$

$$= \frac{30 \times 10^{-6}}{2 \times 7}$$

$$= 2{\cdot}14 \times 10^{-6} \text{ m.}$$

Example 10. *Two identical size spheres A and B at a separation 40 cm. has charge 12 μc each. A third sphere of the same size but uncharged is brought in contact of the first, then brought in contact with the second and then removed. What is the force of repulsion between the spheres A and B?*

Solution. The spheres of identical size share the charges equally. So 12μc charge on sphere 'A' will be equally shared by sphere 'A' and uncharged sphere. Now 6μc charge on uncharged sphere and 12μc charge on sphere 'B' will share equally between the two when brought in contact. So charge on sphere 'A' is now 6 μc and on sphere 'B' is 9μc.

$$q_1 = 6\mu c = 6 \times 10^{-6} \text{ C} ; \; q_2 = 9\mu c = 9 \times 10^{-6} \text{ C}$$

$$r = 0{\cdot}40 \text{ m}, \; k = 9 \times 10^9.$$

$$\therefore \quad F = k\frac{q_1 q_2}{r^2} = \frac{9\times10^9\times6\times10^{-6}\times9\times10^{-6}}{(0.40)^2}$$
$$= 3.0375 \text{ N.}$$

Example 11. *A cube of side 'a' has a charge 'q' at each of its corners. Calculate the potential due to the charges at all the corners at the centre of the cube.*

Solution. The centre of the cube will be at the middle point of the largest diagonal (l) of the cube. Then

$$l^2 = a^2 + a^2 + a^2$$

[since in cube length=breadth=height=a]

$$\therefore \quad l = \sqrt{3}\, a$$

$\therefore$ The distance of the centre of the cube from each corner,

$$r = \frac{l}{2}$$
$$= \frac{\sqrt{3}a}{2}$$

$\therefore$ The potential at the centre of the cube due to the charge q on each of the 8 corners of the cube,

$$V = 8\frac{1}{4\pi\epsilon_0}\frac{q}{r}$$
$$= \frac{2}{\pi\epsilon_0}\frac{q\times2}{\sqrt{3}a}$$
$$\boxed{V = \frac{4q}{\sqrt{3}\pi\epsilon_0 a}}$$

Example 12. *In a hydrogen atom the electron and proton are bound at a distance of 0.53 Å. (a) Calculate the potential energy of the system in eV. (b) What is the minimum work required to free the electrons, given that its kinetic energy in the orbit is half the magnitude of potential energy.*

Solution. (*a*) Potential energy,

$$E_p = k\frac{q_1 q_2}{r}$$
$$= \frac{9\times10^9(-1.6\times10^{-19})(1.6\times10^{-19})}{5.3\times10^{-11}} \text{ J}$$
$$= -\frac{9\times10^9(1.6\times10^{-19})^2}{5.3\times10^{-11}\times1.6\times10^{-19}} \text{ eV}$$
$$= -27.2 \text{ eV.}$$

(*b*) Kinetic energy $E_k = \frac{1}{2}(\,|\,E_p\,|\,)$

$$=\frac{1}{2}\times 27{\cdot}2$$
$$=13{\cdot}6 \text{ eV}.$$

$\therefore$ Total energy of electron,

$$E=E_p+E_k$$
$$=-27{\cdot}2+13{\cdot}6$$
$$=-13{\cdot}6 \text{ eV}.$$

$\therefore$ Minimum work done to free electron,

$$=E_\infty-E$$
$$=0-(-13{\cdot}6)$$
$$=13{\cdot}6 \text{ eV}$$

Example 13. *If one of the two electrons of a H_2 moleclule is removed, we get a hydrogen molecular ion (H_2^+). In the ground state of a H_2^+, the two protons are separated by 1·5 Å and the electron is 1 Å from each proton. Determine the potential energy of the systems.*

Solution. Total potential energy of the system,

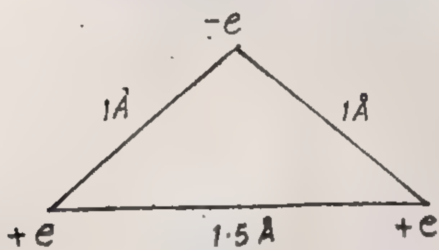


Fig. 1.4.

$$E=E_1+E_2+E_3$$
$$=k\left[\frac{-e\times+e}{(1\times10^{-10})}+\frac{+e\times+e}{(1{\cdot}5\times10^{-10})}+\frac{+e\times-e}{(1\times10^{-10})}\right]$$
$$=k\frac{e^2}{10^{-10}}\left[-1+\frac{2}{3}-1\right]$$
$$=-\frac{4}{3}ke^2\times10^{10}$$
$$=-\frac{4}{3}\times9\times10^9\times(1{\cdot}6\times10^{-19})^2 10^{10} \text{ J}$$
$$=-30{\cdot}72\times10^{-19} \text{ J}$$
$$=\frac{-30{\cdot}72\times10^{-19}}{1{\cdot}6\times10^{-19}} \text{ eV}$$
$$=-19{\cdot}2 \text{ eV}.$$

Example 14. A molecule of a substance has permanent electric dipole moment equal to 10^{-29} cm. A mole of this substance is polarized (at low temperature) by applying a strong electrostatic field of magnitude 10^6 Vm^{-1}. The direction of the field is suddenly changed by an angle of 60°. Calculate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity, assume 100% polarization of the sample.

Solution. Dipole moment, $\rho = 10^{-29}$ cm

Electric field, $E = 10^6$ Vm^{-1}

Avogadro's Number, $N = 6{\cdot}023 \times 10^{23}$

Amount of heat released

$= $ Loss in potential energy of one mole of this substance

$= $ (initial energy $-$ final energy)

$= (N\rho E - N\rho E \cos 60)$

$= N\rho E\,(1 - \cos 60)$

$= (6{\cdot}023 \times 10^{23})(10^{-29})\ 10^6)(1 - \tfrac{1}{2})$

$= 3{\cdot}0115$ J.

EXERCISE 1

[Where necessary the following data may be used : charge of electron $= 1{\cdot}6 \times 19^{-19}$ Coulomb ; mass of electron $= 9 \times 10^{-31}$ kg ; $h = 6{\cdot}63 \times 10^{-34}$ Js ; $K = 9 \times 10^9$ N/Coul ; $C = 3 \times 10^8$ ms^{-1} ; mass of proton $= 1{\cdot}6726 \times 10^{-27}$ kg].

1. Two positive charges are 5 cms apart in air and the force of repulsion is 8 dynes. One charge is twice the other. (*a*) What are the value of the two charges ? (*b*) At what point between the two charges would the total electric field will be zero ?

2. Two insulated metallic spheres charged with $+4\mu C$ and $-6\mu c$ are placed one metre apart. What is the resultant electric field at a point midway between them ?

3. 250 Joule of work must be done in order to move an electric charge of 5 coulomb from a place where potential is V to a place where potential is -20 volt. Find the value of V.

4. A charged metallic sphere 'A' which is free to move is brought near another fixed and charged sphere 'B' at a distance of 8 cm. The resulting repulsion of sphere 'A' is found to be 10 N. If sphere A is touched by an identical uncharged sphere C and sphere B is touched by another identical sphere D. C and D spheres are then removed. If now distance between the spheres A and B is reduced to 5 cm, what will be the resulting repulsion of sphere A ?

5. Compare the electrostatic force of repulsion between two protons in a nucleus with the gravitational force of attraction between them.

6. An electron falls through a distance of 5 cm in a uniform electric field of magnitude $7 \cdot 2 \times 10^4$ Nc^{-1}. Calculate the acceleration of the electron. Calculate the time of fall. If instead of electron proton is falling, what will be the time of fall ?

7. Calculate the velocity of an electron when it passes through a potential difference of 750 V.

8. The voltage across the electrodes of a cathode ray gun is 1000 V. Calculate (*a*) the energy gained by electrons, (*b*) the speed of electrons and (*c*) the momentum of electrons.

9. An electron at rest is accelerated through an electric potential so as to acquire a velocity of $1 \cdot 6 \times 10^7$ ms^{-1}. Calculate the value of electric potential.

10. An electron at rest is accelerated to a velocity of 5×10^7 ms^{-1} by an electric potential of 7100 V. Calculate the charge on electron if its mass is $9 \cdot 11 \times 10^{-31}$ kg.

11. An electron beam moving with a velocity of $1 \cdot 5 \times 10^{-7}$ m s^{-1} passes through a parallel plate condenser. The electric intensity between the plates is 40 V cm^{-1} and the distance is 4 cm. If the length of each plate is 10 cm, calculate the angle of deflection of beam.

12. A glass rod is rubbed with silk. The silk is found to have a negative charge of $4 \mu C$. Calculate the number of electrons transferred (from which to which). Is there a transfer of mass from glass to silk ?

13. Two insulated charged spheres A and B have their centres separated by a distance of $1 \cdot 5$ m (*a*) What is the mutual force of repulsion if the sphere 'A' has the charge double of that on B which is charged with 6×10^{-7}C ? (*b*) What will be its new value, (*i*) if the charge on each sphere is halved and the distance between them is made one-third ? (*ii*) if the two sphere are placed in water having dielectric constant 80.

14. Two spheres A and B each having charge $4 \cdot 6 \times 10^{-7}$C have identical sizes. A third sphere of same size but uncharged is brought in contact with A and then with B and finally removed from both. What is the force of repulsion between A and B if they are separated by a distance of 60 cm ?

15. A charged oil is suspended in uniform electric field of intensity 4×10^4 V m^{-1} so that it neither falls nor rises. Find the charge on the drop if its moss is $9 \cdot 75 \times 10^{-15}$ kg.

16. An oil drop of 25 excess electrons is held stationary under a constant electric field of $2 \cdot 2 \times 10^{-4}$ Vm^{-1} in Millikan's oil drop experiment. The density of oil is $1 \cdot 2$ g cm^{-3}. Calculate the radius of the drop.

17. An oil drop of 24 excess electrons is held stationary under a constant electric field of $1{\cdot}27 \times 10^4$ V m^{-1} in Millikans's oil drop experiment. If the radius of the drop is $9{\cdot}80 \times 10^{-3}$ mm and its density is $1{\cdot}26$ g cm^{-3}, calculate the charge on an electron.

18. Three charges 6μC, -5μC and -3μC are placed at the vertices A, B, C of a equilateral triangle whose side is 8 cm. Find (*a*) The resultant force on the charge at 'A' due to the other two charges at B and C. (*b*) Find the resultant electric field at the centroid of the triangle.

19. Charges of $+10\mu$C each are placed at two opposite corners of a square of side $\sqrt{2}$ cm and -10μC each on the remaining 2 corners. Find the intensity of the electric field and potential at the centre of the square.

20. Two small spherical conductors 54 cm apart have $+10\mu$C and $+20\mu$C charges respectively. Calculate (*a*) the electric force between them, (*b*) the resultant electric field and electric potential at a point midway between them.

21. Out of the two negative charges separated by 18 cm one is having the charge 2·5 times of the other. At which point between the two charges the resultant electric intensity will be zero.

22. A dipole consists of two charges $+4\mu$C and -4μC separated by a certain distance. If they are located at $x=16{\cdot}0$ cm, $y=0$ and $x=-16$ cm, $y=0$ respectively. Calculate the field strength at a point $x=0$, $y=12$ cm.

23. A dipole consists of two charges seperated by 6 cm. The resultant electric field at a point on perpendicular bisector of dipole axis is 10^6 N/C parallel to dipole axis. If the point is 4 cm away from the centre of the dipole axis, what is the charges on dipole ?

24. The positive charges of 6, 12 and 24μC are placed at 3 corners of a square. Find what charge must be placed at the fourth corner in order that the potential at the centre of the square may be zero. Will the electric field at the centre be also zero due to all the four charges. If not, calculate its value if the side of the square is $2\sqrt{2}$ cm.

25. Three charges of q units are placed at the three corners of a equilateral triangle of side 'a'. Calculate the resultant electric fields and potential at the centroid of the triangle.

26. Four equal charges each of $+\sqrt{2}\mu$C are placed one at each corner of a square of 8 cms side. Find the magnitude and direction of electric field and the pontential at the centre of the square.

27. There is a cube of 6 cm side having the charge of $+\sqrt{3}\mu C$ at each of its 8 corners. Find the field and potential at the centre of the cube.

28. There are 3 identical spheres A, B and C of the same size. The spheres A and B are charged with charge of $+16\mu C$ and $-6\mu C$ respectively and are separated by 10 cm. Now the sphere 'C' which is uncharged is brought in contact of the sphere A, then brought in contact of sphere B and then finally removed. What is the force of attraction or repulsion between the spheres A and B ?

29. If in question number 28 above the charges on A and B are (*a*) $+24\mu C, +10\mu C$ (*b*) $+24\mu C$, $-16\mu C$ and if the same process is repeated with the uncharged sphere C. What will be the force of attraction or repulsion between spheres 'A' and 'B' if the distance between each case is 20 cm.

30. Two tiny spheres carrying charges $4.5\ \mu C$ & $3.6\ \mu C$ are located 24 cm apart. Calculate the potential and electric field

    (*a*) at the mid point of the line joining two charges, and

    (*b*) at a point on the perpendicular bisector of the line at a distance 18 cm away from its mid point.

31. A spherical conducting shell of inner radius 12 cm and outer radius 15 cm has a charge $+\pi\mu C$. A charge $0.2\pi\ \mu C$ is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell.?

32. Two charges $+5\mu C$ and $-5\mu C$ are located at points (4 cm, 0, 0) and (–4 cm, 0, 0) respectively.

    (*a*) What is the electrostatic potential at points (6 cm, 0, 0) and (0, 3 cm, 5 cm)

    (*b*) How much work is done in moving a test charge from the point (0, 0, 8 cm) to (0, 0, –7 cm) along the *z*-axis. Does the answer change if the path of the test charge between the same points is not along the *z*-axis ?

33. A molecule of a substance has dipole moment equal to 4×10^{-28} cm. 2 moles of this substance is 100% polarised by applying a strong electrostatic field of $5 \times 10^{5}\ V\ cm^{-1}$. The direction of the field is suddenly changed by 45°. Calculate the heat released by the substance in aligning its dipoles along the new direction of the field.

34. In a hydrogen atom the electron and proton are bound at the distance of 1·06 Å. Calculate the potential energy and kinetic energy of the electron, if the total energy of hydrogen is zero for the above separation of the electron and the proton.

OBJECTIVE TYPE QUESTIONS

35. As one penetrates a uniformly charged sphere, the electric field strength 'E' :
    (*a*) increases
    (*b*) is zero at all points
    (*c*) remains the same as at the surface
    (*d*) decreases. [*PMT 1980*]

36. The magnitude of the electric field strength E is such that an electron placed in it would experience an electric force equal to its weight is given by :
    (*a*) mge (*b*) e/mg
    (*c*) mg/e (*d*) e^2 g/m^2 [*C P.M.T., 1980*]

37. State which of the following is correct :
    (*a*) Joule=coulomb×Volt (*b*) Joule=coulomb÷Volt
    (*c*) Joule=Volt×ampere (*d*) Joule=Volt÷ampere.
    [*C.P.M.T., 1980*]

38. A charge 'q' is placed at the centre of the line joining two equal charges Q. The system of 3 charges will be in equilibrium if *q* is equal to :
    (*a*) −Q/2 (*b*) −Q/4
    (*c*) +Q/4 (*d*) +Q/2 [*J.E.E. I.I.T. 1987*]

39. In Coulomb's law, the constant of proportionality k has the units :
    (*a*) N (*b*) NC^2/m^2
    (*c*) Nm (*d*) Nm^2/C^2

40. 1 Volt equals to
    (*a*) 1 J (*b*) 1 J/C
    (*c*) 1 C/J (*d*) 1 JC.

41. 1 coulomb contains the following number of electrons :
    (*a*) $6{\cdot}25\times10^{18}$ (*b*) $6{\cdot}25\times10^{19}$
    (*c*) $6{\cdot}25\times10^{20}$ (*d*) $1{\cdot}6\times10^{19}$

42. If *q* represents charge on particle and V the potential difference between two points, *q*V represents the magnitude of :
    (*a*) momentum (*b*) power
    (*c*) energy (*d*) torque

43. An electron of mass M kg and charge *e* coulomb travels from rest through a potential difference of *v* Volts. What is its final energy ?

(a) Mey J (b) e/y J

(c) $\frac{ey}{M}$ J (d) ey J

[*B.H.U. Ent. Exam. 1983*]

44. What is the final velocity of electron in above Question No. 42.

(a) $\sqrt{25\text{ Me}}$ ms^{-1} (b) $\left(2\sqrt{\frac{ey}{M}}\right)$ms^{-1}

(c) $(2\text{ My}\sqrt{e})$ ms^{-1} (d) $(2\rho\sqrt{y/M})$ ms^{-1}

45. A hollow metal sphere of radius 5 cm is charged such that the potential on its surface is 10 volts. The potential at the centre of the sphere is [*J.E.E. I.I.T. 1983*]

(a) 10 Volts
(b) zero
(c) Same as at a point 5 cm away from the surface.
(d) Same as at a point 25 cm away from the surface.

46. A charge q_1 exerts some force on a second charge q_2. If a third charge q_3 is brought near, then force of q_1 exerted on q_2 :

(a) will decrease in magnitude
(b) will increase in magnitude
(c) will remain unchanged
(d) not decided. [*C.P.M.T., 1971*]

47. The potential due to a hollow spherical conductor at a point inside it is

(a) Variable
(b) Constant
(c) dependent on the thickness of conductor
(d) none of the above. [*C.P.M.T., 1971*]

48. A charge of 10μC is placed at a distance of 2 m from a charge of 40μC and 4 m from a charge of -20μC. The potential energy of the charge 10μC is

(a) 2·70 J (b) 5·40 J

(c) 4·05 J (d) 1·35 J

49. An electric dipole has charged $+q$ and $-q$ at a separation r. At a distance d from the centre of the dipole along its axis field is proportional to

(a) q/d^2 (b) qr/d^2

(c) q/d^3 (d) qr/d^3

50. If two charges of 1 coulomb each are placed 1 km apart. The force between them will be

(*a*) 9000 N (*b*) ·009 N

(*c*) $1 \cdot 1 \times 10^{-4}$ N (*d*) 10^{-4} N

51. The force between two electrons separated by a distance *r* varies as

(*a*) r^2 (*b*) r

(*c*) r^{-1} (*d*) r^{-2}

Electrostatic Conductors, Capacitors and Dielectrics

IMPORTANT FORMULAE

1. (*a*) Capacitance of a Conductor

$$C=\frac{Q}{V};$$ Q = Charge on the conductor
V = Potential of the conductor

(*b*) Capacitance of a capacitor,

$$C=\frac{Q}{V};$$ Q = Charge on the capacitor
V = Potential difference between the plates of the capacitor

2. Common potential of two capacitors when they are joined together,

$$V=\frac{Q_1+Q_2}{C_1+C_2}=\frac{C_1V_1+C_2V_2}{C_1+C_2}$$

and new charges on two capacitors will be

$$Q_1'=C_1V \text{ and } Q_2'=C_2V$$

3. Energy of a capacitor,

$$E=\frac{1}{2}CV^2=\frac{1}{2}QV=\frac{1}{2}\frac{Q^2}{C}$$

4. Loss of energy when two capacitors are joined together,

$$\Delta E=\frac{1}{2}C_1C_2\frac{(V_1-V_2)^2}{(C_1+C_2)}$$

5. (*a*) For capacitors connected in series,

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots\ldots$$

(*b*) For capacitors connected in parallel,

$$C=C_1+C_2+C_3+\ldots\ldots$$

6. Capacitance of capacitors in farad

(*a*) Parallel plate capacitor

$$C=\frac{1}{k}\left(\frac{k'A}{4\pi d}\right);$$ k' = Dielectric const.
A = Area of plate in m^2
d = distance between the plates in m.
$k=9\times10^9\ Nm^2/c^2$

(*b*) For spherical capacitors

$$C=\frac{1}{k}\left[\frac{k'\,ab}{(a-b)}\right]$$ a = radius of outer sphere in metre
b = radius of interior sphere in metre

(*c*) For cylindrical capacitor (*e.g.* Leyden jar)

$$C=\frac{k'\pi r(r+2h)}{4\pi kd};$$ r = radius of jar in metre
h = height of jar in metre
and d = thickness of jar

$$=\frac{k'r(r+2h)}{4kd}$$

7. The capacitance of a spherical conductor in farad is numerically equal to its radius in metre divided by 9×10^9

$$C\text{ (in farad)}=\frac{r\text{ (in metre)}}{9\times10^9}$$

SOLVED EXAMPLES

Example 1. *A sphere of radius 50 cms carries a charge of 0·01 μC. Calculate the potential of the sphere.*

Solution. $r=50\text{ cm}=0{\cdot}50\text{ m}$

Capacitance, $C=\frac{r}{9\times10^9}=\frac{0{\cdot}5}{9}\times10^{-9}\,f=\frac{5}{9}\times10^{-10}\,f$

Charge, $Q=0{\cdot}01\,\mu C=0{\cdot}01\times10^{-6}\,C=10^{-8}\,C$

$\therefore$ Potential, $V=\frac{Q}{C}$

$$=\frac{9\times10^{-8}}{5\times10^{-10}}$$
$$=180\text{ Volt.}$$

Example 2. *A spherical conductor of radius 18 cm is charged positively with 8×10^{-9} C and then connected with another sphere of radius 9 cm, carrying a negative charge of 4×10^{-9} C. Calculate (a) the potential of each sphere before and after the contact. (b) the charge on each sphere after the contact.*

Solution. Potential of first sphere,

$$V_1=\frac{kQ_1}{r_1}=\frac{9\times10^{-9}\times8\times10^{-9}}{0{\cdot}18}$$
$$=400\text{ Volt}$$

and capacitance, $C_1=\frac{0{\cdot}18}{9\times10^9}=2\times10^{-11}\,f$

Potential of second sphere,

$$V_2=\frac{kV_2}{r_2}$$

$$=\frac{9\times10^{9}\times4\times10^{-9}}{0{\cdot}09}$$
$$=400 \text{ Volt.}$$

and capacitance, $C_2=\dfrac{0{\cdot}09}{9\times10^{9}}=10^{-11}\,f$

$\therefore$ Common potential,

$$V=\frac{(Q_1+Q_2)}{(C_1+C_2)}$$
$$=\frac{(8\times10^{-9}-4\times10^{-9})}{(2\times10^{-11}+1\times10^{-11})}=\frac{400}{3}\text{ Volt}$$
$$=133{\cdot}33 \text{ Volt}$$

$$Q_1'=C_1V=2\times10^{-11}\times\frac{400}{3}=2{\cdot}66\times10^{-9}\text{ C};$$
$$Q_2'=C_2V=10^{-11}\times\frac{400}{3}=1{\cdot}33\times10^{-9}\text{ C}$$

Example 3. *A spherical insulated conductor of capacity 10 pf is charged with 3 μC. Calculate its energy. Also calculate the amount of work done and increase in energy if 2 μC of charge is added to the conductor.*

Solution. $C=10\,pf=10\times10^{-12}\,f=10^{-11}\,f$
$Q=3\,\mu C=3\times10^{-6}\text{ C}$

$\therefore$ Energy, $E_1=\dfrac{1}{2}\dfrac{Q^2}{C}$
$$=\frac{1}{2}\cdot\frac{(3\times10^{-6})^2}{10^{-11}}$$
$$=0{\cdot}45\text{ J}$$

Total charge $=3+2=5\mu C=5\times10^{-6}$ C

$\therefore$ Energy, $E_2=\dfrac{1}{2}\dfrac{(5\times10^{-6})^2}{10^{-11}}$
$$=1{\cdot}25\text{ J}$$

Increase in energy $=E_2-E_1=(1{\cdot}25-0{\cdot}45)=0{\cdot}80$ J

Work done is equal to the increase in energy $=0{\cdot}80$ J

Example 4. *Two spherical conductors of capitance 4 μC and 10 μf have charges 30 μC and 50 μC respectively. If they are joined by a thin metallic wire, calculate the loss of energy.*

Solution. Energy before sharing the charges,

$$E_1=\frac{1}{2}\frac{Q_1^2}{C_1}+\frac{1}{2}\frac{Q_2^2}{C_2}$$
$$=\frac{1}{2}\frac{(30\times10^{-6})^2}{(4\times10^{-6})}+\frac{1}{2}\frac{(50\times10^{-6})^2}{(10\times10^{-6})}$$

$$=(1{\cdot}125\times10^{-4}+1{\cdot}25\times10^{-4})\ \text{J}$$

$$E_1=2{\cdot}375\times10^{-4}\ \text{J}$$

Energy after sharing the charges,

$$E_2=\frac{1}{2}\frac{(Q_1+Q_2)^2}{(C_1+C_2)}$$

$$=\frac{(30\times10^{-6}+50\times10^{-6})^2}{(4\times10^{-6}+10\times10^{-6})}$$

$$=2{\cdot}286\times10^{-4}\ \text{J}$$

$\therefore$ Loss in energy $=E_1-E_2$

$$=2{\cdot}375\times10^{-4}-2{\cdot}286\times10^{-4}$$

$$=0{\cdot}089\times10^{-4}\ \text{J}$$

$$=8{\cdot}9\times10^{-6}\ \text{J}$$

Example 5. *A parallel plate capacitor has its plate of area 220 cm² separated by 0·7 mm thick mica slab. Calculate the capacitance of the capacitor if dielectric constant of mica is 6.* [*A.I.H.S. 1970*]

Solution. $A=6{\cdot}6\ \text{cm}^2=6{\cdot}6\times10^{-4}\ \text{m}^2$

$d=0{\cdot}7\ \text{mm}=7\times10^{-4}\ \text{m}$

$k=6$ and $k'=9\times10^9\ \text{Nm}^2/\text{C}^2$

$$\therefore\quad C=\frac{kA}{4\pi k'd}$$

$$=\frac{6\times6{\cdot}60\times10^{-4}\times7}{4\times22\times9\times10^9\times7\times10^{-4}}$$

$$=50\times10^{-12}\,f$$

$$=50\ pf.$$

Example 6. *Two parallel plates air capacitors have their plate area 100 and 625 cm², and same charge. If the distance between the plates of first capacitor is 1 mm, what should be the distance between the plates of second capacitor so that the potential of two capacitor is same.*

Solution. Since two capacitors have same charge and same potential,

$$C_1=C_2$$

$$\frac{kA_1}{4\pi k'd_1}=\frac{kA_2}{4\pi k'd_2}$$

$$\therefore\quad d_2=\frac{A_2}{A_1}d_1$$

$$=\frac{625}{100}\times1\ \text{mm}$$

$$=6{\cdot}25\ \text{mm}.$$

Example 7. *Three capacitors of 1, 2 and 3 μf are connected with each other as shown in figure below. Calculate the (a) equivalent capacitance of the system. (b) the charge on each capacitor if the system is connected to a 200 volt supply.*

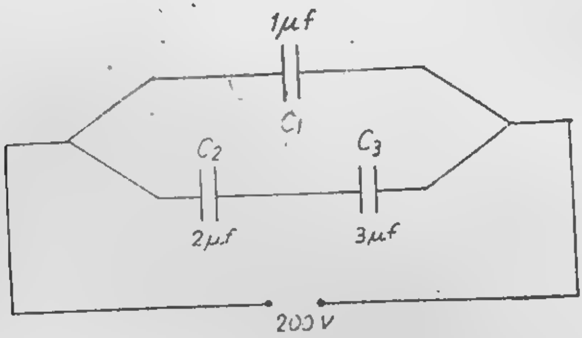


Fig. 2.1.

Solution. (*a*) In the given network C_2 and C_3 are in series, the effective capacitance C′ of these two capacitors will be

$$\frac{1}{C'} = \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{2} + \frac{1}{3}$$

$$\therefore \quad C' = 1\ 2\ \mu f$$

Now C′ and C_1 are connected in parallel, the net equivalent capacitance of the network will be

$$C'' = C' + C_1$$

$$= 1{\cdot}2 + 1$$

$$= 2{\cdot}2\ \mu f$$

(*b*) Charge on capacitor C_1,

$$Q_1 = C_1 V = (1 \times 10^{-6})(200)$$

$$= 200 \times 10^{-6}\ C = 200\ \mu C$$

Charges on capacitors C_2 and C_3 will be same

$$\therefore \quad Q_2 = Q_3 = Q' \qquad \text{(say)}$$

$$Q' = C'V$$

$$= (1{\cdot}2 \times 10^{-6}) \times 200$$

$$= 240 \times 10^{-6} \text{ coulomb}$$

$$= 240\ \mu C$$

Example 8. *A 8 μf capacitor is charged by a 250 V supply. It is then disconnected from the supply and is connected to another uncharged 4 μf capacitor. How much electrostatic energy of the first capacitor is in the form of heat and electromagnetic raditation.*

Solution. Charge on first capacitor,

$$Q_1 = C_1 V_1 = (8 \times 10^{-6})250 = 2 \times 10^{-3}\ \text{C}$$

When first capacitor after charging is connected with second uncharged capacitor, the common potential of the two will be

$$V = \frac{Q'_1}{C_1} = \frac{(Q_1 - Q'_1)}{C_2}$$

$$\therefore \qquad \frac{Q_1'}{8} = \frac{(2 \times 10^{-3} - Q_1')}{4}$$

$$\therefore \qquad Q_1' = \frac{4}{3} \times 10^{-3}\ \text{C}$$

$$\therefore \qquad V = \frac{4 \times 10^{-3}}{3 \times 8 \times 10^{-6}}$$

$$= \frac{1000}{6} = 166{\cdot}67\ \text{Volt.}$$

$\therefore$ Initial energy of the first capacitor will be $\frac{1}{2} Q_1 V_1$

$$\therefore \qquad E_1 = \tfrac{1}{2} \times 2 \times 10^{-3} \times 250$$

$$= 0{\cdot}25\ \text{J}$$

Final energy of two capacitors,

$$E_2 = \tfrac{1}{2} Q'_1 V + \tfrac{1}{2} Q'_2 V$$

where Q'_2 = charge on second capacitor

$$= \tfrac{1}{2}(Q'_1 + Q'_2)\ V$$

$$= \tfrac{1}{2} Q_1 V$$

$$= \tfrac{1}{2} \times 2 \times 10^{-3} \times 166{\cdot}67\ \text{J}$$

$$= 0{\cdot}167\ \text{J}$$

$\therefore$ Loss in energy (which appear in the form of heat and electromagnetic radiation)

$$= E_1 - E_2$$

$$= 0{\cdot}25 - 0{\cdot}167$$

$$= 0{\cdot}083\ \text{J}$$

Example 9. *Obtain the equivalent capacitance of the following network for a 210 V supply.*

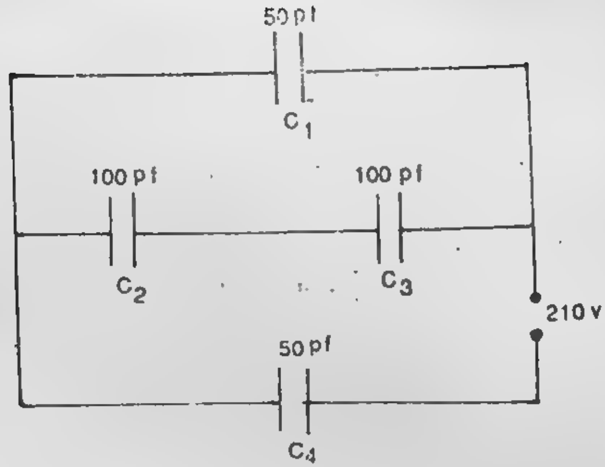


Fig. 2.2.

Also calculate the charge and voltage across each capacitor.

Solution. The circuit of Fig. 2.2 can be redrawn as in Fig. 2.3.

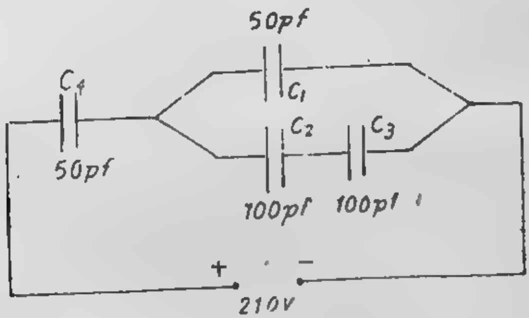


Fig. 2.3.

If effective capacitance of C_2 and C_3 in series be C', then

$$\frac{1}{C'}=\frac{1}{C_2}+\frac{1}{C_3}$$

$$\therefore \qquad \frac{1}{C'}=\frac{1}{100}+\frac{1}{100}$$

or $$C'=50\ pf$$

Now effective capacitance of C_1 and C' in parallel will be

$$C''=C_1+C'$$
$$=(50+50)$$
$$=100\ pf$$

Capacitor C_4 is in series with C'', so if equivalent capacitance of net work be C''',

$$\frac{1}{C'''}=\frac{1}{C''}+\frac{1}{C_4}.$$
$$=\frac{1}{100}+\frac{1}{50}$$

$\therefore$ $$C'''=\frac{100}{3}\ pf$$

Now $$Q_4=C'''V$$
$$=\frac{100}{3}\times 10^{-12}\times 210$$
$$=7\times 10^{-9}\ C$$

$\therefore$ $$V_4=\frac{Q_4}{C_4}=\frac{7\times 10^{-9}}{50\times 10^{-12}}=140\ V$$

$\therefore$ $$V_1=(V-V_4)=(210-140)=70\ V$$

and $$Q_1=C_1V_1=50\times 10^{-12}\times 70=3{\cdot}5\times 10^{-9}\ C$$

Now 70 Volt potential on C_1 will be shared equally by capacitors C_2 and C_3.

$\therefore$ $$V_2=V_3=\frac{70}{2}=35\ V$$

and charges on C_2 and C_3 in series will be same as

$$Q_2=Q_3=C_2V_2\quad(\text{or } C_3V_3)$$
$$=100\times 10^{-12}\times 35$$
$$=3{\cdot}5\times 10^{-9}\ C.$$

Example 10. *A cylindrical capacitor has two coaxial cylinders of length 12 cm and radii 3·1 cm and 3·0 cm. The outer cylinder is earthen and the inner cylinder is given a charge of 0·09 μC. Calculate the capacitance of the capacitor and the potential of the inner cylinder.*

Solution. For cylindrical capacitor,

$$C=\frac{k'r(r+2h)}{4kd}$$

Here $k'=1$ (for air), $r=3{\cdot}0$ cm$=0{\cdot}03$ m, $h=12$ cm$=0{\cdot}12$ m and $d=3{\cdot}1-3{\cdot}0=0{\cdot}1$ cm$=0{\cdot}001$ m and $k=9\times 10^9$ Nm²/C²

$\therefore$ $$C=\frac{1\times 0{\cdot}03(0{\cdot}03+2\times{\cdot}12)}{4\times 9\times 10^9\times{\cdot}001}$$
$$=\frac{0{\cdot}03\times 0{\cdot}27}{4\times 9\times 10^6}$$
$$=2{\cdot}25\times 10^{-10}\ f=215\ pf$$
$$V=\frac{Q}{C}=\frac{0{\cdot}09\times 10^{-6}}{2{\cdot}25\times 10^{-10}}=400\ \text{Volt.}$$

Example 11. *A parallel plate capacitor is to be designed with a voltage rating 2 kV, using a material of dielectric constant 5·5 and dielectric strength about $2 \times 10^7\ Vm^{-1}$. What minimum area of the plates is required to have a capacitance of 700 pf ?*

Solution. For safety maximum field allowed is 10% of the dielectric strength.

$\therefore$ $E = 10\%$ of $2 \times 10^7\ Vm^{-1}$

$E = 2 \times 10^6\ Vm^{-1}$

$V = 2\ KV = 2 \times 10^3\ V$

Now $E = \dfrac{V}{d}$

where d = thickness of the dielectric

$\therefore$ $d = \dfrac{2 \times 10^3}{2 \times 10^6} = 10^{-3}$ m

$C = 700\ pf = 700 \times 10^{-12}\ f$, $k = 9 \times 10^9\ Nm^2/C^2$ and $k' = 5·5$

Now $C = \dfrac{k' A}{4\pi k d}$

$\therefore$ $A = \dfrac{4\pi k d C}{k'}$

$$= \frac{4 \times 22 \times 9 \times 10^9 \times 10^{-3} \times (700 \times 10^{-12})}{7 \times 5·5}$$

$= 144 \times 10^{-4}\ m^2$

$= 144\ cm^2$.

Example 12. *In a Van de Graff generator, the metallic sphere has the potential 12×10^5 V. The dielectric strength of the gas surrounding the sphere is $4 \times 10^7\ Vm^{-1}$. What is the minimum radius required for the sphere ?*

Solution. $V = 12 \times 10^5$ V and $E = 10\%$ of dielectric strength $E = 10\%$ of $4 \times 10^7\ Vm^{-1} = 4 \times 10^6\ Vm^{-1}$

Now $E = \dfrac{V}{r}$, where r is the minimum radius of sphere required

$\therefore$ $r = \dfrac{V}{E}$

$= \dfrac{12 \times 10^5}{4 \times 10^6}$ m

$= 0·3$ m

$= 30$ cm.

Example 13. *A technician requires a capacitance 5μf in a circuit across a potential difference of 500 Volt. A large number of 1 μf capacitors are available to him. Each of which can withstand a potential difference of not more than 150 V. Calculate the minimum*

number of capacitors required and possible arrangement of thes. capacitors.

Solution. Maximum number of capacitors in series in one row $=\frac{500}{150}=3\cdot3=4$ (next whole number)

$\therefore$ The equivalent capacitance C′ of these capacitors will be given by

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\frac{1}{C_4}$$

$$\frac{1}{C}=\frac{4}{1}\ \mu f \quad \text{or} \quad C=\frac{1}{4}\ \mu f$$

Since in parallel, $C'=C_1+C_2+C_3+C_4+\ldots\ldots$

and $\therefore$ $C_1=C_2=C_3=C_4=C$

No. of rows in parallel $=\frac{C'}{C}=\frac{5\ \mu f}{\frac{1}{4}\mu f}=5\times4=20.$

EXERCISE 2

1. A Leyden jar has a radius of 7·5 cm. The height of tin foil is 18 cm and thickness of glass is 2·5 mm. If the value of dielectric constant of glass in 6 4, calculate the capacitance of the Leyden jar.

2. A sphere of radius 9 cm carries a charge of 25 μC. Calculate the capacitance and potential of the sphere. Calculate also the electric field and potential at a distance of 8 cm from the centre.

3. The radius of the earth is about 6400 km. What is its capacitance in microfarad ?

4. A conductor of capacitance 100 *pf* is charged to a potential of 200 V and is then joined to share charges with a conductor of 50 *pf*. Calculate the final charge on each conductor and their potential.

5. Two conductors of capacitance 4*pf* and 6*pf* have charges -24μC and $+60\ \mu$C respectively. If they are joined together, calculate their common potential and charge on each conductor.

6. Two metallic spheres of radius 9 cm and 18 cm have charges 10 μC and 15 μC respectively. Calculate the common potential and loss in energy when two conductors are joined together.

7. A spherical conductor of 4·5 cm is charged to a potential of 100 V. Calculate its capacitance and potential energy. If the conductor is given an additional charge 2×10^{-10} C, calculate the new potential increase in energy.

8. A capacitor of 5 μf capacitance is charged to a potential of 250 V. It is disconnected from supply and then connected in parallel with an uncharged capacitor of 1·25 μf. Find the common potential and the charge on each capacitor.

9. A parallel plate capacitor has its two plates of 10 cm × 10 cm each and a mica slab of 1 mm thick in between the plates. Calculate the capacitance of the capacitor if dielectric constant for mica is 6.

10. A Leyden jar has its radius 7 cm and the height of the metallic foil on the jar is 14 cm. If the glass of jar is 3 mm thick and has dielectric constant 6·4, calculate the capacitance of the Leyden jar.

11. At what distance should the circular plate each 10 cm in radius of a parallel plate capacitor with a dielectric of dielectric constant 3 be placed in order to have the capacitance 100 *pf*.

12. There are 3 capacitors of capacitance 1 μf, 2 μf and 3 μf. After joining the second and third capacitors in series the combination is joined in parallel with the first capacitor. What is the equivalent capacitance of the network ?

13. There are 4 capacitors each of 5 μf. After connecting the three capacitors in series with each other, the combination is connected in parallel with fourth capacitor which is already connected to a 200 Volt supply. Calculate the equivalent capacitance of the network and the charge on each capacitor.

14. A network of 5 capacitors is connected to a 250 volt supply as shown in Fig. 2.4 (*a*), 2.4 (*b*) and 2·4 (*c*) in 3 different ways. Calculate the following in each case if $C_1 = C_2 = C_3 = C_4 = 200 pf$ and $C_5 = 400\ pf$.

(*a*) the equivalent capacitance of network.

(*b*) the charge and voltage across each capacitor.

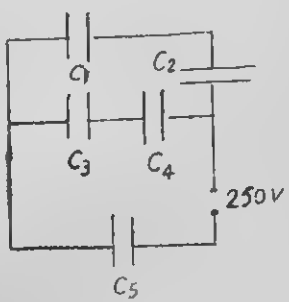


Fig. 2.4 (a)

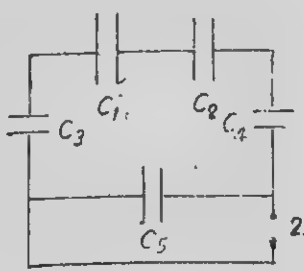


Fig. 2.4 (b)

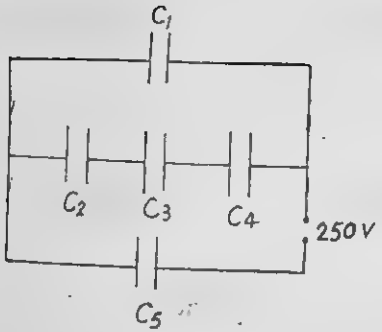


Fig. 2.4 (c)

15. A 500 *pf* capacitor is charged by a 200 Volt battery. How much electrostatic energy is stored by the capacitor. The capacitor is disconnected from the battery and connected to another 500 *pf* capacitor. What is the energy lost in the form of heat and electromagnetic radiation ?

16. Three capacitors of 8, 8 and 4 μf are connected with 15 Volt supply in series. Calculate the charge on capacitor of 4 μf.

17. An electric technician requires a capacitance of 4 μf in a circuit across a potential difference of 2KV. A large number of 2 μf capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires a minimum number of capacitors.

18. What is the area of the plates of a 7 *pf* parallel plate capacitor, given that the separation between the plates is 0·2 mm.

19. The plates of a parallel plate capacitor have an area 990 cm^2 each and are separated by 3·5 mm. The capacitor is charged by connecting it to a 200 V supply.

    (*a*) What is the capacitance and charge on the capacitor ?

    (*b*) How much electrostatic energy in stored by the capacitor ?

20 In a spherical capacitor the radius of inner and outer sphere is 9 cm and 10 cm. respectively. If the capacitor is given a charge of 12 μC and the space between the concentric spheres is filled with an oil of dielectric constant 60, calculate (*a*) the capacitance of the capacitor. (*b*) Compare the capacitance of this capacitor with that of an isolated sphere of radius 18 cm.

21. A parallel plate capacitor is to be designed with a voltage rating 1·5 KV, using a material of dielectric constant 2·0 and dielectric strength about 10^6 Vm^{-1}. What minimum area is required to have a capacitance of 350 *pf* ?

22. In a Van de Graff generator the metallic sphere has a potential of $10^6 V$. The dielectric strength of the gas surrounding the

sphere is 2×10^7 Vm^{-1}. What is the minimum radius of the sphere required ?

OBJECTIVE TYPE QUESTIONS

23. 64 charged drop are put together. If each small drop has the capacitance 'C', potential 'V' and charge '*q*', the charge on bigger drop will be

(*a*) *q* (*b*) 4*q*

(*c*) 16*q* (*d*) 64*q*.

24. The capacitance of the bigger drop in above problem (Q. No. 23) will be :

(*a*) C (*b*) 4 C

(*c*) 16 C (*d*) 64 C.

25. The potential of the bigger drop in above problem (Q. No. 23) will be

(*a*) V (*b*) 4 V

(*c*) 16 V (*d*) 64 V.

26. The potential difference (in Volts) between the plates of 20 μf capacitor whose charge is 0·01 coulomb is

(*a*) 5000 (*b*) 50

(*c*) 2000 (*d*) 500.

27. Two identical capacitors joined in series give equivalent capacitance of 4 μf. The capacitance of each of the capacitor is

(*a*) 8 μf (*b*) 6 μf

(*c*) 2 μf (*d*) 1 μf

28. The capacitance of a parallel plate capacitor depends on :

(*a*) the type of metal used

(*b*) the thickness of plates

(*c*) the potential applied across the plates

(*d*) the separation between the plates.

29. The energy stored in a condenser of capacity *c* which is given a charge Q is :

(*a*) $\frac{1}{2}$ Q/V (*b*) $\frac{1}{2}$ QV

(*c*) $\frac{1}{2}$ QV^2 (*d*) $\frac{1}{4}$ QV.

30. A parallel plate air capacitor is immersed in an oil of dielectric constant 2. The field between the plate is :

(*a*) increased proportional to 2

(*b*) decreased proportional to $\frac{1}{2}$

(*c*) increased proportional to $\sqrt{2}$

(*d*) decreased proportional to $\frac{1}{\sqrt{2}}$

31. A capacitor having capacitance C, consists of parallel plates with oil (dielectric constant=3) between them. What will its capacitance be if oil is drained out and the separation is made two times what it originally was ?

(*a*) 6 C (*b*) C/3

(*c*) C/6 (*d*) 3 C/2.

32. A capacitor of capacitance 50 μf is charged to 10 Volts. Its energy is

(*a*) $2{\cdot}5 \times 10^{-3}$ J (*b*) $2{\cdot}5 \times 10^{-4}$ J

(*c*) 5×10^{-3} J (*d*) $1{\cdot}25 \times 10^{-8}$ J.

33. Referring to the figure below, the effective capacitance betbeen A and B is :

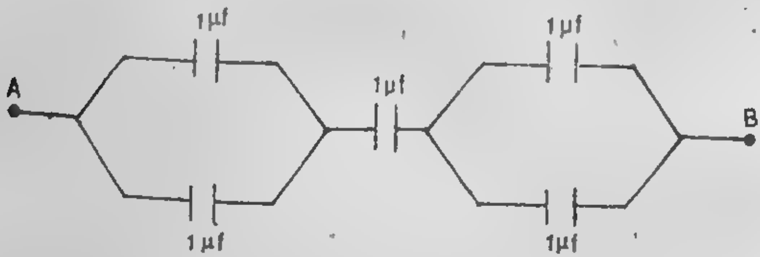


Fig. 2.5.

(*a*) 0·5 μf (*b*) 1·5 μf

(*c*) 2·0 μf *d*) 2·5 μf.

34. The capacitors C_1 and C_2 are connected in parallel. If a charge '*q*' in given to the assembly, the charge gets shared. The ratio of the charge on capacitor C_1 to the charge on capacitor C_2 is :

(*a*) $\frac{C_1}{C_2}$ (*b*) $\frac{C_2}{C_1}$

(*c*) $C_1 . C_2$ (*d*) $\frac{1}{C_1 C_2}$

35. The equivalent capacitance of the network given below is $1 \mu f$. The value of C is :

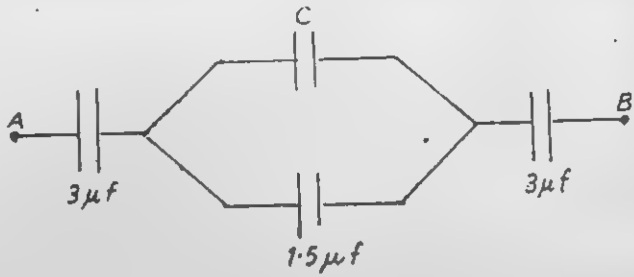


Fig. 2.6.

(*a*) 0·5 μf (*b*) 1·5 μf (*c*) 2 μf (*d*) 3 μf.

□ □

Current Electricity

IMPORTANT FORMULAE

1. Ohm's law : $V=IR$
   where V=Potential difference across the conductor
   I=The current flowing through the conductor
   R=Resistance of the conductor
2. Drift Velocity,
   $$v=\frac{eE}{m}\,t^{-}$$
   e=charge on electron
   E=Intensity of electric field
   t^{-}=Average relax time
   and m=mass of the electron
3. Resistivity (ρ) of the material of a wire is given by
   $$\rho=R\,\frac{A}{l}$$
   where A=Area of cross-section of the wire
   l=length of the wire
   $$=\frac{m}{ne^2t}$$
   n=no. of free electrons per unit volume
4. The electric current.
   $$I=enAv$$
5. $R_2=R_1\,[1+\alpha\,(t_2-t_1)]$ where α=temp. coefficient of resistance.
6. The temperature dependance of resistivity in insulating substance is given by
   $$\rho(T)=\rho_0(T)\ \rho(E_g/K_BT)$$
   where K_B=Boltzmann constant
   E_g=Positive energy
   e=exponential constant=2·718.
7. (a) The current in a circuit having a battery of emf E and internal resistance r and an external resistance R is given by
   $$I=\frac{E}{(R+r)}$$

(*b*) Potential difference (P.D.) across external resistance,

$$V_{ext}=IR$$

or $$V_{ext}=\frac{ER}{(R+r)}$$

(*c*) Also the terminal voltage of battery *i.e.* P.D. on external resistance,

$$V_{ext}=(E-Ir)$$

8. (*a*) The equivalent resistance of the resistances connected in series is given by

$$R=R_1+R_2+R_3+\ldots\ldots$$

(*b*) The equivalent resistance of the resistances connected in parallel is given by

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\ldots\ldots$$

9. Kirchoff's Law :

(*a*) **First Law :** It states that at any junction of several circuit elements, the sum of current entering the junction must equal the sum of the currents leaving it (Fig. 3.1).

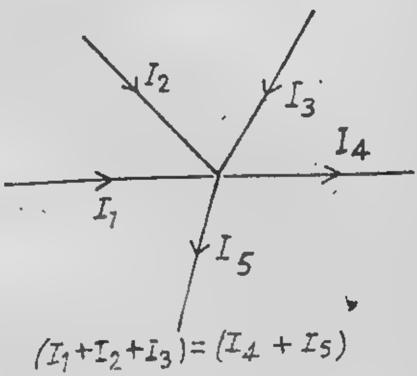


Fig. 3.1.

(*b*) **Second Law** : The algebraic sum of the emf's in any loop of the network is equal to the algebraic sum of IR products in it, (Fig. 3.2).

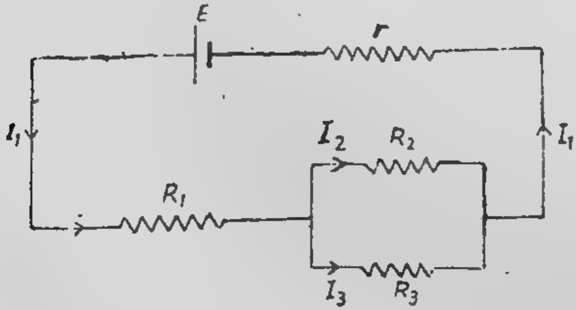


Fig. 3.2.

$$E=I_1R_1+I_2R_2+I_1r$$

or $$E=I_1R_1+I_3R_3+I_1r$$

10. (a) The shunt resistance (R_s) required to convert a galvanometer of resistance R_g into an ammeter of range I amp. is given by

$$R_s=\frac{I_g}{I-I_g}R_g$$ where I_g=the current through galvanometer for its full scale deflection.

and the effective resistance of ammeter,

$$R_{eff}=\frac{R_sR_g}{R_s+R_g}$$

(b) The series resistance (R) required to convert a galvanometer into a voltmeter of range V volts is given by

$$R=\left(\frac{V}{I_g}-R_g\right)$$

and the effective resistance of the voltmeter,

$$R_{eff}=(R+R_g)$$

11. Potentiometer :

(a) If we get null point in the galvanometer at length l_1 and l_2 with the two batteries of emf's E_1 and E_2 respectively,

$$\boxed{\frac{E_1}{E_2}=\frac{l_1}{l_2}}$$ provided the current through the wire remains constant and the wire is of uniform cross section.

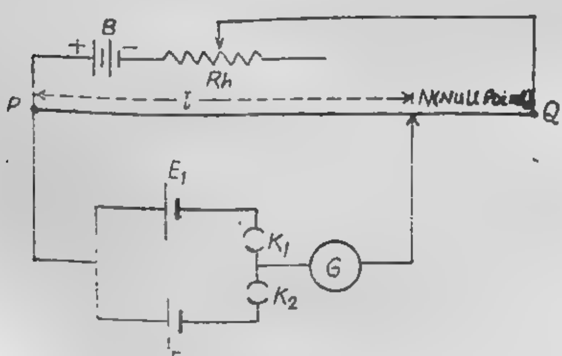


Fig. 3.3.

(*b*) The internal resistance of a battery,

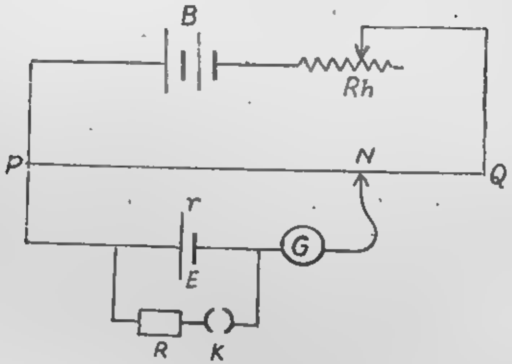


Fig. 3.4.

$$r=\left(\frac{l_1}{l_2}-1\right)R$$

where l_1 = balance point with battery alone

l_2 = balance point with battery when it is shunted with resistance 'R'.

(*c*) The ratio of the two resistance in series with the battery,

$\frac{R_1}{R_2}=\frac{l_1}{l_2}$ where l_1 and l_2 are the balance points with resistances R_1 and R_2 respectively.

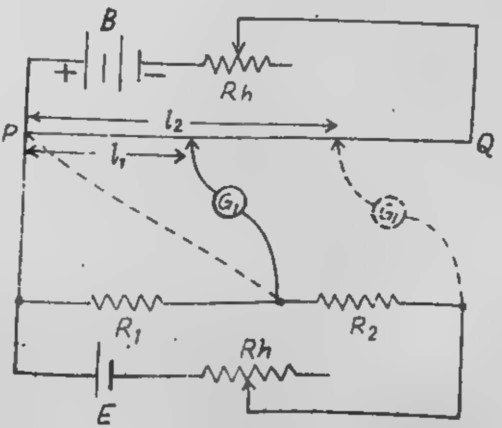


Fig. 3·5.

12. Principle of meter bridge (wheatstone's bridge) : For the null point in galvanometer in meter bridge or wheatstone's bridge,

$$\frac{P}{Q}=\frac{R}{S}$$

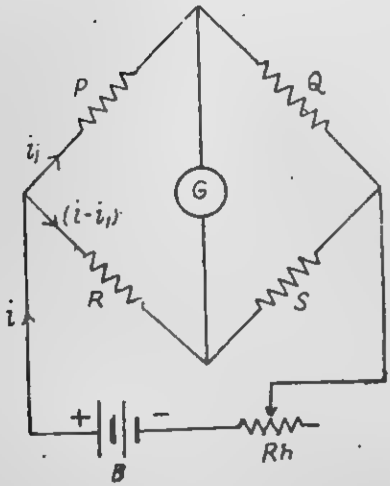


Fig. 3.6.

SOLVED EXAMPLES

Example 1. *How many electrons pass through a lamp in 2 minutes if the current is 300 mA.*

Solution.

$$I=\frac{Q}{t}=\frac{ne}{t}$$ where n=no. of electrons

e=charge on an electron

$\therefore \quad n=\frac{I\times t}{e}$

Now $\quad I=300\ mA=0.3\ A$

$t=2$ minutes$=120\ S$

and $\quad e=1.6\times10^{-19}\ C$

$\therefore \quad n=\frac{0.3\times120}{1.6\times10^{-19}}$

$=225\times10^{18}$ electrons.

Example 2. *The electron in the hydrogen atom circles around the proton with a speed of $2.18\times10^6\ ms^{-1}$ in an orbit of radius 5.3×10^{-11} m. Calculate the equivalent current.*

Solution. Circumference of the orbit$=2\pi r$

$=2\times3.14\times5.3\times10^{-11}$ m

$=3.3284\times10^{-10}$ m

If the electron will go around the proton n times in one second,

$V=2\pi nr$

$$n=\frac{V}{2\pi r}=\frac{2\cdot18\times10^{6}}{3\cdot3284\times10^{-10}}=6\cdot55\times10^{15}$$

$$I=\frac{ne}{t}$$

$$=\frac{1\cdot6\times10^{-19}\times6\cdot55\times10^{15}}{1}$$

$$=1\cdot048\times10^{-3}\text{ A}$$

$$=1\cdot048\text{ mA.}$$

Example 3. *A rod of silver 1 m long and $1\cdot0\times10^{-4}$ m^2 area of cross-section has one free electron per atom. Calculate (a) the number of free electrons in the rod (b) the drift velocity of electrons if the current through the rod is 4 ampere. (Given density of silver= $10\cdot5\times10^3$ kg m^{-3} and atomic wt. of silver=108).*

Solution. (*a*) Volume of the rod,

$$V=A\times l$$

$$=(1\cdot0\times10^{-4})\times1$$

$$=10^{-4}\text{ m}^3$$

Mass of silver rod,

$$M=Vd$$

$$=10^{-4}\times10\cdot5\times10^{3}$$

$$=1\cdot05\text{ kg}$$

Now 108 kg silver contains number of silver atoms

$$=6\cdot023\times10^{26}$$

$\therefore$ 1·05 kg silver contains$=\dfrac{60\cdot23\times10^{26}\times1\cdot05}{108}=5\cdot855\times10^{23}$

Since one atom has one free electron

$\therefore$ No. of free electrons in given rod

$$N=5\cdot855\times10^{23}$$

(*b*) No. of free electrons per unit volume

$$n=\frac{N}{V}$$

$$=\frac{5\cdot855\times10^{23}}{10^{-4}}$$

$$n=5\cdot855\times10^{27}$$

$$I=4\text{ amp.}$$

$$e=1\cdot6\times10^{-19}\text{ C}$$

and $\quad A=10^{-4}\text{ m}^2$

$\therefore$ By the relation $I = enAV_d$, the drift velocity,

$$V_d = \frac{I}{enA}$$

$$= \frac{4}{1.6\times10^{-19}\times5.855\times10^{27}\times10^{-4}}$$

$$= 4.27\times10^{-5}\ \mathrm{ms^{-1}}.$$

Example 4. *Eight lead acid type of secondary cells each of emf 1.8 volts and internal resistance 0.012 Ω are joined in series to supply a current to a resistance of 8.0 ohms (a) What is the current drawn from the supply and its terminal voltage (b) If the emf of the cell reduces to 1.6 volt after a long use and its internal resistance becomes 320 ohms, what maximum current can be drawn from a cell ?*

Solution. (*a*) $$I = \frac{nE}{R+nr}$$

$$= \frac{8\times1.8}{8+8\times.012}$$

$$= \frac{1.8}{1.012} = 1.778\ \text{amp}$$

Terminal voltage,

$$V = IR$$

$$= 1.778\times8$$

$$= 14.224\ \text{volt.}$$

(*b*) Maximum current,

$$I_{max} = \frac{E}{(R+r)}$$

$$= \frac{1.6}{(8+320)}$$

$$= 0.004878\ \text{A.}$$

Example 5. *A storage battery is marked a capacity of 5.0 Ah at 1 h discharge rate (a) what does this signify ? Would the cell provide 15 A for 20 minutes ? (b) which type of cell would you require for supplying (i) a current of 50 A for 25 S and (ii) a current of 50 mA occasionally ?*

Solution. (*a*) A cell marked with 5.0 Ah at 1 h discharge rate will supply a maximum current of 5 A for 1 hour. It can't supply a current more than 5 A even for a period lesser than 1 hour. However if we draw less current say 1 A from this cell, it will work for 5 hours. So the cell cannot provide 15 A current at all even for a period less than 20 S.

(*b*) (*i*) A secondary cell called storage battery or accumulator say lead accumulator with very low resistance can supply a current of 50 A for 25 S.

(*ii*) A primary cell like dry cell (*e.g.*, eveready Philips, Geep marked etc.) can be used to get a current of 50 mA occasionally.

Example 6. *In a discharge tube, the number of hydrogen positive ions i e. protons drifting a cross section in 1S is $1{\cdot}0\times 10^{18}$, while the number of electrons drifting in the opposite direction across another cross section in 1 S is $2{\cdot}7\times 10^{18}$. If the discharge tube is given a voltage of 220 V, what is the effective resistance of the tube?*

Solution. Total no. of charges (either proton or electrons) drifting in 1S, $n=(1{\cdot}0\times 10^{18}+2{\cdot}7\times 10^{18})$

$$=3{\cdot}7\times 10^{18}$$

Now $I=\dfrac{ne}{t}$ where e=magnitude of charge on electron or proton

Now $e=1{\cdot}6\times 10^{-19}$ C

$t=1$ S

$$\therefore \quad I=\frac{(3{\cdot}7\times 10^{18})\times(1{\cdot}6\times 10^{-19})}{1}$$

$$I=0{\cdot}592 \text{ A}$$

$$\therefore \quad R=\frac{V}{I}=\frac{220}{0{\cdot}592}=371{\cdot}6\ \Omega.$$

Example 7. *A platinum wire has resistance of 10 ohms at 0°C and 20 ohms at 273°C. Find the value of temperature coefficient of resistivity.*

Solution. $R_2=R_1\,[1+\alpha\,(t_2-t_1)]$

$$20=10\,[1+\alpha\,(273-0)]$$

$$\frac{20}{10}=[1+273\,\alpha]$$

$$\therefore \quad \alpha=\frac{1}{273}=0{\cdot}003663°C^{-1}$$

Example 8. *Two wires of equal length, one of the aluminium and the other of iron have the same resistance which of the two wires is lighter? Hence explain why aluminium wires are preferred for overhead power cables (relative density of Al=2·7 and of iron=8·0 and resistivity of Al=$2{\cdot}7\times 10^{-8}$°C^{-1} and of iron=10×10^{-8}°C^{-1}).*

Solution. $R=\rho\dfrac{l}{A}=\rho\dfrac{l}{\pi r^2}$

$$\therefore \quad R_1=R_2$$

$$\rho_1\frac{l}{\pi r_1^2}=\rho_2\frac{l}{\pi r_2^2}$$

$$\therefore \quad \frac{\rho_1}{\rho_2}=\frac{r_1^2}{r_2^2}$$

$$\therefore \quad \frac{M_1}{M_2}=\frac{\pi r_1^2\, l\, d_1}{\pi r_2^2\, l\, d_2}$$

$$=\frac{\rho_1}{\rho_2}\times\frac{d_1}{d_2}$$

$$=\frac{2{\cdot}7\times10^{-8}}{10\times10^{-8}}\times\frac{2{\cdot}7}{8}=91:1000$$

$\therefore$ Aluminium wire is lighter.

Since aluminium wire is lighter so it is preferred for overhead power cables.

Example 9. *Calculate the resistivity of the material of a wire 1·1 m long, 0·4 mm in diameter & having a resistance 2·1 ohms.*

Solution.

$$l=1{\cdot}1 \text{ m}, \; r=\frac{0{\cdot}4}{2} \text{ mm}=0{\cdot}2\times10^{-3} \text{ m}$$

$$R=2{\cdot}1\ \Omega$$

$$\therefore \quad \rho=\frac{RA}{l}=\frac{R\pi r^2}{l}$$

$$=\frac{2{\cdot}1\times{\cdot}22\times({\cdot}2\times10^{-3})^2}{7\times1{\cdot}1}$$

$$=24\times10^{-8} \text{ ohm m}$$

Example 10. *A wire of 200 g having resistance 10 ohms is drawn so as to make its length double, calculate the charge in resistance of wire ?*

Solution.

$$R=\rho\frac{l}{A}$$

$$\therefore \quad 10=\rho\frac{l}{A} \qquad \ldots(1)$$

when length of the wire on drawing becomes double, the area of cross-section becomes half so that volume $(l\times A)$ of wire may remain constant. So the new resistance of wire,

$$R'=\rho\frac{2l}{(A/2)}$$

$$=4\,\rho\frac{l}{A}$$

$\therefore$ By eq. (1),

$$R'=4\times10=40\Omega.$$

So the resistance of wire has increased by $(40-10)=30\Omega$.

Example 11. *Two identical cells of emf 2·00 V each joined in parellel provide supply to an external circuit consisting of two resis-*

tance 15Ω each joined in parallel. A very high resistance voltmeter reads the terminal voltage of the cells to be 1·8 V. Calculate the internal resistance of each cell.

Solution. E.M.F. of each cell$=1{\cdot}5$ V

E.M.F. of 2 cells in parellel, $E=1{\cdot}5$ V

Equivalent resistance of 2 resistors of 15Ω in parellel,

$$\frac{1}{R}=\frac{1}{15}+\frac{1}{15}$$

$$R=7{\cdot}5\Omega$$

Terminal potential difference,

$$V=F-Ir$$

where r=Equivalent internal resistance of two cells in parellel

$$\because \quad Ir=E-V$$

$$=2{\cdot}0-1{\cdot}8$$

$$Ir=0{\cdot}2 \text{ V}$$

But $$I=\frac{E}{(R+r)}$$

$$\therefore \quad \frac{E}{(R+r)}r=0{\cdot}2$$

$$\frac{2{\cdot}0\,r}{(7{\cdot}5+r)}=0{\cdot}2$$

$$\therefore \quad r=\frac{1{\cdot}5}{1{\cdot}8}=\frac{5}{6}\Omega$$

Now $$\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}$$

$\because$ $r_1=r_2=r_0$ (say), internal resistance of each cell

$$\therefore \quad \frac{1}{r}=\frac{1}{r_0}+\frac{1}{r_0}$$

$$\therefore \quad r_0=2r$$

$$=\frac{2\times5}{6}\Omega$$

$$r_0=1{\cdot}67\ \Omega.$$

Example 12. *Give the 3 resistances of 2Ω, 3Ω and 4Ω, how will you connect them to get (a) maximum resistance (b) minimum resistance (c) 5·2Ω (d) 4·33Ω.*

Solution. (*a*) The resistances connected in series give maximum resistance.

$$\therefore \quad R=R_1+R_2+R_3$$

$$=2+3+4$$

$$=9\Omega$$

(*b*) The resistances connected in parellel give the minimum resistance.

$$\therefore \quad \frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

$$=\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$$

$$\therefore \quad \frac{1}{R}=\frac{6+4+3}{12}$$

$$R=\frac{12}{13}\Omega$$

(*c*) Join 2Ω and 3Ω resistances in parellel and the combination in series with 4Ω.

$$\therefore \quad R=\left(\frac{2\times 3}{2+3}+4\right)$$

$$=5{\cdot}2\Omega$$

(*d*) Join 2Ω and 4Ω resistances in parellel and combination in series with 3Ω.

$$\therefore \quad R=\left(\frac{2\times 4}{2+4}+3\right)$$

$$R=4{\cdot}33\Omega$$

Example 13. *Find out the equivalent resistance of the following networks between A and B, where the value of resistance of each resistor written is in ohms.*

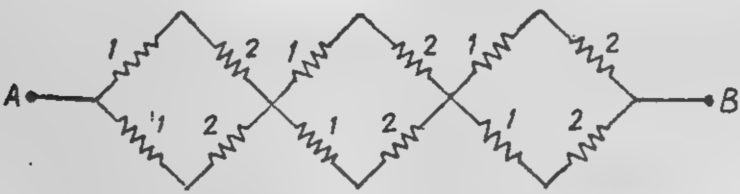


Fig. 3.7.

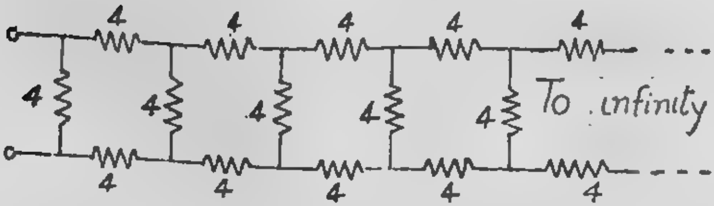


Fig. 3.8.

Solution. (*i*) For Fig. 3.7 Each pair of 1Ω & 2Ω are connected in series. So equivalent resistance, R=1+2=3 Ω.

Each 3Ω resistance is connected in parellel with another 3Ω resistance and there are such 3 combinations. So net equivalent resistance,

$$R'=\left(\frac{3\times 3}{3+3}\right)\times 3$$

$$=4{\cdot}5\Omega$$

(*ii*) For Fig. 3·8,

Let R is the equivalent resistance of the whole network consisting of infinite resistances. So if you cut away first section of 3 resistors, left out network will also have the resistance R. So equivalent circuit will be the one as shown in Fig. 3.9.

Now resistances of arms CD, DE and EF are in series and this

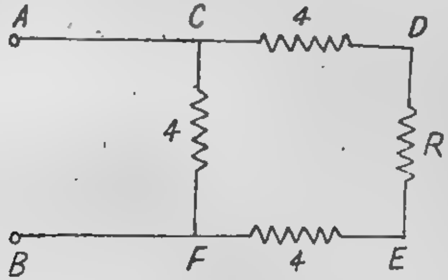


Fig. 3.9.

combination is in parellel with the resistance of arm CF. So equivalent resistance R of whole network will be given by

$$R=\frac{4\,(4+4+R)}{4+(4+4+R)}$$

$$R\,(12+R)=32+4R$$

$$\therefore\quad R^2+8R-32=0$$

$$R=\frac{-8\pm\sqrt{8^2-4\times 1\times(-32)}}{2\times 1}$$

$$=\frac{-8\pm 8\sqrt{3}}{2}=(-4\pm 6{\cdot}93)$$

$$=2{\cdot}93\Omega$$

(*iii*) For Fig. 3.10,

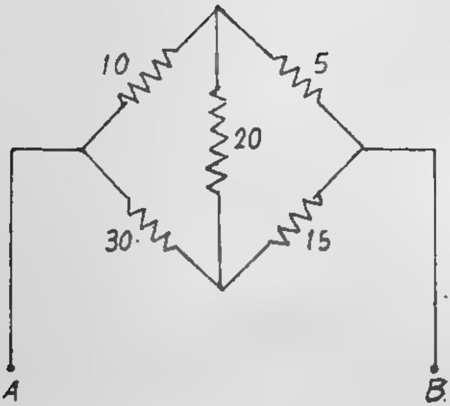


Fig. 3.10.

Since $\frac{10}{5}=\frac{30}{15}$, wheatstone's bridge as balanced so the resistance at the centre (here 20Ω) is not considered is no current flows through this arm.

∵ In the circuit 10Ω is connected in series with 5Ω and 30Ω in series with 15Ω. These two combinations of resistances $R_1=10+5=15\Omega$ & $R_2=30+15=45\Omega$ respectively are connected in parellel with each other. So equivalent resistance of the circuit,

$$R=\frac{15\times45}{(15+45)}$$

$$R=11\cdot25\ \Omega$$

(*iv*) For Fig. 3.11,

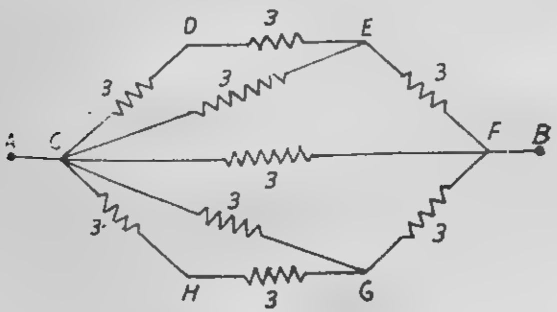


Fig. 3·11.

The resistance of arm CDE=3+3=6Ω

This arm CDE of 6Ω is joined in parellel with arm CE of 3Ω, so net resistance,

$$R=\frac{6\times 3}{(6+3)}=2\Omega$$

Now this resistance R=2Ω is joined in series with arm EF of resistance 3Ω. So net resistance of arm CDEF

$$R'=R+3$$
$$=2+3$$
$$R'=5\Omega$$

Similarly the net resistance of arm CHGF will also be 5Ω.

Now these two 5Ω resistances are joined in parellel with 3Ω resistance of arm CF. So equivalent resistance R″ of the whole circuit will be given by

$$\frac{1}{R''}=\frac{1}{5}+\frac{1}{5}+\frac{1}{3}$$
$$\frac{1}{R''}=\frac{11}{15}$$
$$R''=\frac{15}{11}=1{\cdot}\dot{3}\dot{6}\ \Omega$$

Example 14. *Determine the current in each branch of the following network :*

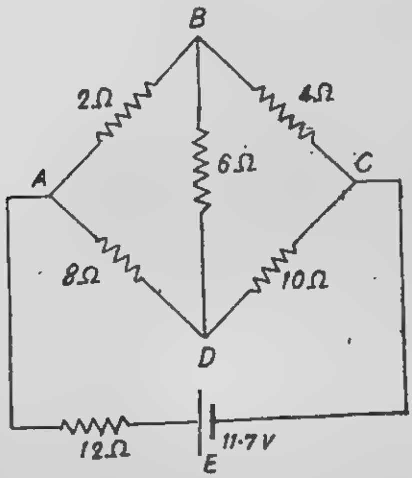


Fig. 3.12

Solution. Let the current in different branches of the network be I, I_1, I_2, as shown in Fig. 3.13.

Using Kirchoff's second law,

(*i*) For loop ABDA,

$$2I_1+6I_2-8(I-I_1)=0$$
$$2I_1+6I_2-8I+8I_1=0$$
$$1I_2+6I_2-8I=0$$
$$5I_1+3I_2-4I=0 \qquad \text{...(1)}$$

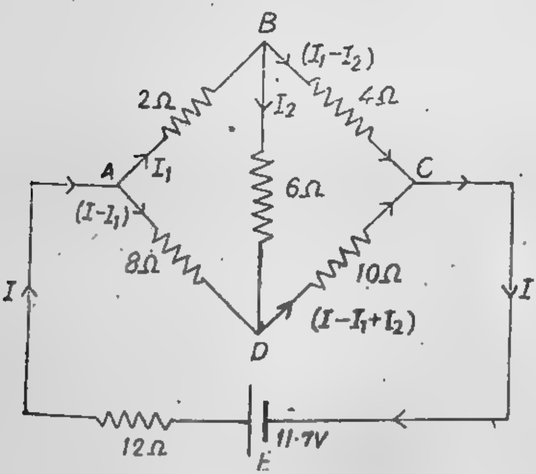


Fig. 3.13.

(*ii*) For loop BCDB,

$$4(I_1-I_2)-10(I-I_1+I_2)-6I_2=0$$
$$4I_1-4I_2-10I+10I_1-10I_2-6I_2=0$$
$$14I_1-20I_2-10I=0$$
$$7I_1-10I_2-5I=0 \qquad \text{...(2)}$$

(*iii*) For loop ADCEA,

$$8(I-I_1)+10(I-I_1+I_2)+12I=11.7$$
$$8I-8I_1+10I-10I_1+10I_2+12I=11.7$$
$$30I-18I_1+10I_2=11.7 \qquad \text{...(3)}$$

On adding Eq^n (2) and Eq^n (3), we get

$$25I-11I_1=11.7 \qquad \text{...(4)}$$

From eq^n (1) and (2),

$$50I_1+30I_2-40I=0$$
$$21I_1-30I_2-15I=0$$
$$\overline{71I_1-55I=0}$$

$$I_1=\frac{55}{71}\ I \qquad \text{...(5)}$$

$\therefore$ From eq^n (4) and (5),

$$25I-11\times\frac{55}{71}I=11{\cdot}7$$

or $71\times25I-11\times55I=11{\cdot}7\times71$

$$I=\frac{11{\cdot}7\times71}{1170}=0{\cdot}71\text{A}$$

$$\therefore \quad \boxed{I=0{\cdot}71\text{A}}$$

$\therefore$ From eq^n (5),

$$I_1=\frac{55}{71}\times0{\cdot}71$$

$$\therefore \quad \boxed{I_1=0{\cdot}55\text{A}}$$

$\therefore$ From eq^n (1)

$$5\times0{\cdot}55+3I_2-4\times0{\cdot}71=0$$

$$3I_2=2{\cdot}84-2{\cdot}75$$

$$I_2=\frac{0{\cdot}09}{3}$$

$$\therefore \quad \boxed{I_2=0{\cdot}03\text{A}}$$

$\therefore$ The current in $AB=I_1=0{\cdot}55\text{A}$,

The current in $AD=(I-I_1)=(0{\cdot}71-0{\cdot}55)=0{\cdot}16\text{A}$,

The current in $BD=I_2=0{\cdot}03\text{A}$,

The current in $BC=(I_1-I_2)=(0{\cdot}55-0{\cdot}03)=0{\cdot}52\text{A}$,

The current in $DC=(I-I_1+I_2)=(0{\cdot}71-0{\cdot}55+0{\cdot}03)$
$=0{\cdot}19\text{A}$,

and The total current$=I=0{\cdot}71\text{A}$

Example 15. *In a metre bridge (see Fig. 3·14), the balance point is found to be 24·8 cm from the end A (a) Calculate the resistance of Y if the resistance X is of 12 Ω, (b) Calculate the balance point of the bridge if X and Y are interchanged.*

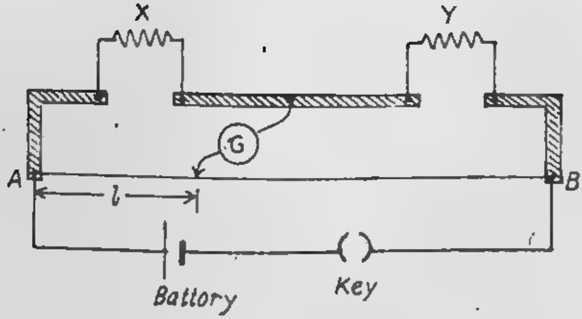


Fig. 3.14.

Solution. In a metre bridge having a wire of uniform cross-section,

$$\frac{X}{Y}=\frac{l}{(100-l)}$$

(*a*) $\therefore$

$$Y=\frac{(100-l)}{l}\,X$$

$$=\frac{(100-24\cdot8)}{24\cdot8}\times12$$

$$=36\cdot4\Omega$$

(*b*)

$$\frac{Y}{X}=\frac{l}{(100-l)}$$

$\therefore$

$$Y=\frac{l}{(100\ ml)}\,X$$

$$36\cdot4=\frac{l}{(100-l)}\times12$$

$\therefore$

$$l=75\cdot2\text{ cm.}$$

Example 16. *A galvanometer has an internal resistance of 4Ω. It gives maximum deflection for a current of 50 mA. Show how this instrument can be converted into (a) an ammeter with a maximum range of 2·5 A (b) a Voltmeter with a maximum reading of 2·5 Volt, (c) Find the resistance of the ammeter and the voltmeter.*

Solution.

(*a*) The shunt resistance (R_s) required to convert a galvanometer into an ammeter,

$$R_s=\frac{I_g}{(I-I_g)}\,R_g$$

Now $\quad I_g=50\text{ mA}=0\cdot05\text{A}$

$\quad I=2\cdot5\text{A}$

and $\quad R_g=4\ \Omega$

$$\therefore \qquad R_s = \frac{0.05}{(2.5-0.05)} \times 4$$

$$= \frac{4}{49} \Omega \text{ in parallel}$$

(*b*) The series resistance (R) required to convert a galvanometer into a voltmeter,

$$R = \frac{V}{I_g} - R_g$$

$$= \frac{2.5}{0.05} - 4$$

$$= 50 - 4 = 46\Omega \text{ in series.}$$

(*c*) The resistance of ammeter (R_a) is given by

$$\frac{1}{R_a} = \frac{1}{R_g} + \frac{1}{R_s}$$

$$\therefore \qquad R_a = \frac{4 \times \frac{4}{49}}{(4 + \frac{4}{49})}$$

$$= \frac{4}{50}$$

$$= 0.08 \ \Omega$$

The resistance of voltmeter (R_v) is given by

$$R_v = (R_g + R)$$

$$= (4 + 46)$$

$$R_v = 50 \ \Omega$$

Example 17. *A voltmeter reads 3 V at full scale deflection. It is graded at full scale deflection as 2000 Ω/V. (a) How will you convert it into a voltmeter that reads 10V at full scale deflection (b) Will you prefer this voltmeter to one that is graded 2500 Ω/V ?*

Solution. (*a*)

$$I_g = \frac{V}{R_g}$$

$$= \frac{1 \text{ V}}{2000 \ \Omega}$$

$$= 0.0005 \text{ A}$$

The series resistance (R) required to increase its range from 3 V to 10 V,

$$R = \frac{V}{I_g}$$

$$= \frac{(10-3)}{0.0005}$$

$$= 14{,}000 \ \Omega$$

The 'resistance per volt' of the new meter is the same as before.

(*b*) The higher the 'resistance per Volt' of the meter, the lesser is the current it draws from the circuit and the better it is. So the meter graded 2500 Ω/V will be more accurate than the one graded as 2000 Ω/V.

Example 18. *A storage battery of emf 6 volt and internal resistance 0·3Ω is being charged by a 100 V d.c. supply using a series resistor of 24·7 Ω (a) What is the terminal voltage of the battery during charging ? (b) What is the purpose of having a series resistor in the charging circuit ?*

Solution.

$$I=\frac{V}{(R+r)}$$

$$=\frac{100}{(24{\cdot}7+0{\cdot}3)}$$

$$I=4\text{ A}$$

At the time of charging a back emf develops in a storage battery. So in equation of terminal voltage (V) of battery, the value of emf (E) will be taken as negative.

$$\therefore \quad V=(-E)-Ir$$

$$=-6-4\times 0{\cdot}3$$

$$V=-7{\cdot}2 \text{ Volt}$$

The –ive sign is showing that instead of getting a Voltage from the battery you are applying a voltage on the battery.

(*b*) The series resistance prevents the current through the battery to be extraordinary high. The high current through the battery is quite dangerous and may damage it.

Example 19. *A d.c. supply of 80V is connected to a large resistance X. A voltmeter of resistance 20 kΩ placed in series in the circuit reads 5 V. Calculate the value of X, (b) Why the voltmeter instead of ammeter is used to measure a large resistance ?*

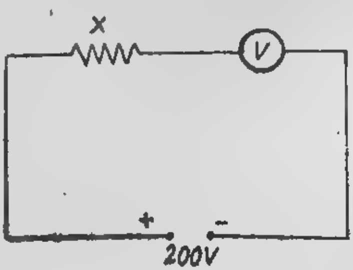


Fig. 3 15.

Solution. (*a*) *According to circuit in Fig. 3.15,*

$$I=\frac{V}{R}$$

$$=\frac{5}{20{,}000}=2{\cdot}5\times 10^{-4}\text{A}$$

Now $\qquad V=I(R+X)$

$\therefore$
$$X=\frac{V}{I}-R$$
$$=\frac{80}{2\cdot5\times10^{-4}}-20{,}000$$
$$=320{,}000-20{,}000$$
$$=300{,}000\ \Omega$$
$$=300\ \text{k}\ \Omega.$$

(*b*) The value of current in the circuit is too small to be measured by an ammeter due to very high resistance X in the circuit. That is why a voltmeter is used instead of a Voltmeter.

Example 20. *A cylindrical aluminium shell is made of two coaxial cylinders of inner radius 0·8 cm and outer radius 1·2 cm and length 2·5 cm. Calculate the resistance of the conductor (between the cylinders), given that resistivity of aluminium is $27\times10^{-8}\ \Omega m$.*

Solution. Let us consider a cylindrical shell of inner radius r and thickness dr,

Then,
$$A=2\pi rl$$
and
$$l=dr$$
(l is the length of the conductor and not the cylinder)

$\therefore$ The resistance of the conductor,
$$dR=\rho\frac{l}{A}$$
$$=\rho\frac{dr}{2\pi rl}$$

$\therefore$ The total resistance of the conductor between the cylinders of radius r_1 and r_2 will be
$$R=\int_{r_1}^{r_2} dR$$
$$=\frac{\rho}{2\pi l}\int_{r_1}^{r_2}\frac{dr}{r}$$
$$=\frac{\rho}{2\pi l}\log\frac{r_2}{r_1}$$

$\therefore$
$$R=\frac{2\cdot7\times10^{-8}}{2\times3\cdot14\times0\cdot025}\log\frac{1\cdot2}{0\cdot8}$$
$$=17\cdot197\times10^{-8}\times\log 1\cdot5$$

$$=17{\cdot}197\times0{\cdot}1761\times10^{-8}\ \Omega$$
$$=3{\cdot}028\times10^{-8}\ \Omega.$$

Example 21. *In the given circuit,*

$$E_1=3E_2=2E_3=6 \text{ Volts}$$
$$R_1=2R_4=6 \text{ ohms}$$
$$R_3=2R_2=4 \text{ ohms}$$
$$C=5\ \mu f.$$

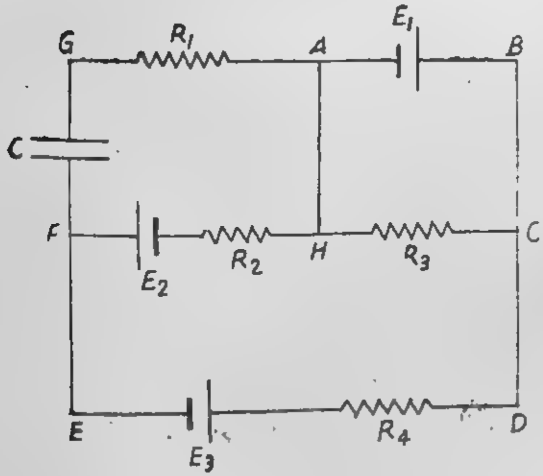


Fig. 3.16.

Find the current in R_3 and the energy stored in the capacitor.

Solution. Applying Kirchhoff's law across the loop ABCHA,

$$iR_3=E_1$$

$$\therefore \qquad i=\frac{E_1}{R_3}$$

$$=\frac{6}{4}$$

$$\boxed{i=1{\cdot}5\text{A}}$$

For the loop ABCDEFHA,

$$I(R_2+R_4)=(E_1-E_2-E_3)$$ ($\because$ there is no current in the loop FGA because the capacitor blocks the d.c.).

$$\therefore \qquad I=\frac{(6-2-3)}{(2+3)}$$
$$=\frac{1}{5}\text{ A}$$
$$=0{\cdot}2\text{ A}$$

The P.D. (Potential Difference) between the two plates of the capacitor, *i.e.* between the points F and A,

$$V=E_2+R_2I$$
$$=2+2\times0{\cdot}2$$
$$\therefore \qquad V=2{\cdot}4\text{ V}$$

Now $\qquad C=5\ \mu f.$
$$=5\times10^{-6}f$$

$\therefore$ The energy stored in the capacitor,

$$E=\tfrac{1}{2}CV^2$$
$$=\tfrac{1}{2}(5\times10^{-6})(2{\cdot}4)^2$$
$$=14{\cdot}4\times10^{-6}\text{ J.}$$

Example 22. *Calculate the steady current in the 6 Ω resistor in the Fig. 3.17, the internal resistance of the battery is negligible.*

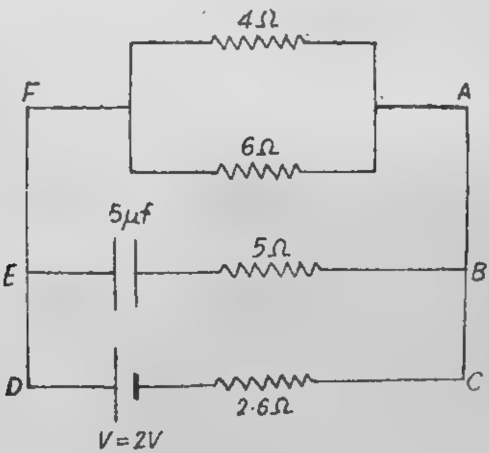


Fig. 3.17.

Solution. The capacitor does not allow d.c. to pass through it, so does $5\mu f$ capacitor in the arm EB of the circuit. So we may think as there is no arm like EB in the circuit.

Now effective resistance of 4 Ω and 6 Ω resistances in parallel,

$$R=\frac{4\times6}{(4+6)}$$
$$=2{\cdot}4\ \Omega$$

This $R=2{\cdot}4\ \Omega$ resistance is joined with $2{\cdot}6\ \Omega$ resistance in series. So equivalent resistance of the circuit,

$$R'=2{\cdot}4+2{\cdot}6$$
$$=5{\cdot}0\ \Omega$$

$$\therefore \qquad I=\frac{V}{R'}$$
$$=\frac{2}{5}$$
$$=0{\cdot}4\ A$$

$\therefore$ The current in $6\ \Omega$ resistance,

$$I_1=\frac{4}{(4+6)}\times 0{\cdot}4$$

$$\boxed{I_1=0{\cdot}16\ A}$$

EXERCISE 3

1. The resistances of $4\ \Omega$, $8\ \Omega$ and $12\ \Omega$ are connected in series and the potential difference 6 Volt is applied across the extreme ends. Calculate the current in the circuit.
2. Three cells each of emf 2 V and internal resistance $1\ \Omega$ are connected in series with an external resistance of $47\ \Omega$. (*a*) What is the current in the circuit, (*b*) If the cells are connected in parellel with the external resistance, will the current in the circuit change ? If yes, what will be the new value of the current ?
3. An equilateral triangle is formed with each side having a resistance of $1\ \Omega$. What is the eff[illegible]ve resistance across any side ?
4. The resistance of two conductors joined in series is $8\ \Omega$ and in parellel is $1{\cdot}5\ \Omega$. Calculate the value of each resistance.
5. If a wire is stretched to double its length, prove that its resistance becomes 4 times.
6. An aluminium wire and a copper wire of the same length have the same resistance. What is the ratio of their radii ? (The resistivity of Al and cu are $2{\cdot}7\times10^{-8}\ \Omega$ m and $1{\cdot}7\times10^{-8}\ \Omega$ m respectively).
7. How many electrons pass per minute through a given cross-section of a wire in which a current of 100 mA is flowing ?
8. A copper wire of cross-section 1 mm in diameter carries a current of 2 A. If there are 8×10^{28} electrons per cubic metre, calculate the drift velocity of the electrons.

9. The resistivity of copper is $1.72 \times 10^{-8}\ \Omega$ m. Calculate the length of copper wire of radius 1 mm such that the resistance offered is $0.2\ \Omega$.

10. A copper wire 1 mm in diameter carries a current of 12 A. Find the potential difference between two points in the wire which are 2 m apart (Resistivity of Cu$=1.7 \times 10^{-8}\ \Omega$ m).

11. One kilogram of copper is drawn into a wire (*a*) 1 mm thick and (*b*) 0.5 mm thick. Compare their resistances.

12. 100 g of aluminium wire of resistance $16\ \Omega$ is drawn so that its length becomes one and a half times. Calculate the change in resistance of the wire.

13. The same current passess through a metre of copper wire 1 mm in diameter and two metres of thinner copper wire. The P.D. between the ends of the first wire is 1 Volt and that between the ends of the other is 20 Volts. Find the radius of the thinner wire.

14. If the resistance of a wire of length 2.4 m and radius 0.2 mm is found to be $5.0\ \Omega$, calculate the resistivity of the material of the wire.

15. The length of the tungsten filament of a lamp 200 Volts, 40 Watt is 20 cm. Calculate the diameter of the wire, if the resistivity of its material is 5×10^{-8} ohm m.

16. The emf of a cell is 2 Volts and the P.D. between its plates is 1.6 Volts when it is connected in series with a resistance of $10\ \Omega$. Find the internal resistance of the cell.

17. A battery of voltage 2.2 Volts delivers 0.5 A current through a resistance of 4.3 ohms. Calculate the internal resistance of the cell and the terminal voltage of the battery.

18. A galvanometer of resistance $99\ \Omega$ gives full scale deflection with a current of 10^{-4} A. How will you convert it to
(*a*) an ammeter of range 0 to 1 A.
(*b*) a voltmeter of range 0 to 1 V.

19. An ammeter of range 1 A has a resistance of $0.1\ \Omega$. What shunt must be used to increase its range upto 10 A ?

20. A galvanometer of resistance $10\ \Omega$ gives full scale deflection with a current of 5 mA. How will you convert it into a voltmeter of range 0—10 Volts ?

21. A galvanometer of resistance $30\ \Omega$ gives full scale deflection with a current of 20 mA. How will you convert it into (*a*) an ammeter of range 0—5 A, (*b*) a voltmeter of range 0—5 V and (*c*) Calculate in each case the effective resistances.

22. A galvanometer has an internal resistance $1.0\ \Omega$. It gives maximum deflection for a current of 50 mA. (*a*) Show how

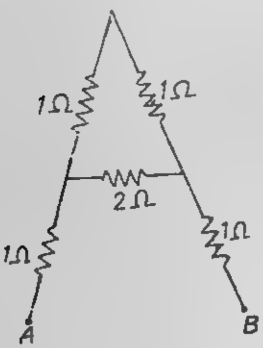
this instrument can be converted into (*i*) an ammeter with a maximum reading of 2·5 A, (*ii*) a voltmeter with a maximum reading of 2·5 V, (*b*) Calculate also the effective resistance in each case.

23. If in the Question No. 22 above, galvanometer resistance $= 5\ \Omega$, current for full scale deflection $= 200$ mA and ammeter range $= 0-3$ A and voltmeter range $= 0-3$ V, calculate each of the quantities in the question above.

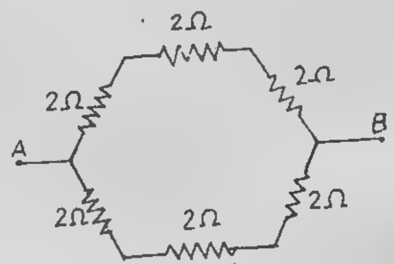
24. A galvanometer of 120 Ω resistance give full scale deflection with a current of 0·0005 A. Find the value of shunt that must be connected in parellel so that it can read a maximum current of 5 A. What is the resistance of the new ammeter ? [*A.I.S.S.E. 1979*]

25. A galvanometer has a full scale deflection with a current of 100 m A. The resistance of the galvanometer is 100 Ω. Convert the galvanometer into (*a*) voltmeter reading up to 200 Volts (*b*) ammeter reading up to 1·0 A. [*D.S.S.E. 1979*]

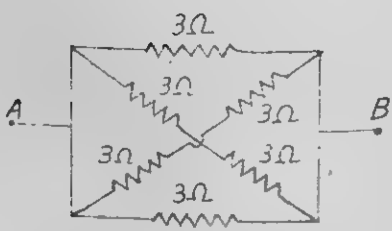
26. Find out the equivalent resistance of the following networks between A and B in figures *a* to *f*.



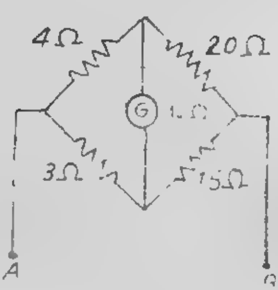
(*a*)



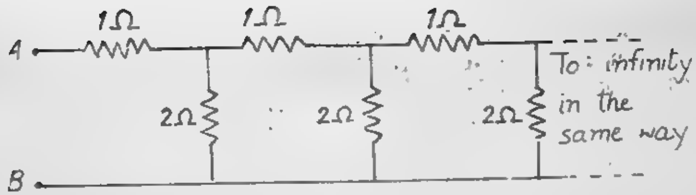
(*b*)



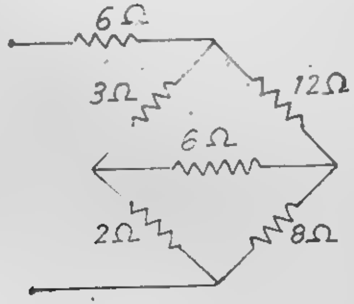
(*c*)



(*d*)



(e)



(f)

Fig. 3.18.

27. A electric heater uses nichrome for its heating element. When a negligibly small current passes through it, its resistance at room temperature 25°C is found to be 68 Ω. When the heater is connected to a 240 V supply the current settles after a few seconds to a steady value of 3 A. What is the temperature coefficient of nichrome if its steady temperature 1050°C.

28. The resistance of the element in an electric toaster at 30°C is found to be 90 Ω. When it is connected to 250 V supply the current settles down to 2·5 A. What is the steady temperature of the element ? (Given temperature coefficient of resistivity of nichrome is $1{\cdot}7\times10^{-4}\ ^\circ C^{-1}$).

29. Calculate the current through 5 Ω, 6 Ω and 8 Ω resistances in the circuit in Fig. 3.19. The emf of battery is 4 V and its internal resistance is 2Ω as shown in Fig. 3.19.

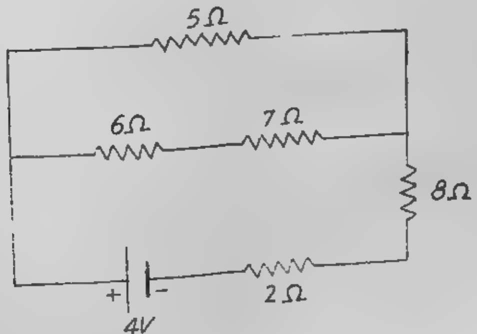


Fig. 3.19.

30. Calculate the current I, and I_1 in the following network of resistances. The emf of battery is 10 Volts and its internal resistance is 2 ohms.

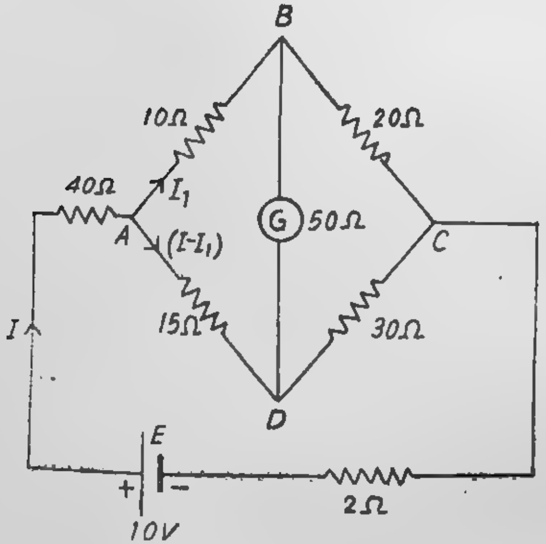


Fig. 3.20.

31. Calculate the current in each branch of the following network :

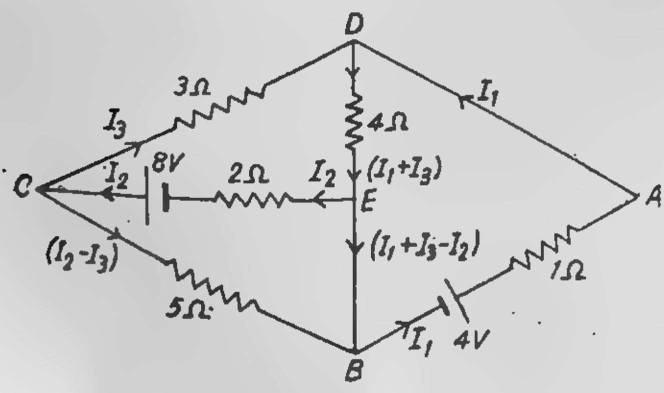


Fig. 3.21.

32. Calculate the currents in different branches of the following network.

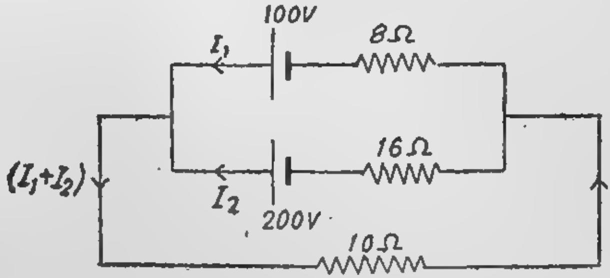


Fig. 3.22.

33. Find out the current in each branch of the circuit shown in Figure 3.23.

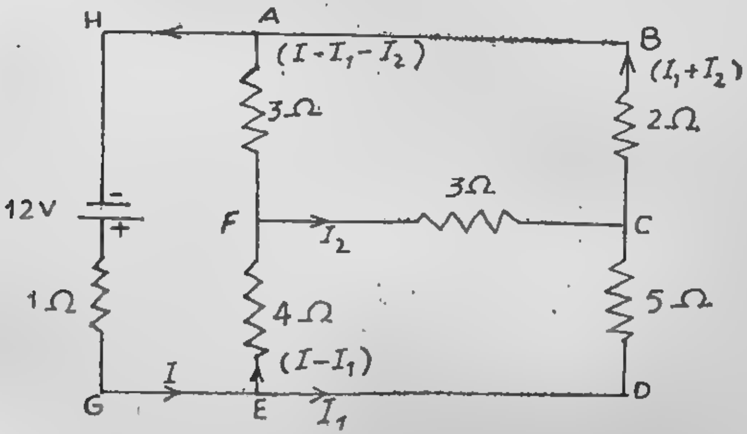


Fig. 3.23.

34. Find the equivalent resistance of the following networks between the points A and B in Fig. 3.24 (*a*) and (*b*).

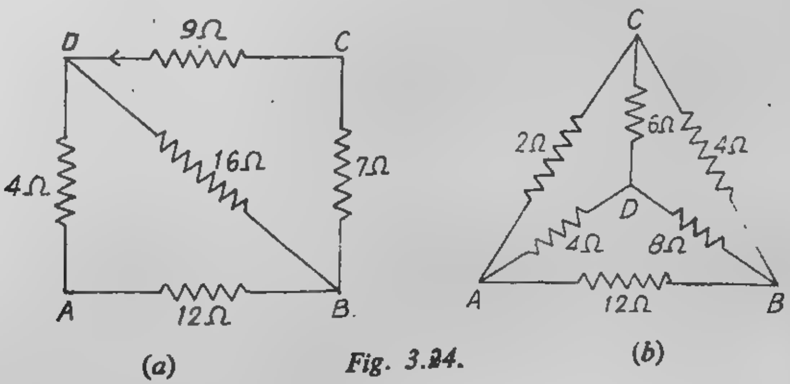
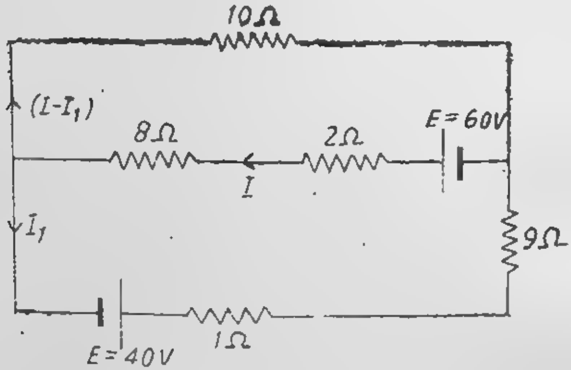


Fig. 3.24.

35. Find the currents in different branches of the following networks in Fig. 3.25 (*a*) and (*b*).



(a)

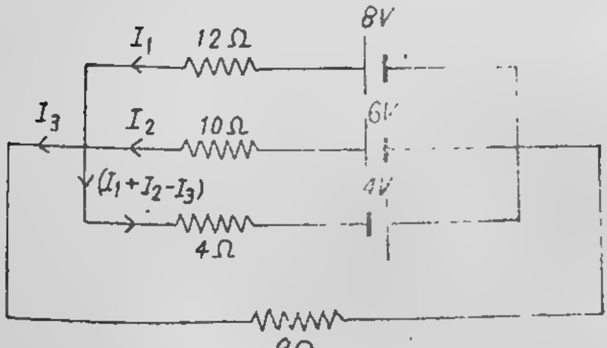


Fig. 3.25 (*b*)

36. Find the reading of the ammeter in the circuit in Fig. 3.26.

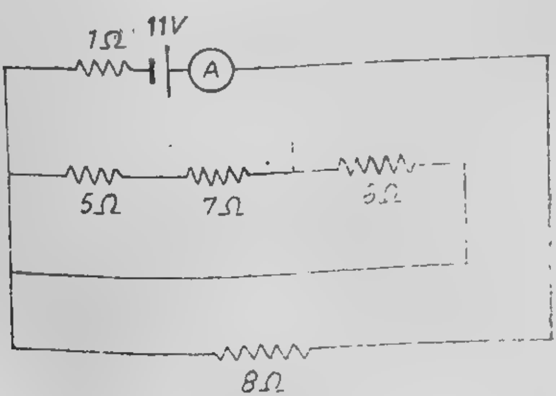


Fig. 3.26

37. A high resistance voltmeter in the circuit in Fig. 3.27 reads 50 Volts when connected across 100 Ω resistance. Calculate (*a*) the resistance of the voltmeter (*b*), the reading of the voltmeter when it is connected across 80 Ω resistance.

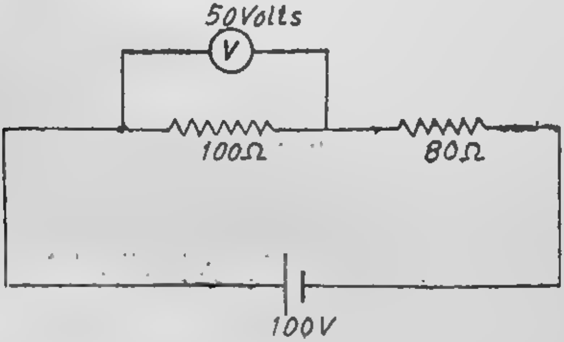


Fig. 3.27.

38. A part of circuit in a steady state along with the currents flowing in the branches, the value of resistances etc. is shown in the Fig. 3.28. Calculate the energy stored in the capacitor.
[*I.I.T. JEE 1986*]

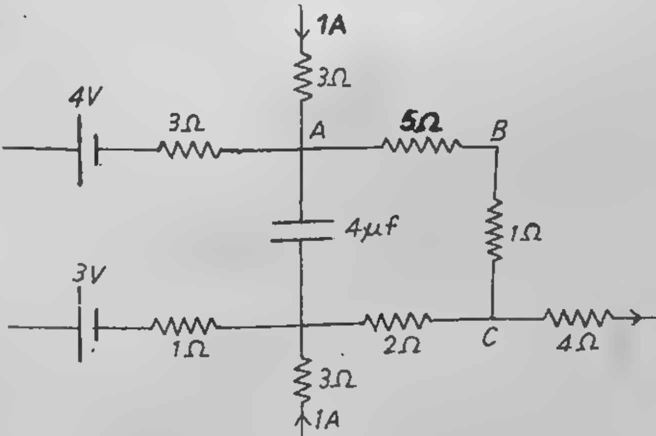


Fig. 3.28.

39. An infinite ladder network of resistances is constructed with 1Ω and 2Ω resistances as shown in Fig. 3.29. The 6 V battery between A and B has negligible resistance (*a*) Show that the effective resistance between A and B is 2 ohms, (*b*) What is the current that passes through the 2 ohms resistance nearest to the battery ? [*I.I.T. JEE 1987*]

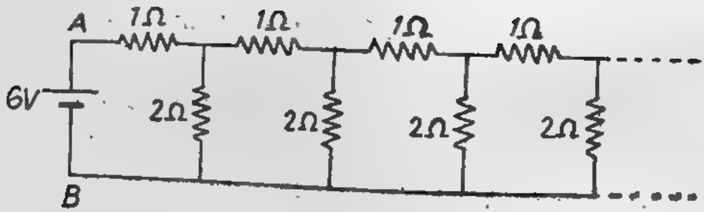


Fig. 3.29.

40. In a metre bridge, the wire of unknown resistance is connected in left hand side gap and a resistance of 40 Ω is introduced in the circuit from a resistance box connected in the right hand side gap (*a*) Calculate the unknown resistance if the balance point is obtained at 40 cm, (*b*) If the wire and resistance box are interchanged, what will be the position of the balance point ?

41. In a metre bridge a 9 Ω resistance is joined in L.H.S. gap and 16 Ω resistance in R.H.S. gap. What will be the position of balance point.

42. If the resistance of a copper wire is 5·15Ω at 40°C and 6·25Ω at 100°C, calculate the temperature coefficient of resistivity.

43. In a potentiometer the balance point is obtained at 215 cm when the Laclanche cell in the auxillary circuit is open and 161 cm when the cell is closed through a resistance of 20Ω. Calculate the internal resistance of the cell.

44. A cylindrical copper shell is made of two coaxial cylinders of inner radius 1·2 cm and outer radius 1·8 cm and length 2·8 cm. Calculate the resistance of the conductor (between the cylinders). Given that resistivity of copper $1{\cdot}7 \times 10^{-8}$ Ω m.

OBJECTIVE TYPE QUESTIONS

45. The most commonly used wire in heating appliances is made of

(*a*) aluminium (*b*) nickel
(*c*) tin (*d*) nichrome.

46. Nichrome is an alloy of :

(*a*) lead and tin (*b*) nickel and lead
(*c*) nickel and chromium (*d*) lead and aluminium

47. The unit of resistivity is :

(*a*) $\Omega \times m$ (*b*) $\Omega\, m^{-1}$
(*c*) Ω (*d*) Ω^{-1}

48. If the length of a wire on stretching it becomes 3 times, its resistance will become :

(a) 3 times (b) 6 times
(c) 9 times (d) $\frac{1}{3}$ times.

49. If the length of a wire is doubled and its cross-section is doubled, then resistance of wire :
(a) remains same (b) decreases
(c) increases (d) undecided.

50. If the radius of a wire is doubled, its resistance will become :
(a) twice (b) four times.
(c) half (d) one fourth.

51. A battery of internal resistance 0·5 Ω has on emf 3 V in an open circuit. If it is connected to an external resistance of 2·5 Ω, the terminal potential of battery will be :
(a) 3 V (b) 2·5 V
(c) 0·5 V (d) none of them.

52. No current flows between two charged bodies when connected if they have same :
(a) copacity (b) potential
(c) charge (d) none of the above

[*C.P.M.T. 1971*]

53. A piece of wire of resistance 4 Ω is bent through 180° at its mid-point and the two twisted together. Will the resistance be :
(a) 1 Ω (b) 2 Ω
(c) 8 Ω (d) 54 Ω

54. Number of electrons that constitute one ampere of current is :
(a) 265×10^{16} (b) 625×10^{13}
(c) $4·8 \times 10^{10}$ (d) 625×10^{16}

55. The unit of resistance is :
(a) ohm (b) ohm cm^{-8}
(c) ohm (d) ohm cm^{-3}

56. The example of a non-ohmic resistance is :
(a) Copper wire (b) Carbon resistance
(c) Tungsten wire (d) Diode.

57. Two resistance r_1 and r_2 $(r_1 > r_2)$ are joined in parellel. The equivalent resistance R is such that :
(a) $R > (r_1 + r_2)$ (b) $R < (r_1 + r_2)$
(c) $r_1 < R > r_2$ (d) none of the above.

58. The resistivity of a wire varies with its :
(a) length (b) cross-section
(c) mass (d) material.

59. What is the resistance of the following circuit between A and B

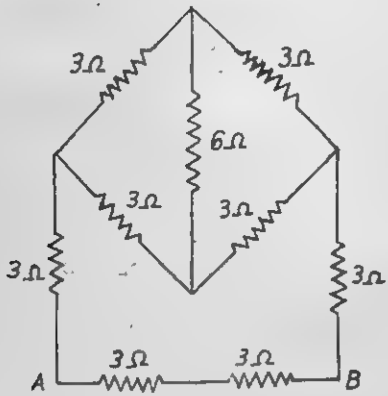


Fig. 3.30.

(*a*) 2·4 Ω (*b*) 3·6 Ω
(*c*) 6 Ω (*d*) 18 Ω.

60. Drift velocity V_d of an electron *e* varies with intensity of electric field E as :

(*a*) $V_d \propto E$ (*b*) $V_d \propto \frac{1}{E}$

(*c*) $V_d = \text{const}$ (*d*) $V_d \propto E^{1/2}$

61. An electric bulb rated for 500 Watts at 100 Volts is used in a circuit having a 200 Volts supply. The resistance R that one must put in series with the bulb, so that bulb delivers 500 Watt is :

(*a*) 40 ohms (*b*) 20 ohms
(*c*) 10 ohms (*d*) 5 ohms.

[*JEE I.I.T., 1987*]

62. The current '*i*' in the circuit is :

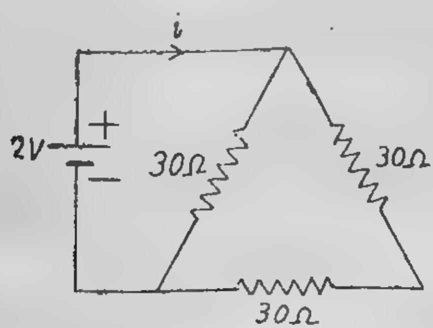


Fig. 3.31.

(*a*) $\frac{1}{45}$ amp (*b*) $\frac{1}{15}$ amp

(*c*) $\frac{1}{10}$ amp (*d*) $\frac{1}{5}$ amp.

63. Electromotive force represents :
(*a*) force (*b*) work
(*c*) energy per unit charge (*d*) energy

64. The unit of conductance is :
(*a*) ohm (*b*) sieman
(*c*) henry (*d*) farad.

65. The unit of copacitance is :
(*a*) ohm (*b*) sieman
(*c*) henry (*d*) farad.

66. The resistivity of a wire 1·1 m long, 0·4 mm in diameter and having total resistance of 4·2 Ω will be :
(*a*) $4{\cdot}97 \times 10^{7}$ Ω cm (*b*) 48×10^{-6} Ω cm
(*c*) 48×10^{6} Ω cm (*d*) none.
[*M.A.M.C. 1983*]

67. Three resistances of 2 Ω each are arranged in a triangle. The resistance in ohms between any two corners is :
(*a*) $\frac{4}{3}$ Ω (*b*) 3 Ω
(*c*) 4 Ω (*d*) 6 Ω. [*B.H.U. 1983*]

68. The internal resistance of which of the following instrument is the highest :
(*a*) galvanometer (*b*) ammeter
(*c*) voltmeter (*d*) milliammeter.
[*B.H.U. 1983*]

69. What is the resistance between A and B in Fig. 3.2 on page 33 :
(*a*) 5 Ω (*b*) 10 Ω
(*c*) 15 Ω (*d*) 20 Ω.

□□

Thermal And Chemical Effects of Currents

IMPORTANT FORMULAE

1. Electric Power,

   P(in Watts)=V(in Volts)×I (in Amperes)

   $\therefore \quad P = VI = \frac{V^2}{R} = I^2R$

2. (*a*) Electric energy,

   E(in kwh)=P(in kilowatts)×t(in hours)

   (*b*) Cost of electric energy=E(in kwh)×Rate

3. Joule's law of heating effect of current : Heat produced in Joules in a resistance 'R' due to the electric current I flowing through it is

   $H = I^2Rt.$

4. (*a*) At neutral temperature,

   $\frac{dE}{dT} = 0,$ E=*emf* of thermocouple

   T=Temperature of hot junction in kelvin.

   (*b*) Peltier coefficient,

   $$\pi = T\frac{dE}{dT}$$

   (*c*) Thomson coefficient,

   $$\sigma = T\frac{d^2E}{dT^2}$$

   (*d*) The temperature of inversion (θ_i) is always as much above the neutral temperature (θ_n) as the cold jounction (θ_c) is below it.

   $$\theta_n = \frac{\theta_i + \theta_c}{2}$$

5. **Faraday's law of chemical effect of Current :**

(i) mass of ions deposited in kg,

$m = ZIt$, Z=Electrochemical equivalent in kg C^{-1}

I=current in amperes.

t=time in seconds.

(ii) When the same current passes through various electrolytes for the same time, the masses of various ions (m) deposited are proportional to their chemical equivalent weights (w) *i.e.*

$$m_1 : m_2 : m_3 : \ldots\ldots = w_1 : w_2 : w_3 : \ldots\ldots$$

6. *Electrochemical equivalent* (Z).

(a) $$Z = \frac{\text{Atomic wt. of element}}{\text{Valency} \times \text{Faraday constant}}$$

(Faraday constant=96500 coulomb mol^{-1})

(b) $$\text{Z of an element} = \frac{\text{Z of } H_2 \times \text{At.wt of element}}{\text{Valency of element}}$$

SOLVED EXAMPLES

Example 1. *The element of a heater in rated 660 watt. If it works at 220 volts, calculate the resistance of the element and current drawn by it.*

Solution.

$$P = V \times I$$

$$660 = 220\ I$$

$$\therefore \quad I = 3\ A$$

$$\therefore \quad R = \frac{V}{I}$$

$$= \frac{220}{3}$$

or $$R = 73.3\ \Omega$$

Example 2. *A generator in supplying power to a factory by the cables of resistance 20 Ω. If the generator is generating 50 Kw power at 5000 volts, what is the power and P.D. received by the factory ?*

Solution.

$$I = \frac{P}{V}$$

$$= \frac{50{,}000 \text{ watt}}{5000 \text{ volt}}$$

$$= 10\ A$$

Drop of potential on cables, $V' = IR$

$$= 10 \times 20$$
$$= 200 \text{ Volts}$$

$\therefore$ P.D. received by the factory, $V'' = (V - V')$

$$= (5000 - 200)$$
$$= 4800 \text{ Volt.}$$

$\therefore$ Power recevied by the factory $= V'' I$

$$= 4800 \times 10$$
$$= 48000 \text{ watt}$$
$$= 48 \text{ Kw.}$$

Example 3. *An electric motor of 2 H.P. works at 220 V for 6 hours daily. It delivers mechanical power of 1200 watt.*

(*a*) *What is the current drawn by motor ?*

(*b*) *Calculate the cost of electric energy at the rate of 30 paise per unit for 30 days.*

(*c*) *What is the efficiency of the motor ?*

(*d*) *How much energy is lost as heat in 6 hours ?*

Solution.

(*a*) $P = 2 \text{ H.P.} = 2 \times 746 = 1492 \text{ Watt}$

$$\therefore \quad I = \frac{P}{V}$$
$$= \frac{1492}{220}$$
$$= 6{\cdot}78 \text{ A.}$$

(*b*) $E(\text{in } kwh) = P(\text{in } kw) \times t(\text{in hr})$

$$= \left(\frac{1492}{1000}\right)(6 \times 30)$$
$$= 268{\cdot}56 \text{ kwh or unit.}$$

$$\therefore \quad \text{Cost} = E \times R$$
$$= 268{\cdot}56 \times 30 \text{ paise}$$
$$= \text{Rs. } 8{\cdot}06.$$

(*c*) $\text{Efficiency} = \dfrac{\text{Power output}}{\text{Power input}} \times 100$

$$= \frac{1200}{1492} \times 100$$
$$= 80{\cdot}429\ \%$$

(*d*) Power lost $= 1492 - 1200 = 292$ watt.

$\therefore$ Energy lost as heat $= P \times t$

$$= 292 \times (6 \times 3600) \text{J}$$

$$=63{\cdot}072\times10^5\ \text{J}$$

$$=\frac{63{\cdot}072}{4{\cdot}18}\times10^5\ \text{cal}$$

$$=15\times10^5\ \text{cal.}$$

Example 4. *In how much time 1 kg of water at 20°C in an electric kettle rated 1 kw, 220 V will get boiled ?*

Solution.

Heat produced in calorie in the electric kettle,

$$H=\frac{P\times t}{4{\cdot}18}$$

But $$H=mS\Delta T$$

$\therefore$ $$\frac{P\times t}{4{\cdot}18}=mS\Delta T$$

$\therefore$ $$t=\frac{mS\Delta t\times4{\cdot}18}{P}$$

Now $m=1$ kg, S=1000 Cal/kg (sp. heat of water)

$$\Delta T=(100-20)=80°\ C$$

and $$P=1\ \text{kw}=1000\ \text{Watt.}$$

$\therefore$ $$t=\frac{1\times1000\times80\times4{\cdot}18}{1000}$$

$$=334{\cdot}4\ \text{s}$$

$$=5\ \text{mt}\ 34{\cdot}4\ \text{s.}$$

Example 5. *How much current will be required by an electric bulb of 60 watt working at 220 V. Calculate the resistance of filament also. If 20% energy is converted into light, calculate the rate of heo produced.*

Solution.

The current drawn by the electric bulb,

$$I=\frac{P}{V}$$

$$=\frac{60}{220}=\frac{3}{11}\ A$$

$\therefore$ The resistance of the filament of the bulb,

$$R=\frac{V}{I}$$

$$=\frac{220}{3}\times11$$

$$=806{\cdot}67\ \Omega$$

The electric energy converted into heat

$$=(100-20)=80\%$$

$$\therefore \quad H=\frac{80}{100}E$$

$$=\frac{4}{5}\times(P\times t)$$

$$\therefore \quad \frac{H}{t}=\frac{4}{5}\times 60$$

$$=48 \text{ J/S}$$

or Heat produced per sec.

$$=\frac{48}{4\cdot 18}=11\cdot 48 \text{ cal/s}$$

Example 6. *Find out the ratio of heat produced in the four arms of wheatstone's bridge shown in Fig. 4.1.*

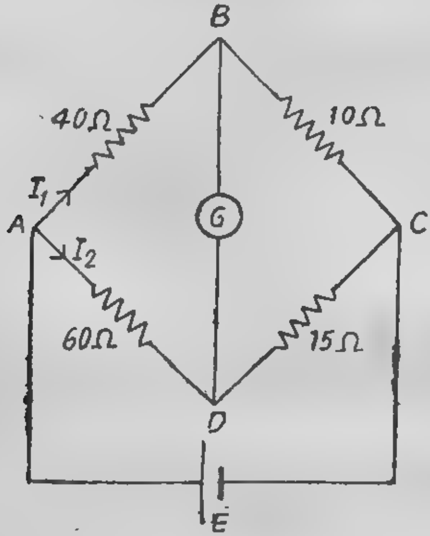


Fig. 4.1.

Solution.

Since $\dfrac{40}{10}=\dfrac{60}{15}$,

the Wheatstone's bridge is balanced and hence

P.D. across AB=P.D. across AD

$$I_1\times 40=I_2\times 60$$

$$\frac{I_1}{I_2}=\frac{60}{40}=\frac{3}{2}=1\cdot 5$$

$\therefore \qquad I_1 = 1.5\, I_2$

Heat produced in arm AB, $H_1 = I_1^2 \times 40 \times t$ Joule

$$= (1.5\, I_2)^2 \times 40\, t$$
$$= 90\, I_2^2\, t$$

Heat produced in arm BC,

$$H_2 = I_1^2 \times 10 \times t$$
$$= (1.5\, I_2)^2 \times 10 \times t$$
$$= 22.5\, I_2^2\, t$$

Heat produced in arm AD,

$$H_3 = I_2^2\, 60\, t$$
$$= 60\, I_2^2\, t$$

Heat produced in arm DC,

$$H_4 = I_2^2 \times 15 \times t$$
$$= 15\, I_2^2\, t$$

$$\therefore \quad H_1 : H_2 : H_3 : H_4 = 90 : 22.5 : 60 : 15$$
$$= 6 : 1.5 : 4 : 1.$$

Example 7. *A dry cell of emf 1.5 V and internal resistance 0.20 Ω is connected across a resistor in series with a very low resistance ammeter. When the circuit is switched on, the ammeter reading settles to a steady value of 2.5 A. What is the*

(*a*) *rate of chemical energy consumption of the cell?*

(*b*) *rate of energy dissipation inside the cell?*

(*c*) *rate of energy dissipation in the resistor?*

(*d*) *power output of the source?*

Solution. (*a*) Rate of chemical energy consumption of the cell

$$= EI$$
$$= 1.5 \times 2.5$$
$$= 3.75 \text{ watt.}$$

(*b*) Rate of dissipation energy inside the cell

$$= I^2 r$$
$$= 2.5^2 \times 0.2$$
$$= 1.25 \text{ watt.}$$

(*c*) Rate of energy dissipation in the resistor

$$= (3.75 - 1.25)$$
$$= 2 \text{ watt.}$$

(*d*) Power output of the source will be the same as the rate of energy dissipation in the resistor = 2 watt.

Example 8. *An electric motor operating on a 200 V d.c. supply draws a current of 15 A. If the efficiency of the motor is 40%, calculate the resistance of the windings of the motor.*

Solution. The rate of energy dissipated as heat

$$
\begin{aligned}
P &= (100-40) \\
&= 60\% \text{ of power consumed} \\
&= \frac{60}{100} \times 200 \times 15 \\
&= 1800 \text{ Watt.}
\end{aligned}
$$

But $\qquad P = I^2R$

$\therefore \qquad I^2R = 1800$

$\therefore \qquad R = \dfrac{1800}{I^2} = \dfrac{1800}{12^2} = 12.5\ \Omega.$

Example 9. *A series battery of 4 lead accumulators each of emf 2.1 V and internal resistance 0.4 Ω is charged by 80 V d.c. supply. What series resistance should be used in charging circuit in order to limit the current to 8.0 A? Using the required resistor, calculate the power supplied by the d.c. source and d.c. energy stored in the battery in 10 minutes.*

Solution. Let R be the resistance in series.

$\therefore$ Charging current, $\quad 8 = \dfrac{80 - 4 \times 2.1}{R + 4 \times 0.4}$

$\therefore \qquad R = 7.35\ \Omega$

$\therefore$ Power supplied by the d.c. source,

$$
\begin{aligned}
P &= V \times I \\
&= 80 \times 8 \\
&= 640 \text{ watt.}
\end{aligned}
$$

Power dissipated as heat,

$$
\begin{aligned}
P' &= I^2(R+r) \\
&= 8^2(7.35 + 4 \times 0.4) \\
&= 64 \times 8.95 \\
&= 572.8 \text{ watt.}
\end{aligned}
$$

$\therefore$ Energy stored in the battery in 10 min,

$$
\begin{aligned}
E &= (P - P')\, t \\
&= (640 - 572.8)(10 \times 60) \\
&= 4.032 \times 10^4 \text{ J}
\end{aligned}
$$

Example 10. *A battery of emf E and internal resistance r is connected across a pure resistance R. Show that the power output of the battery is maximum when R=r. Determine also maximum power output.*

Solution. Power output,

$$P = I^2R$$

$$= \left(\frac{E}{R+r}\right)^2 R$$

$$P = \frac{E^2R}{(R+r)^2} \quad \ldots(1)$$

Now for power 'P' to be maximum $(R+r)^2$ must be minimum.

Now $(R+r)^2 = (R-r)^2 + 4Rr$, will be minimum only when

$$R = r.$$

$\therefore$ Maximum power output from eqn. (1)

$$P_{max.} = \frac{E^2R}{(R+R)^2}$$

$$P_{max.} = \frac{E^2}{4R} \quad \text{or} \quad \frac{E^2}{4r}$$

Example 11. *Show that power output of electric motor is maximum when the back emf is one half the source emf provided the resistance of the windings of the motor is negligible.*

Solution. Since external resistance R is negligible, the current in the motor will be

$$I = \frac{E-E'}{(r+0)}$$

where E=emf of source and E′=back emf.

$$I = \frac{E-E'}{r} \quad \ldots(1)$$

We know power output of a source is maximum when R=r (see example 10) so the current from source through external resistance 'R',

$$I = \frac{E}{R+r} = \frac{E}{r+r} = \frac{E}{2r}.$$

Now since R=0 (the resistance of the winding is negligible) power output of the motor

=power output of the source

$\therefore$ Power output of the motor is maximum if

$$I = \frac{E}{2r}.$$

$\therefore$ Equation (1) becomes

$$\frac{E}{2r}=\frac{E-E'}{r}$$

$\therefore$ $$E'=\frac{E}{2}.$$

Example 12. *An ribbon of Nichrome has length 6·3 cm, width 2·7 mm and thickness 0·05 mm. The respective values for a ribbon of constantan are 8·4 m, 1·2 mm, 0·025 mm. Which of the two ribbons corresponds to a greater rate of heat production, for a fixed voltage supply (Resistivity of nichrome and constantan are $4{\cdot}8\times10^{-7}$ and 10×10^{-7} respectively).*

Solution. Rate of heat production

$$\frac{H}{t}=I^2 R$$

or $$h=\left(\frac{v}{R}\right)^2 R$$

where h = Rate of heat production

or $$h=\frac{v^2}{R}=\frac{v^2 A}{\rho l} \qquad \left[\because \quad R=\rho\frac{l}{A}\right]$$

$\therefore$ $$\frac{h_1}{h_2}=\frac{v^2 A_1/\rho_1 l_1}{v^2 A^2/\rho_2 l_2}$$

$$=\frac{A_1 \rho_2 l_2}{A_2 \rho_1 l_1}$$

$$=\frac{2l_1(b_1+t_1)\ \rho_2\ l_2}{2l_2\ (b_2+t_2)\ \rho_1\ l_1}$$

$$=\frac{(b_1+t_1)\ \rho_2}{(b_2+t_2)\ \rho_1}$$

$$=\frac{(2{\cdot}7+{\cdot}05)10\times10^{-7}}{(1{\cdot}2+{\cdot}025)\ 4{\cdot}8\times10^{-7}}$$

$$=\frac{2{\cdot}75\times10}{1{\cdot}225\times4{\cdot}8}=\frac{27{\cdot}5}{5{\cdot}88}$$

$$=1375:294$$

Example 13. *A fuse wire with radius of 0·12 mm blows at 12A. What should be the radius of another fuse made of the same material which will below at 25A.*

Solution. The temperature of the fuse will increase upto a certain temperature θ such that heat lost by radiation per second.

= Heat produced per second

$$h\times2\pi rl=I^2 R$$

where h is the rate of loss of heat per sec. per unit area

or $$2\pi r l h = I^2 \frac{\rho l}{\pi r^2} \qquad [\because A = \pi r^2]$$

$$r^3 = \frac{\rho}{2\pi^2 h} I^2$$

Both the fuse of the same material will blow at same temperature, so 'h' will remain constant

$$\therefore \quad r^3 = kI^2$$

$$\frac{r_2^3}{r_1^3} = \frac{I_2^2}{I_1^2}$$

$$\therefore \quad r_2 = r_1 \left[\left(\frac{I_2}{I_1}\right)^2\right]^{\frac{1}{3}}$$

$$= 0{\cdot}12 \left[\left(\frac{25}{12}\right)^2\right]^{\frac{1}{3}} = 0{\cdot}12\,[4{\cdot}3403]^{\frac{1}{3}}$$

Now let $$x = 4{\cdot}3403^{\frac{1}{3}}$$

$$\log x = \tfrac{1}{3} \log (4{\cdot}3403)$$

$$= \tfrac{1}{3} \times 0{\cdot}6375 = 0{\cdot}2125$$

$$x = \text{Antilog}\,(0{\cdot}2125)$$

$$= 1{\cdot}631$$

$$\therefore \quad r_2 = 0{\cdot}12 \times 1{\cdot}631 \text{ mm}$$

$$= 0{\cdot}19572 \text{ mm}$$

Example 14. *An electric bulb is marked 200 Watt and 250 V. If the supply voltage drops to 180 V, calculate the light and heat energy produced in 10 minutes.*

Solution. Resistance of the filament of bulb,

$$R = \frac{V}{I}$$

$$= \frac{V}{P/V}$$

$$= \frac{V^2}{P}$$

$$= \frac{250 \times 250}{200}$$

$$= 312{\cdot}5\ \Omega$$

Now bulb is working at 180V, so heat and light energy produced in 10 min or 600 S,

$$H = I^2 R t$$

$$= \left(\frac{V}{R}\right)^2 R t$$

$$= \frac{V^2}{R} t$$

$$= \frac{180 \times 180 \times 600}{312 \cdot 5}$$
$$= 66208 \text{ J}$$

Example 15. *There are two electric bulbs rated 60 W, 110 V and 100 W, 110 V. If they are connected in series with 220 V d.c. supply, (a) find out the bulb, if any, which will fuse ? (b) what will happen if the two bulbs are connected in parallel with the same supply ?*

Solution. (*a*) We know that

$$I = \frac{P}{V} \text{ and } R = \frac{V}{I}$$

$\therefore$
$$I_1 = \frac{60}{110} = \frac{6}{11} \text{ A} = 0 \cdot 55 \text{ A}$$

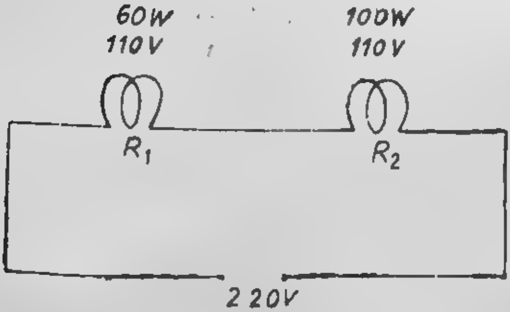


Fig. 4.2.

and
$$R_1 = \frac{V_1}{I_1} = \frac{110}{6} \times 11 = 201 \cdot 6 \ \Omega$$

Now
$$I_2 = \frac{100}{110} = \frac{10}{11} \text{ A} = 0 \cdot 91 \text{ A}$$

and
$$R_2 = \frac{V_2}{I_2} = \frac{110 \times 11}{10} = 121 \Omega$$

$\therefore$ The current through the two bulb in series in the circuit will be

$$I = \frac{V}{(R_1 + R_2)}$$
$$= \frac{220}{(201 \cdot 6 + 121)}$$
$$I = 0 \cdot 68 \text{ A}$$

$\therefore$ We find $I > I_1$ and $I < I_2$.

A bulb get fused if the current which flows through it is more than permissible current (I_1 or I_2).

$\therefore$ The bulb of 60 W will fuse and that of 100 W will light up with less light.

(*b*) Both the bulbs will fuse.

Example 16. *Calculate the strength of the current that will deposit 0·5 g of silver on a spoon in 6 minutes (Given that E.C.E. of Ag=0·001118g c⁻¹).*

Solution. We know

$$m = Z\,I\,t$$

$$0{\cdot}5 = 0{\cdot}001118 \times I \times (6 \times 60)$$

$$\therefore \quad I = 1{\cdot}24 \text{ A}$$

Example 17. *Two voltameters are connected in series and arranged to deposit copper and silver respectively. How much silver will be deposited in the silver voltameter when 0·8 g of copper is deposited in the copper voltameter (Given that chemical equivalent weight=32 for Cu and 108 for Ag).*

Solution.

$$\frac{m_2}{m_1} = \frac{w_2}{w_1}$$

$$\frac{m_2}{0{\cdot}8} = \frac{108}{32}$$

$$\therefore \quad m_2 = 2{\cdot}7 \text{ g}$$

Example 18. *A steady P.D. of 2·05 V is maintained across two platinum electrodes placed in a solution of $CuCl_2$. At the end of 5 minutes, the mass of copper deposited on the cathode is found to be 3·84 g. If the back emf of the voltmeter is 1·29 V, calculate its resistance. (Faraday Constant=96500 c mol⁻¹ and atomic mass of Cu=63·5).*

Solution.

$$Z = \frac{\text{Atomic weight}}{\text{Valency} \times \text{Faraday Constant}}$$

$$= \frac{63{\cdot}5}{2 \times 96500}$$

$$m = Z\,I\,t$$

$$\therefore \quad I = \frac{m}{Z\,t}$$

$$= \frac{3{\cdot}84 \times 2 \times 96500}{63{\cdot}5 \times ({\cdot}5 \times 60)}$$

$$= 38{\cdot}9 \text{ A.}$$

$\therefore$ The resistance of the voltameter,

$$R = \frac{V}{I}$$

$$= \frac{(2{\cdot}05 - 1{\cdot}29)}{38{\cdot}9}$$

$$= 0{\cdot}0195\ \Omega$$

Example 19. *A lamp rated 80W-200 V is connected in series with a copper voltameter. The combination is connected with a 100 volt D.C. supply. If 0·066 g copper is deposited on cathode in 30 minutes. Calculate (a) P.D. across the lamp. (b) rate of energy consumed in the combination. (c) What will be the amount of ions deposited on cathode in next half hour if the lamp is short circuited.*

[*E.C.E. of Cu*$=0{\cdot}00033$ *g* c^{-1}].

Solution.

(*a*)

$$m = Z\,I\,t$$

$$I = \frac{m}{Z\,t}$$

$$= \frac{0{\cdot}066}{0{\cdot}00033 \times (30 \times 60)}$$

or

$$I = \frac{1}{9}\ \text{A}$$

The resistance of the filament of lamps

$$R = \frac{V^2}{P}$$

$$= \frac{200 \times 200}{80}$$

or

$$R = 500\ \Omega$$

$\therefore$ P.D. across the lamps, $V' = R\,I$

$$= 500 \times \frac{1}{9}$$

$$= 55{\cdot}55\ \text{Volts}$$

(*b*)

$$P = V\,I$$

$$= 100 \times \frac{1}{9}$$

$$= 11{\cdot}11\ \text{watts.}$$

(*c*) The P.D. across voltameter

$$= (V - V')$$

$$= (100 - 55{\cdot}55)$$

$$= 44{\cdot}45\ \text{V}$$

$\therefore$ Internal Resistance of voltameter

$$r = \frac{44{\cdot}45}{1/9}$$

$$r = 400\ \Omega$$

Now if the lamp is short circuited,

$$I' = \frac{V}{R} = \frac{100}{400} = 0{\cdot}25\ \text{A}$$

$$\therefore \quad m' = Z\, I'\, t'$$
$$= 0{\cdot}00033 \times 0{\cdot}25 \times (\tfrac{1}{2} \times 60 \times 60)$$
$$= 0{\cdot}1485 \text{ g.}$$

Example 20. *A hollow tube of copper of internal curved surface area 150 cm². is to be electroplated with 0·01 mm thick silver layer. A battery of 12 volts and internal resistance 0·5 Ω is connected with the voltmeter and a resistance of 3 5 Ω all in series. If the P.D. across the voltameter remains constant at 10 V, calculate the time required to deposit this layer of silver. (Density of silver = 10·5 g cm⁻³ and E.C E. of silver = 0·001118 g c⁻¹).*

Solution. If 'R_0' be the resistance of the voltameter,

$$I = \frac{E}{(R + r)}$$

$$\therefore \quad I = \frac{12}{(3{\cdot}5 + R_0 + 0{\cdot}5)} = \frac{12}{(4 + R_0)} \qquad \text{...(1)}$$

For voltameter alone,

$$V = I\, R_0$$
$$10 = I\, R_0$$

$\therefore$ From equation (1),

$$10 = \frac{12}{(4 + R_0)}\, R_0$$

$$\therefore \quad R_0 = 20\ \Omega$$

$\therefore$ By Equation (1)

$$I = \frac{12}{(4 + 20)}$$
$$= \frac{12}{24}$$
$$I = 0{\cdot}5 \text{ A}$$

mass of silver ions deposited,

$$m = V \times \rho$$
$$= (A \times x)\, \rho$$
$$= (150 \times 0{\cdot}001)\, 10{\cdot}5$$
$$m = 1{\cdot}575 \text{ g}$$
$$Z = 0{\cdot}001118 \text{ g } c^{-1}$$

$$\therefore \quad m = Z\, I\, t$$

$$\therefore \quad t = \frac{m}{Z\, I}$$
$$= \frac{1{\cdot}575}{0{\cdot}001118 \times 0{\cdot}5}$$
$$= 2818 \text{ S}$$
$$= 46 \text{ mt. } 58 \text{ S.}$$

Example 21. *Near room temperature, the thermo emf of copper-constantan couple is 40 $\mu V\ ^\circ C^{-1}$. What is the smallest temperature difference that can be detected with single such couple and a galvanometer of 80 Ω resistance couple of detecting currents as low as 10^{-6} A.*

Solution. Least P.D. that can be detected with the galvanometer,

$$V = R\,I$$
$$= 80 \times 10^{-6} \text{ Volts.}$$

and $$V_0 = 40\ \mu V = 40 \times 10^{-6} \text{ volts}$$

$\therefore$ Smallest temp. difference detectable

$$= \frac{V}{V_0}$$
$$= \frac{80 \times 10^{-6}}{40 \times 10^{-6}}$$
$$= 2^\circ C$$

Example 22. *In a given thermo couple of Cu-Fe, the temperature of inversion is 580°C. When the temperature of the cold junction is 20°C. Calculate the neutral temperature of the thermocouple.*

Solution. $\theta_c = 22^\circ C$, $\theta_i = 580^\circ C$

$\therefore$ Neutral temperature,

$$\theta_n = \frac{(\theta_i + \theta_c)}{2}$$
$$= \frac{(580+20)}{2}$$

$\therefore$ $$\theta_n = 300^\circ C$$

Example 23. *The emf of a copper-constantan thermocouple, one junction of which is kept at 0°C, is given by*

$$E = \alpha\theta + \tfrac{1}{2}\beta\theta^2$$

Calculate (a) the neutral temperature (b) Peltier cofficient and Thomson coefficient at 800°C. ($\alpha = 41 \mu V^\circ C^{-1}$ & $\beta = 0.041\ \mu V^\circ C^{-2}$).

Solution. $\theta^\circ C = (T - 273)$ K

$\therefore$ $E = \alpha\,(T-273) + \tfrac{1}{2}\beta\ (T-273)^2$

$\therefore$ $$\frac{dE}{dT} = \alpha + \beta\ (T-273) \qquad ...(1)$$
$$= \alpha + \beta\theta$$

At neutral temp.,

$$\frac{dE}{dT} = 0$$

$\therefore$ $$\alpha + \beta\theta = 0$$

$$\therefore \quad \theta = -\frac{\alpha}{\beta}\,^\circ C$$

$$= -\frac{41}{0.041}$$

$$|\theta| = 1000^\circ C$$

Peltier coefficient,

$$\pi = T\frac{dE}{dT}$$

$\therefore$ **From eqn. (1),**

$$\pi = T\,[\alpha + \beta\,(T - 273)]$$
$$= (\theta + 273)(\alpha + \beta\theta)$$
$$= (800 + 273)(41 + 0.041 \times 800)$$
$$= 1073 \times 73.8$$
$$= 79187.4\ \mu V$$
$$= 0.0791874\ V$$

Thomson coefficient,

$$\rho = T\frac{d^2E}{dT^2}$$
$$= T\beta$$
$$= (800 + 273) \times 0.041$$
$$= 43.993\ \mu V\,^\circ C^{-1}$$

EXERCISE 4

1. An electric heater is rated 2Kw. If the voltage is 250 V, what is the value of current and resistance of the element of heater ? How much time is required to boil 1 kg of water at 10°C ?
2. An electric heater consists of 20 m length of mangnin wire of 0.23 mm^2 cross–sectional area. Calculate the voltage of the heater when the P.D. across the heater is 200 V.
   (Resistivity of mangnin $= 4.6 \times 10^{-7}\ \Omega m$) [*D.B.S.S.E. 1980*]
3. An electric bulb is rated 250 V and 100w. (*a*) What is the resistance of the bulb ? (*b*) What will be the cost of lighting this bulb for 4 hours daily in the month of April ? The electricity costs 30 paisa per unit.
4. An electric radiator has a resistance of 20 Ω. What will be loss of heat radiation in calories per minute if it works at 210 volt d.c. supply ($J = 4.2\ J\ cal^{-1}$).
5. An electric iron is rated 220 V, 1Kw. What is the resistance of iron when it is hot If the electricity cost 30 paise per unit, for how many hours the iron can be used in 5 rupees ?
6. An electric radiator consumes 2Kw power at 220 volts. Calculate (*a*) the current drawn (*b*) the resistance of the radiator

(*c*) the cost of using it for 5 hours at the rate of 30 paise per unit.

7. A 2 H.P. electric motor, 660 watt heater and an electric iron which draws 5A current all are used at 220 volts. (*a*) How much total current all the 3 draw ? (*b*) What is the cost for running them for 4 hrs at the rate of 25 paise per unit ?

8. A heater of 500W, 220 V is used to boil 750 g of water at 20°C. If 40% of energy goes waist, how much time will be required for the job (J=4·2 J/cal).

9. 60 Ω resistance immerson rod is dipped in 4·20 kg of water. How much temperature will rise in 1 minute if it draws 7A current from the supply ? (J=4.2 J/cal).

10. An electric kettle of 1 kw, 250 V is used to prepare 2 cups of tea by boiling 450 g of water at 20°C. (*a*) How much time will be required for the job ? (*b*) What is the cost if the rate of electricity is 20 paisa per unit ?

11. How can you use safely 40 W, 40 V bulbs (which is common in railway compartment) at your domestic supply line of 220V ?

12. The two lamps L_1 and L_2 are connected in 2 different ways with the power supply of 220 V as shown in figure 4.3 : (*a*) and (*b*).

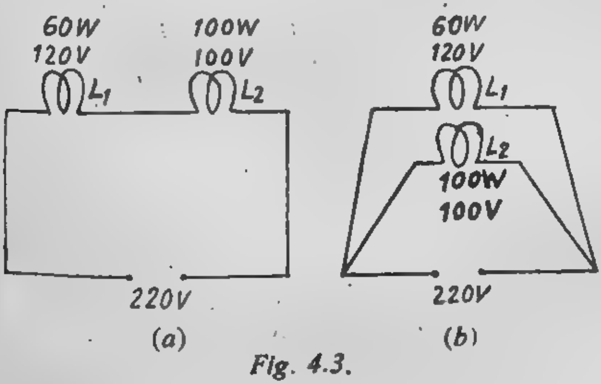


Fig. 4.3.

Make the necessary calculations to find out in each case if any of the bulb will fuse ? What will happen if the two bulbs are connected in parallel with a supply of 120 V ?

13. A tap supplies water at 27°C. We require a continuous supply of water at the temperature of human body (98·0°F or 37°C) at the rate of 1 kg of water per minute (*a*) How much power will be consumed by an electric geyser joined with water pipeline ? (*b*) How much current will be drawn by its element if works at 210 Volt supply ? [*I.I.T. 1964*]

14. The two bulbs rated 60 W, 220 V and 40 W, 220 V are connected in series with 220 Volt supply. Calculate the rate of heat produced in each bulb. (J=4·2 J/cal).

15. Find out the rate of heat production in the four arms of Wheatstone's bridge shown in Fig. 4.4.

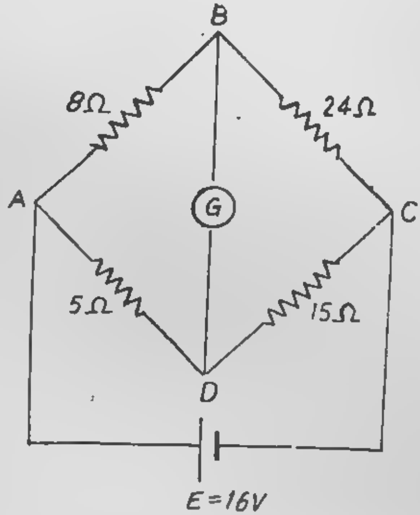


Fig. 4.4.

16. A generator generates power at 220 V and supplies it to a village at a far distance by transmission lines. The village receive the power at 200 V and requires only 40 A current. Calculate (*a*) the resistance of transmission line (*b*) the energy consumed (in kwh) in village in 10 hours and (*c*) the energy waisted in the transmission line in the same time.

17. A generator is supplying power to a factory by the cables of the resistance 33 Ω. If it generates 100 kw power at 600 Volts, what is the power and P.D received by the factory ?

18. If 25% energy in an electric bulb rated 100 watt 250 V converts into light energy, calculate the rate of heat produced.

19. An electric motor of 3 H.P. works at 210 V for 8 hours daily. It delivers 1500 watt mechanical power (*a*) What is the efficiency of the motor ? (*b*) How much energy is lost as heat per second ?

20. An electric motor of 1·5 H.P. raises water from a tank of 1 m × 1 m × 1 m full of water to a reservoir 10 m high in 1 minute (*a*) What is the efficiency of the motor ? (*b*) How much energy is lost as heat in 2 minutes ?

21. A dry cell of emf 6·0 Volt and internal resistance 0·40 Ω is connected across a resistor in series with an ammeter of very low resistance. The ammeter reading settles down to a steady value of 5·2 A. What is

(*a*) the rate of chemical energy consumption of the cell ?
(*b*) the rate of energy dissipated inside the cell ?
(*c*) the rate of dissipation of energy in the resistor ?
(*d*) the power output of the source ?

22. If $E=10$ V, $r=0.5\ \Omega$ and $I=7.1$ A respectively in above Question Number 21, calculate the values from (*a*) to (*d*).

23. A series battery of 5 lead accumlators each of emf 2 00 V and internal resistance $0.3\ \Omega$ is charged by a 120 V d.c. supply. What series resistance should be used in charging circuit in order to limit the current of 8 A ? Using the required resistor, calculate the power supplied by d.c. source and d.c. energy stored in the battery in 12 minutes.

24. If in above Question Number 23, $N=6$, emf$=1.8$ V, $r=0.25\Omega$, $V=100$ Volts, $I=9.0$ A and $t=15$ minute respectively. Calculate all the quantities as in Q. No. 23.

25. An electric motor operating at 220 V d.c. supply draws a current of 11A. If the efficiency of the motor is 70%, calculate the resistance of the winding of the motor.

26. If in above Question Number 25, $V=210$ Volts, $I=10.5$ A, $\eta=60\%$, calculate the resistance and drop of the potential of the windings of the motor.

27. A battery of emf 1.5 V and internal resistance $0.2\ \Omega$ is connected across a pure resistance. Calculate the maximum power output.

28. A d.c. battery of emf 100 V is connected across an electric motor whose windings has negligible resistance. What should be the value of back emf so that the power output of electric motor is maximum ?

29. An electric bulb is marked 100 W and 240 V. If the supply voltage drops to 150 V, calculate the heat and light energy produced in 15 minutes.

30. Two fuse wires of the same material blow at 8A and 27A. What is the ratio of the diameters of the two wire ?

31. A fuse wire with radius 0.09 mm blows at 10 A. What current will fuse another wire of the same material of radius 0.16 mm ?

32. Find the strength of current which deposits 0.777 g of metallic copper in 30 minutes in a copper voltameter. (The E.C.E. of copper$=0.000332\ gC^{-1}$).

33. A silver ornament weighing 200 g is to be electroplated with 2.0% of its weight by gold If the current is 2 A and E.C.E. of gold is $0.00068\ gC^{-1}$, how long will it take to deposit the required weight of gold ?

34. A current of 5 A is passed through a silver voltameter for 1 hour and 40 minutes. Find the inerease in the weight of the cathode. (Equivalent weight of silver is 108 and E.C.E. of hydrogen is ·0000103 gC^{-1}).

35. A steady current was passed for 25 minutes through a silver voltameter and ammeter in series when 0·6708 g of silver was deposited. The ammeter reads 0·45 A. Find the error if any in the reading of the ammeter (Equivalent weight of silver =108 and Faraday constant=96500 C $mole^{-1}$).

36. A spoon has its total surface area equal to 10 cm^2. Calculate the thickness of silver layer deposited on it in 4 hour 48 S in a silver voltameter when the current flowing through it is 1·2 A (E.C.E. of silver 0·001118 and density of silver $r=10·5$ g cm^3).

37. A battery of e m f. 1·2 V and internal resistance 0·1 Ω is connected with a resistor of 5 Ω and a silver voltameter in series. If the mass of silver deposited on cathod in $\frac{1}{2}$ hour is 0·18 g, calculate (*a*) the resistance of the voltameter and (*b*) the number of cells required to make the current in the voltameter double. How will you connect these cells ?

38. Three voltameters are connected in series. If 0·5 g copper is deposited in copper voltameter, how much silver will be deposited in silver voltameter and how much hydrogen and oxygen will be deposited in water voltameter in the same time by the same current ? (E.C.E. of copper, silver, hydrogen and oxygen respecively are 31·5, 108, 1, 8).

39. A steady current is passed for a certain time through three voltmeters (Cu electrodes in $CuSO_4$), a silver voltameter (silver electrodes in $AgNO_3$) and an iron voltameter (iron electrodes in $FeCl_3$). The mass of iron deposited on the cathode of iron voltameter is found to be 34 6 g. Calculate the masses of copper and silver deposited on the respective cathode of the other two voltmeter during the same time (At. wt. of Cu, Ag, and Fe are 63·4, 108 and 56 respectively).

40. A steady P.D. of 1·8 V is maintained across two platinum electrodes in the solution of silver nitrate. The mass of silver deposited on the cathode is 8 minutes is found to be 1·128 g. If the back e.m.f. of the voltameter is 0·8 V, calculate its resistance (Atomic weight of silver=108 and Faraday constant =96500 C mol^{-1}).

41. A lamp rated 100 W and 240 V is connected in series with a copper voltameter. The combination is connected with a 150 volt d.c. supply. If 0 088 g copper is deposited on cathode in 40 minutes, calculate (*a*) P.D. across the lamp (*b*) the rate of energy consumed in the combination and (*c*) what will be the amount of ions deposited on cathode in next 20 minutes

if the lamp is short circuited. (E.C.E. of of copper $=0.00033$ gC^{-1}).

42. Near room temperature, the thermo e.m.f of copper constant couple is $40\ \mu V°C^{-1}$. What is the smallest temperature difference that can be detected with single such couple and a galvanometer of resistance 200 Ω capable of detecting currents as low as 1 μA.

43. In a given thermocouple of Fe-Cu, the temperature of cold junction is 10°C while the neutral temperature is 300°C. What is the temperature of inversion ?

44. In a given thermo couple the temperature of inversion is 520°C. What is the neutral temperature if the temperature of the cold junction is 20°C ?

45. The junctions of a thermocouple are maintained at 0°C and 300°C. Find the thermo e.m f. in the couple if it is given by

$$E=\alpha\theta+\beta\theta^2$$

($\alpha=122\ \mu V°C^{-1}$ and $\beta=0.04\ \mu V°C^2$)

46. A copper wire having cross sectional area of $0.5\ mm^2$ and length of 0·1 m is initially at 25°C and is thermally insulated from its surrounding. If a current of 10 A is set up in the wire, (*a*) find the time in which wire will start melting. The change in resistance with temperature may be neglected (*b*) what will the time be, if the length of the wire is doubled. (Given density of copper $=9\times10^3\ kgm^{-3}$, sp. heat $=0.09$ kcal $kg^{-1}°C^{-1}$, melting point $=1075°C$ and resistivity $=1.6\times10^{-8}$ Ωm). (*I.I.T. 1979*)

47. The emf. of a thermocouple, one junction of which is kept at 0°C is given by

$$E=\alpha\theta+\beta\theta^2.$$

Find out the value of neutral temperature and the value of Peltier and Thomson coefficients.

OBJECTIVE TYPE QUESTIONS

48. The coil of a heater is cut into two equal halves. If one of them is used as heater, the heat produced will become :

(*a*) half (*b*) double
(*c*) one-fourth (*d*) four times.

49. The wire of resistance 20Ω is stretched to double its length. I ts new resistance will be

(*a*) 10Ω (*b*) 20Ω
(*c*) 40Ω (*d*) 80Ω.

50. Two wires of resistance R_1 and R_2 are in series in a circuit. If $R_2 > R_1$, the heating would be more across :

(a) R_2 (b) R_1
(c) Same for both (d) can not be decided.

51. If the current is flowing through a 10 ohms resistor, then indicate in which case maximum heat will be generated :

(a) 5 amp. in 2 minutes (b) 4 amp. in 3 minutes
(c) 3 amp. in 6 minutes (d) 2 amp. in 5 minutes.

52. The amount of charge in coulombs required to deposit one gm. equivalent in electrolysis is :

(a) 48×10^{-10} (b) 9608
(c) 96490 (d) 6×10^{23}

53. A bulb is marked 220 V and 60 watt. Its filament when glowing has resistance :

(a) $220 \times 60\Omega$ (b) $220 \div 60\Omega$
(c) $(220)^2 \times 60\Omega$ (d) $(220)^2 \div 60\Omega$

54. If 0·1 amp. current flow for 10^{-6} S through $CuSO_4$ solution, the number of Cu ions collected is about :

(a) 6×10^{11} (b) 3×10^{11}
(c) $1{\cdot}5 \times 10^{10}$ (d) $0{\cdot}75 \times 10^{10}$

55. In the following circuit if heat evolved in 10Ω resistance is 10 calories. The heat evolved across 4Ω resistance will be :

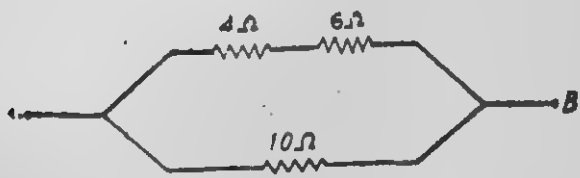


Fig. 4.5.

(a) 10 cal (b) 4 cal
(c) 5 cal (d) 40 cal.

56. If the two wires of resistance R_1 and R_2 are connected in parallel in a circuit and $R_1 > R_2$, the heating would be more across

(a) R_1 (b) R_2
(c) same for both (d) can not be decided.

57. If the diameter of a wire is doubled, its resistance will become :
(*a*) half (*b*) double
(*c*) one fourth (*d*) four times.

58. The power supplied by a battery of internal resistance 'r' to a pure external resistance 'R' will be maximum if :
(*a*) $R > r$ (*b*) $r > R$
(*c*) $R = r$ (*d*) can not be decided.

59. The maximum power supplied by a battery of emf 'E' and internal resistance 'r' to a pure external resistance is :
(*a*) E^2/r (*b*) $E^2/2r$
(*c*) $E^2/3r$ (*d*) $E^2/4r$.

60. An electric bulb is marked 100 W, 250V. It will get fused if it is given a voltage :
(*a*) 300V (*b*) 250V
(*c*) 200V (*d*) 150V.

61. How many minimum no. of bulbs each marked 60W, 40V connected in series with 240V mains supply can work safely ?
(*a*) 2 (*b*) 4
(*c*) 6 (*d*) 8.

□□

UNIT 5

Magnetic Effect of Current

IMPORTANT FORMULAE

1. **Biot Sovard Law—The magnetic field dB due to a current element dl, at a point P, distance r from it is given by**

$$dB=\frac{\mu_0 I}{4\pi}\frac{dl \sin\theta}{r^2}$$

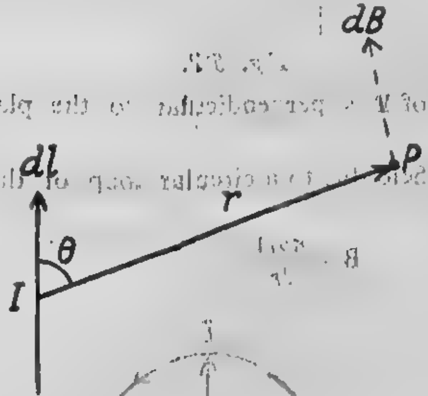


***Fig.* 5·1.**

where μ_0 = Permeability of the medium = $4\pi\times10^{-7}$ T-A^{-1} for vacuum or air

I = the current through the element

θ = Angle between the direction of dl & r.

The direction of dB is perpendicular to the plane containing dl & r.

2. **The magnetic field due to a straight (infinitely) long current carrying conductor I at a point P, perpendicular distance r away from it is given by**

$$B=\frac{\mu_0 I}{2\pi r}$$

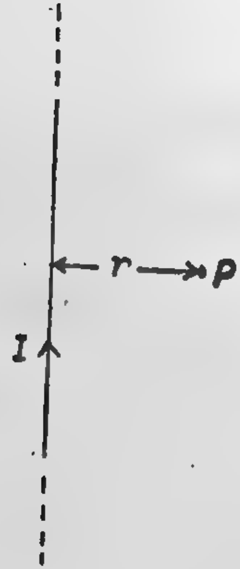


Fig. 5·2.

The direction of B is perpendicular to the plane containing $\vec{I}I$ and r.

3. The magnetic field due to a circular loop of the wire at the centre, P,

$$B = \frac{n\mu_0 I}{2r}$$

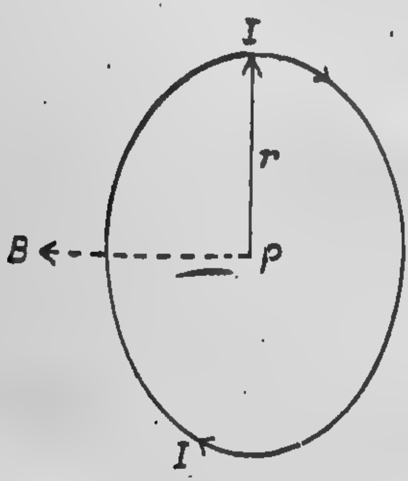


Fig. 5·3.

n=No. of turns in the loop
r=radius of the circular loop.

The direction of B is perpendicular to the plane of the loop.

4. The magnetic field at a point P along the axis of a circular loop of the wire,

$$B=\frac{n\mu_0 I r}{4\pi(r^2+a^2)^{3/2}}$$

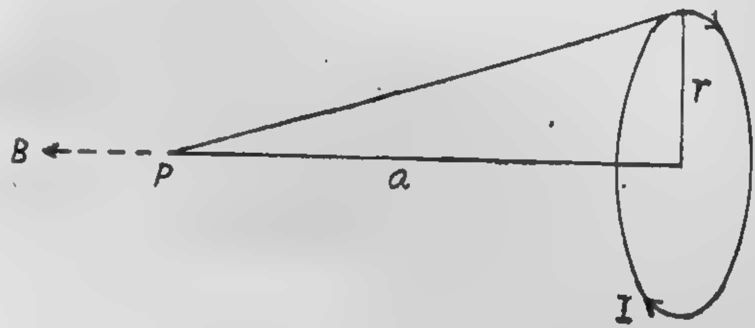


Fig. 5·4.

5. Magnetic field due to a solenoid (*a* long coil)

$$B=\mu_0 n I$$

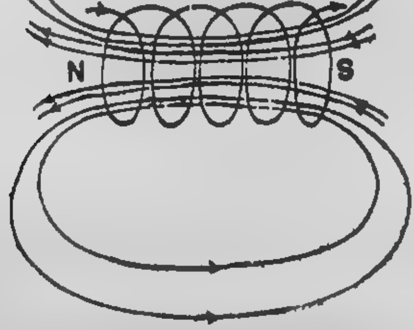


Fig. 5·5.

n=No. of turns per unit length.

6. **Force on a conductor of length l, carrying a current I and placed in a magnetic field B is given by**

$$F = IlB \sin \alpha$$

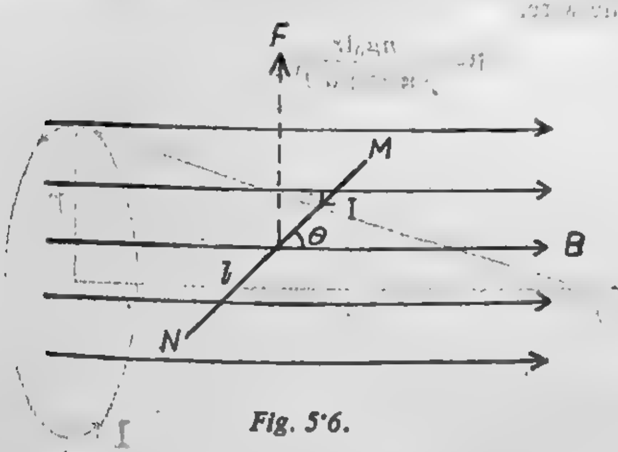


Fig. 5·6.

where α = Angle between the directions of I and B = (180 − θ)

The direction of F is perpendicular to the plane containing I and B.

7. The current through a conductor of cross section 'A', is given by

$$I = enAv$$

where e = Charge on an electron

n = No. of free electrons per unit volume

and v = velocity of electrons

8. The force of attraction or repulsion per unit length between two straight conductors carrying the current I_1 and I_2 at a distance r from each other is given by

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{r}$$

9. The force on a charge particle 'q' moving with velocity 'v' in a magnetic field 'B', $F = q\,vB \sin \theta$; θ = Angle between v and B.

10. Lorentz force (the total force) on a charge q moving with a velocity v in an electric field E and magnetic field B is :

$$F = q\,(E + vB)$$

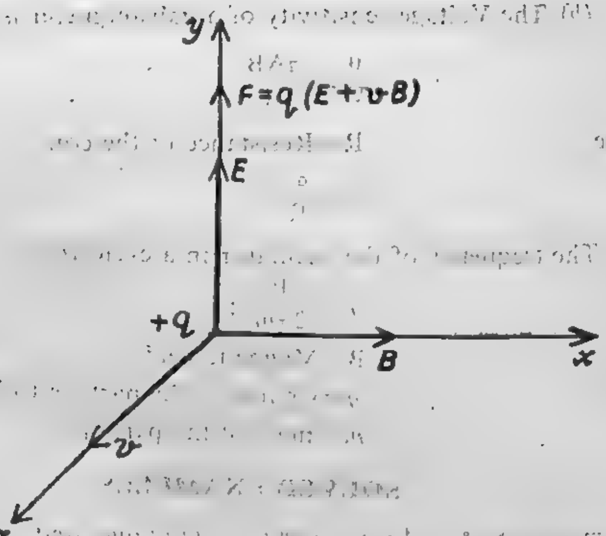


Fig. 5·7.

11. The magnetic dipole moment m of a circular loop of area A and carrying the current I is given by

$$m=\mathrm{IA}$$

12. The torque τ on a loop of wire of area A carrying a current I and placed in a magnetic field B,

$$\tau=n\mathrm{IBA}\sin\theta\ ;$$

θ = Angle which the normal to the loop makes with the field.

and n = No. of turns in the loop.

13. (*a*) The current sensitivity of a galvanometer is given by

$$\alpha=\frac{\theta}{\mathrm{I}}=\frac{n\mathrm{AB}}{\mathrm{K}}$$

where K = Torsion constant of the wire or the spring *i.e.*, restoring torque per unit radian twist.

θ = Angle by which the coil turns

I = Current through the coil

A = Area of each turn in the coil

n = No. of turns in the coil

B = Magnetic field intensity.

(*b*) The Voltage sensitivity of a galvanometer is given by

$$\frac{\theta}{V}=\frac{nAB}{KR}$$

where R=Resistance of the coil

$$=\frac{a}{R}$$

14. The frequency of the oscillator in a cyclotron,

$$f=\frac{Bq}{2\pi m};$$

B=Magnetic field

q=Charge of the particle to be accelerated

m=mass of the particle.

SOLVED EXAMPLES

Example 1. *A vertical wire is quite long and carries a current of 2A. (a) Find the magnitude and direction of the magnetic field produced at a point 5 cm away from the wire. (b) What force does the magnetic field exert on an electron at the same point moving with a velocity 100 kms⁻¹ parallel to the wire.*

Solution. (*a*) $\mu_0=4\pi\times10^{-7}$ TmA^{-1},

I=2A, r=5 cm.=0·05 m

$$\therefore \quad B=\frac{\mu_0 I}{2\pi r}$$

$$=\frac{4\pi\times10^{-7}\times2}{2\pi\times\cdot05}$$

or $B=8\times10^{-6}$ Tesla, in a direction $\perp$ to the plane containing the wire and the point.

(*b*) $F=qv.\,B\sin\theta$

Now $q=1\cdot6\times10^{-19}$ c,

v=100 Kms^{-1}=10^5 ms^{-1} and $\theta=90°$

$$\therefore \quad F=1\cdot6\times10^{-19}\times10^5\times8\times10^{-6}\times\sin 90$$

$$=12\cdot8\times10^{-20}\text{ N}$$

Example 2. *A horizontal wire 0·1 long carries a current of 5A. Find the magnitude and direction of the magnetic field which can support the weight of the wire of mass 3 g.*

Solution. $F=IlB$

$F=5\times0\cdot1\times B$

$=0\cdot5$ B Newton

$$W = mg$$
$$= (3 \times 10^{-3})\,(9{\cdot}8)$$
$$W = 29{\cdot}4 \times 10^{-3} \text{ Newton}$$

But given $\quad F = W$

$\therefore \quad 0{\cdot}5B = 29{\cdot}4 \times 10^{-3}$

$$B = 0{\cdot}0588 \text{ Tesla.}$$

Example 3. *Two cocentric circular coils A and B of radii 12 cm and 8 cm respectively lie in the same vertical plane containing east-west direction. The coil A has 30 turns and carries a current of 10A and coil B has 50 turns and carries a current of 14A. An observer looking at the two coils from north finds that the current in coil A is clockwise and that in B is anti clockwise. Calculate the magnitude and direction of the resultant magnetic field at their common centre.*

Solution. $\quad B = \dfrac{n\mu_0 I}{2r}$

$\therefore$ Field at the centre of coil 'A'

$$B_1 = \frac{30 \times (4\pi \times 10^{-7})\ 10}{2 \times 0{\cdot}12}$$
$$= 5\pi \times 10^{-3} \text{ T towards south.}$$

Similarly field at the centre of coil 'B'.

$$B_2 = \frac{50 \times (4\pi \times 10^{-7}) \times 14}{2 \times {\cdot}08}$$
$$= 1{\cdot}75\pi \times 10^{-3} \text{ T towards north.}$$

Since the centre of the two coils A and B is at the same point, the resultant field at this point (common centre) will be :

$$B = B_1 - B_2$$
$$= (5\pi \times 10^{-3} - 1{\cdot}75\pi \times 10^{-3})$$
$$= 3{\cdot}25\pi \times 10^{-3}$$
$$= 3{\cdot}25 \times 3{\cdot}14 \times 10^{-3}$$
$$= 10{\cdot}205 \times 10^{-3} \text{ T towards south.}$$

Example 4. *A magnetic field of 200 Gauss is required which is uniform in a region of linear dimension about 12 cm and area of cross section 10 cm^2. The maximum current carrying capacity of a given solenoid is 20 A and the maximum number of turns per unit length that can be wound round its nonferromagnetic core is at the most 1000 turns m^{-1}. What should be the particulars of the solenoid for the required purpose ?*

Solution. There may be number of set of the particulars of

the solenoid for the required purpose, *e.g.*, one of the particulars may be :

1. No. of turns per unit length,

$$n = 900 \text{ turns m}^{-1}$$

2. Then $\quad I = \frac{B}{\mu_0 N} \qquad [\because B = \mu_0 nI]$

$$= \frac{200 \times 10^{-4}}{(4\pi \times 10^{-7}) \times 900} \qquad [\because 1 \text{ Gauss} = 10^{-4} \text{ Tesla}]$$

$$= 17.6 \text{ A.}$$

$\therefore$ The current in the solenoid $\simeq 18$ A.

3. The length of the solenoid $= 5 \times 12 \simeq 60$ cm.

4. $\quad r = \sqrt{\frac{A}{\pi}} \qquad [\because A = \pi r^2]$

$$= \sqrt{\frac{10 \times 7}{22}}$$

$$= 1.78 \text{ cm}$$

$\therefore$ The radius of the solenoid $\simeq 2$ cm.

Example 5. *Consider two parallel coaxial circular coils of equal radius 'r' and number of turns 'n', carrying equal current 'I' in the same direction and separated by a distance 'r'. Calculate the field around the mid point between the coils over a distance that is small compared to 'r'.*

Solution. The field in a small region of the length $2l$ about the mid point between the coils,

$$B = \frac{n\mu_0 I r^2}{2(r^2 + a^2)^{3/2}}$$

$$= \frac{n\mu_0 I r^2}{2}\left[\frac{1}{\left\{r^2 + \left(\frac{r}{2} + l\right)^2\right\}^{3/2}} + \frac{1}{\left\{r^2 + \left(\frac{r}{2} - l\right)^2\right\}^{3/2}}\right]$$

$$B = \frac{n\mu_0 I r^2}{2}\left[\frac{1}{\left(r^2 + \frac{r^2}{4} + rl + l^2\right)^{3/2}} + \frac{1}{\left(r^2 + \frac{r^2}{4} - rl + l^2\right)^{3/2}}\right]$$

neglecting l^2 as l is very small,

$$B = \frac{n\mu_0 I r^2}{2} \frac{1}{\left(\frac{5r^2}{4}\right)^{3/2}}\left[\frac{1}{\left(1 + \frac{4l}{5r}\right)^{3/2}} + \frac{1}{\left(1 - \frac{4l}{5r}\right)^{3/2}}\right]$$

$$=\frac{n\mu_0 I r^2}{2}\,\frac{8}{5\sqrt{5}r^3}\left[\left(1+\frac{4l}{5r}\right)^{-3/2}+\left(1-\frac{4l}{5r}\right)^{-3/2}\right]$$

$$=\frac{4n\mu_0 I}{5\sqrt{5}r}\left[\left(1-\frac{3}{2}\times\frac{4}{5}\,\frac{l}{r}\right)+\left(1+\frac{3}{2}\times\frac{4}{5}\,\frac{l}{r}\right)\right]$$

neglecting $\left(\frac{l}{r}\right)^2$ and higher terms as $\left(\frac{l}{r}\right)<<<1.$

$$=\frac{8}{5\sqrt{5}}\,\frac{n\mu_0 I}{r}$$

$$\therefore\ B=0{\cdot}72\,\frac{n\mu_0 I}{r}$$

Example 6. *A toroid has a non-ferromagnetic core of inner radius 20 cm and outer radius 21 cm around which 2000 turns of a wire are wound. If the current in the wire is 10 A, calculate the magnetic field (a) outside the toroid, (b) inside the core of the toroid, (c) in the empty space surrounded by the toroid.*

Solution. (*a*) The field outside the toroid is found to be zero.

(*b*) The field inside the core of a toroid is given by

$$B=\frac{\mu_0 n I}{2\pi r}.$$

where r=Mean radius of core

$$=\frac{(4\pi\times10^{-7})\times2000\times10}{2\pi\left(\frac{20+21}{2}\right)}$$

$$=\frac{40\times10^{-4}}{20{\cdot}5}$$

$$=1{\cdot}95\times10^{-4}\text{ T.}$$

(*c*) The field in the empty space of the toroid is zero.

Example 7. *Two wires A and B have the same length equal to 40 cm, and carry a current of 12 A each. Wire A is bent into a circle and wire B is bent into a square (a) which wire produces a greater magnetic field at the centre. (b) calculate the magnetic field in each case at the centre.*

Solution. The magnetic field due to the current in a conductor of length l at a point equidistant from its ends at a distance 'x' from its centre is given by

$$B=\frac{\mu_0 I}{2\pi x}\cdot\frac{l}{(l^2+4x^2)^{1/2}}$$

$\therefore$ Field at the centre of a square wire of side l will be :

$$B=4\times\frac{\mu_0 I}{2\pi\left(\frac{l}{2}\right)}\frac{l}{\left[l^2+4\left(\frac{l}{2}\right)^2\right]^{1/2}}$$

$$=\frac{4}{\sqrt{2}}\frac{\mu_0 I}{\pi l}$$

$$B=2\sqrt{2}\frac{\mu_0 I}{\pi l}$$

or $$B=\frac{8\sqrt{2}\,\mu_0 I}{\pi\,(4l)}$$

or $$B=\frac{8\sqrt{2}}{\pi}\frac{\mu_0 I}{L}$$

where L = Total length of the wire

Magnetic field at the centre of the circular wire of radius 'r',

$$B'=\frac{\mu_0 I}{2r}$$

$$=\frac{\mu_0 I\pi}{2\pi r}$$

or $$B'=\frac{\pi\mu_0 I}{L}$$

$\therefore$ $$\frac{B}{B'}=\frac{8\sqrt{2}}{\pi^2}$$

$$=\frac{8\times 1{\cdot}414}{3{\cdot}14\times 3{\cdot}14}$$

or $$\frac{B}{B'}=\frac{11{\cdot}312}{1{\cdot}8596}$$

$\therefore$ $B>B'$.

(*b*) (*i*) Field at the centre of square wire,

$$B=\frac{8\sqrt{2}}{\pi}\frac{\mu_0 I}{L}$$

$$=\frac{8\sqrt{2}}{\pi}\frac{(4\pi\times 10^{-7})12}{0{\cdot}40}$$

$$=96\sqrt{2}\times 10^{-6}$$

$$=1{\cdot}358\times 10^{-4}\text{ T}.$$

(*ii*) Field at the centre of the circular wire,

$$B'=\frac{\pi\,\mu_0 I}{L}$$

$$=\frac{3.14\,(4\times 3.14\times 10^{-7})\times 12}{0.40}$$

$$=1.183\times 10^{-4}\text{ T.}$$

Example 8. *An electron emitted by a heated cathode and accelerating through a potential difference of 2KV enters in a uniform magnetic field 0·1 T. Determine the trajectory of the electron if the field (a) is acting perpendicular to the initial velocity (b) makes an angle of 30° with the initial velocity.*

Solution. Energy gained by an electron in an electric field,

$$\tfrac{1}{2}mv^2=eV$$

$$\therefore \qquad v=\sqrt{\frac{2eV}{m}}$$

$$=\sqrt{\frac{2\times 1.6\times 10^{-19}\times 2\times 10^3}{9\times 10^{-31}}}$$

$$=\frac{8}{3}\times 10^7\text{ ms}^{-1}$$

(*a*) Here $\theta=90°$, so $\sin\theta=\sin 90=1$.

$\therefore$ $F=qvB\sin\theta$ becomes

$F=qvB$

This force acting $\perp$ to the plane containing v & B makes the electron to move in a circular path of radius r given by

$$qvB=\frac{mv^2}{r}$$

$$r=\frac{mv}{qB}$$

$$=\frac{\left(9\times 10^{-31}\right)\left(\frac{8}{3}\times 10^7\right)}{1.6\times 10^{-19}\times 0.1}$$

$$=1.5\times 10^{-3}\text{ m}$$

$$=1.5\text{ mm.}$$

(*b*) Here $\theta=30°$, so $\sin\theta=\sin 30=\frac{1}{2}$.

$\therefore$ $F=qvB\sin\theta$ becomes

$$F=qvB\times\tfrac{1}{2}$$

$\therefore$ In this case

$$\frac{1}{2}\,qvB=\frac{mv^2}{r}$$

or $$r=\frac{2mv}{qB}$$

$\therefore$ $$r=2\times1{\cdot}5 \text{ mm.}$$
$$=3 \text{ mm.}$$

The other component of velocity $V\cos\theta=\frac{8}{3}\times10^7\times\cos 30$

$=2{\cdot}3\times10^7 \text{ ms}^{-1}$ makes the plane of the circular orbit of electrons path advance along the magnetic field axis *i.e.* the actual path of the electron is helix (See fig. 5 8).

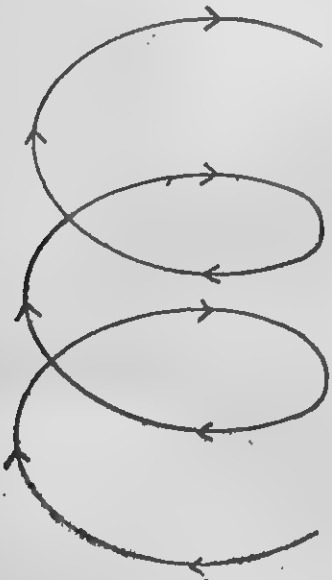


Fig. 5·8.

Example 9. *A cyclotron's oscillating frequency is 5MHz (a) What should be the operating magnetic field for accelerating deutrons? (b) What is the kinetic energy in MeV of the deutron if the radius of dees is 56 cm. (mass of deutron$=3{\cdot}3\times10^{-27}$ kg, Charge$=1{\cdot}6\times10^{-19}$C).*

Solution. (*a*) We know that

$$f=\frac{Bq}{2\pi m} \quad \text{...(i)}$$

$$B=\frac{2\pi mf}{q}$$

$$=\frac{2\times 3.14\times(3.3\times 10^{-27})\times(5\times 10^{6})}{1.6\times 10^{-19}}$$

$$=0.648 \text{ T}$$

(*b*) We know that

$$Bqv=\frac{mv^2}{r}$$

$$\therefore \quad v=\left(\frac{Bq}{m}\right)r$$

$$\therefore \quad \text{or } v=2\pi f r \qquad \left[\because \ f=\frac{Bq}{2\pi m}\right]$$

$$\therefore \quad K.E.=\tfrac{1}{2}mv^2$$

$$=\tfrac{1}{2}m(2\pi fr)^2$$

$$=2m(\pi fr)^2$$

$$=2\left(3.3\times 10^{-27}\right)\left(\frac{22}{7}\times 5\times 10^{6}\times 0.56\right)^2$$

$$=5.11\times 10^{-13}\text{ J}$$

$$=\frac{5.11\times 10^{-13}}{1.6\times 10^{-13}}\text{ MeV}$$

$$=3.19\text{ MeV}$$

Example 10. *An uniform magnetic field at 2·5 T exists in a cylindrical region of radius 12 cm, its direction parallel to the axis along east to west. A wire carrying a current of 10 A in the north to south direction passes through the region. What is the magnitude and direction of the force on the wire if*

(*a*) *the wire intersects the axis.*

(*b*) *the wire is turned from N-S to north-east and south-west direction by 60°.*

Solution. I=10 A, $l=2r=2\times 12=24$ cm$=0.24$ m.
and B=2·5 T

(*a*) Here $\theta=90°$,

$\therefore$ Magnitude of force,

$$F=BIl\sin\theta$$

$$=2.5\times 10\times 0.24\sin 90$$

$$=6\text{N, vertically downward}$$

(*b*) Here again $\theta=90°$

$$\therefore \quad F=2.5\times 10\times 0.24\sin 90$$

$$=6\text{N, vertically downward}$$

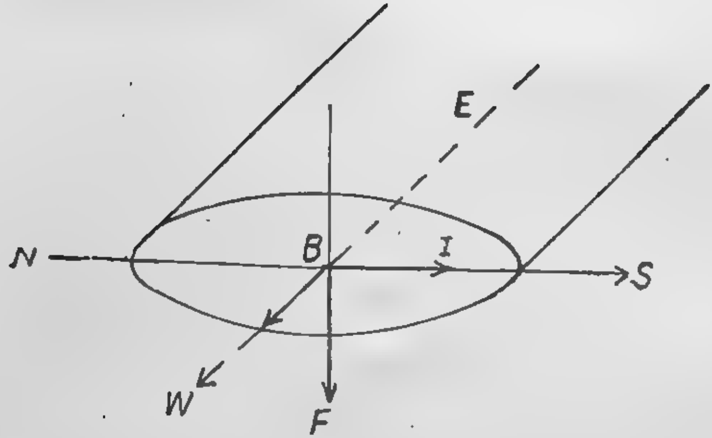


Fig. 5

Example 11. *On a smooth plane inclined at 60° with the horizontal, a thin current carrying metallic conductor is placed parallel to the horizontal. If a uniform magnetic field of 2T is acting in vertical direction, what should be the value of current in the conductor to make it stationary on the inclined plane (mass per unit length of the conductor=0·1 kg m^{-1}).*

Solution :

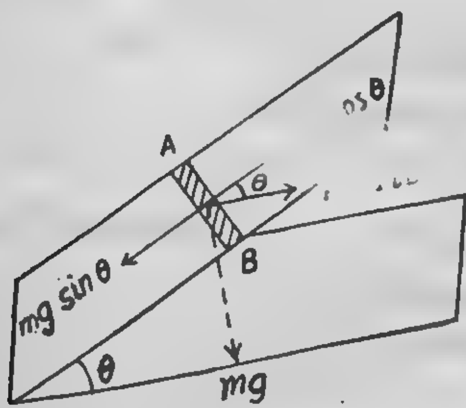


Fig. 5·10.

For the conductor AB in Fig. 5·9 to be stationary,

$$IBl \cos \theta = mg \sin \theta$$

$$\therefore \quad I = \frac{mg \tan \theta}{Bl}$$

$$=\frac{m/l \times g \times \tan\theta}{B}$$
$$=\frac{0{\cdot}1 \times 9{\cdot}8 \times \tan 60}{2}$$
$$=0{\cdot}85 \text{ A}$$

Example 12. *Given a uniform magnetic field of 250 G in north-south direction and a 55 cm long wire with a current carrying capacity of 8A (a) What is the shape and orientation of the loop made of this wire which yields maximum turning effect on the loop ? (b) What is the magnitude of the maximum torque ?*

Solution. (*a*) For a given perimeter, a circle encloses maximum area for any two dimensional shape, so shape of the *loop is circular.*

For maximum turning effect, the angle between normal to the plane of the loop and magnetic field should be 90° as $\tau = IAB \sin\theta$, so loop is oriented with its *plane in north-south direction.*

(*b*) ∴ Maximum torque,

$$\tau = IAB \sin 90$$

or

$$\tau = IAB$$

Now

$$2\pi r = 55$$
$$r = \frac{55 \times 7}{2 \times 22} = \frac{35}{4} = 8{\cdot}75 \text{ cm.}$$

∴ Area of loop,

$$A = \pi r^2$$
$$= \frac{22}{7} \times (8{\cdot}75)^2$$

or

$$A = 240{\cdot}625 \text{ cm}^2$$

∴

$$A = 240{\cdot}625 \times 10^{-4} \text{ m}^2,\ B = 250 \text{ G}$$
$$= 250 \times 10^{-4} \text{ T}$$

∴

$$\tau = IAB$$
$$= 8 \times (240{\cdot}625 \times 10^{-4})\ (250 \times 10^{-4})$$
$$= 4{\cdot}8125 \times 10^{-5} \text{ Nm.}$$

Example 13. *A short conductor of length 4 cm. is placed parallel to a long conductor of length 2·0 m near its centre. The conductors carry the currents 2A and 5A respectively in the opposite direction. What is the total force experienced by the long conductor when they are 2 cm apart ?*

Solution. The magnetic field at short conductor due to the current in long conductor will be

$$B = \frac{\mu_0 I}{2\pi r}$$
$$= \frac{(4\pi \times 10^{-7}) \times 5}{2 \times \pi \times 0{\cdot}02}$$
$$= 5 \times 10^{-5} \text{ T}$$

$\therefore$ Force on the short conductor,

$$F = BIl$$
$$= (5 \times 10^{-5}) \times 2 \times 0.04$$
$$= 4 \times 10^{-6}\ N$$

By Newton's third law of motion, to every action there is an equal and opposite reaction.

$\therefore$ Force (total force) experienced by the long conductor $= 4 \times 10^{-6}$ N, repulsive since the currents are in opposite directions.

Example 14. *A flat copper strip width 1·2 cm and thickness 0·8 mm carries a current of 180 A. A magnetic field of 3·0 T is applied perpendicular to the flat face of the strip. The Hall emf developed across the width of the strip is measured to be 15·4 μV Calculate the number density of free electrons in the metal.*

Solution. We know

$$eE = evB \qquad \text{(Hall Effect)}$$

and
$$I = neAv$$

$\therefore$
$$\frac{eE}{I} = \frac{evB}{neAv}$$

or
$$n = \frac{IB}{eEA}$$

Now $I = 180$ A, $B = 3.0$ T, $e = 1.6 \times 10^{-19}$ C, $A = .012 \times .0008\ m^2$

and E (Hall *e.m.f.* per unit width) $= \dfrac{15.4 \times 10^{-6}}{0.012}\ Vm^{-1}$

$\therefore$
$$n = \frac{180 \times 3 \times 0.012}{1.6 \times 10^{-19} \times 15.4 \times 10^{-6} \times 0.012 \times 0.0008}$$
$$= 2.74 \times 10^{29}$$

Example 15. *Compare the current sensitivity and voltage sensitivity of the following moving coil galvanometers having the identical spring.*

Galvanometer—A : $n = 25$, $A = 3 \times 10^{-3} m^2$, $B = 0.4$ T and *$R = 25\ \Omega$*

B : $n = 20$, $A = 2.5 \times 10^{-3} m^2$, $B = 0.35$ T and *$R = 40\Omega$*

Solution. We know that the current sensitivity *i.e.* deflection per unit current $= \dfrac{nAB}{k}$

$\therefore$
$$\frac{\text{Current sensitivity of A}}{\text{Current sensitivity of B}} = \frac{n_1A_1B_1/k}{n_2A_2B_2/k}$$

or
$$\frac{\alpha_1}{\alpha_2} = \frac{n_1}{n_2} \times \frac{A_1}{A_2} \times \frac{B_1}{B_2}$$
$$= \frac{25}{20} \times \frac{3 \times 10^{-3}}{2.5 \times 10^{-3}} \times \frac{0.4}{0.35}$$

$$\therefore \quad \frac{\alpha_1}{\alpha_2}=12:7$$

$$\frac{\text{Voltage sensitivity of A}}{\text{Voltage sensitivity of B}}=\frac{\alpha_1/\alpha_2}{R_1/R_2}$$
$$=\frac{12}{7}\times\frac{40}{25}$$
$$=96:35$$

Example 16. *A solenoid 80 cm long and of radius 5 cm has 4 layers of windings of 400 turns each. A 2·5 cm long wire of mass 3 g lies inside the solenoid near its centre normal to its axis, both the wire and axis of solenoid are in horizontal plane. If the current through the wire is 5·0 A, what value of current in the windings of the solenoid can support the weight of the wire ?*

Solution. We know $F=IlB$

Here $F=mg$

$\therefore$ $IlB=mg$

$$B=\frac{mg}{Il}$$
$$=\frac{(3\times10^{-3})\,9{\cdot}8}{5\times(2{\cdot}5\times10^{-2})}$$
$$=0{\cdot}2532\text{T}$$

For a solenoid, $B=\mu_0 nI$

$$\therefore \quad I=\frac{B}{\mu_0 n}$$

Now $n=\dfrac{4\times400}{0{\cdot}80}=2000$

$$\therefore \quad I=\frac{0{\cdot}2352}{(4\pi\times10^{-7})\times2000}$$
$$=93{\cdot}5\text{ A}$$

Example 17. *A long straight wire carries a current of 5A. An electron travelling at a distance of 4 cm from the wire has a speed of 5000 kms^{-1}. What force acts on the electron if its velocity is (a) parallel to the wire (b) at right angle to the wire away from it (c) at right angle to the wire but at a constant distance from it.*

Solution. The magnetic field due to the current in long straight wire,

$$B=\frac{\mu_0 I}{2\pi r}$$
$$=\frac{(4\pi\times10^{-7})\times5}{2\pi\times0{\cdot}04}$$
$$=2{\cdot}50\times10^{-5}\text{ T},$$ perpendicular to the plane containing the wire.

(*a*) $\theta = 90°$

$\therefore$ $F = qvB \sin \theta$

$= (1.6 \times 10^{-19})\ (5000 \times 10^{3})\ (2.5 \times 10^{-5})\ (\sin 90)$

$= 2 \times 10^{-17}$ N, perpendicular to the current

(*b*) $\theta = 90°$

$\therefore$ $F = 2 \times 10^{-17}$ N, parallel to the current

(*c*) $\theta = 0$

$\therefore$ $F = qvB \sin 0$

$= 0$

Example 18. *A rectangular loop of sides 27 cm × 12 cm carrying a current of 12 A is placed with its longer side parallel to a long straight conductor 3 cm apart carrying a current of 20 A. What is the net force on the loop.*

Solution. The force on the length AB of the loop,

$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi a} l, \text{ away from MN}$$

The force on the length CD of the loop,

$$F_2 = \frac{\mu_0 I_1 I_2}{2\pi(a+b)} l \text{ towards MN}$$

The forces on breadths BC and AD will be equal and opposite to each other and hence cancel away. So net force on the loop,

$$F = F_1 - F_2$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} l \left[\frac{1}{a} - \frac{1}{(a+b)} \right]$$

$$= \frac{\mu_0 I_1 I_2\, lb}{2\pi a\,(a+b)}$$

$$= \frac{4\pi \times 10^{-7} \times 12 \times 20 \times 0.27 \times 0.12}{2\pi \times 0.03 \times (0.03 + 0.02)}$$

$F = 3.456 \times 10^{-4}$ N, repulsive

(away from the long conductor.)

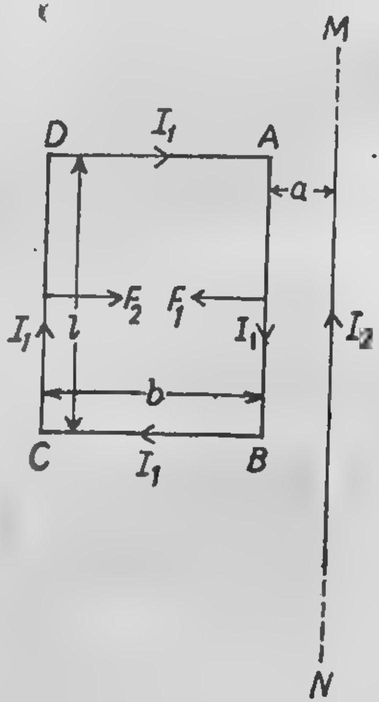


Fig. 5.11

If the current in the long conductor MN is in downward direction, force will be attractive.

EXERCISE 5

1. A long straight wire carries a current 2A. An electron travels with a velocity of 4.0×10^{4} ms^{-1} parallel to the wire 0.1 m away from it and in a direction opposite to the current. What

is the magnetic field acting on the electron and the force experienced by it.

2. Two long parallel wire carry currents of 3A and 4A respectively in opposite directions. If the separation between them is 0·1 m., find the force exerted by one over the other. Is the force attractive or repulsive ?

3. The currents through two long parallel conductors are 4 A and 5 A respectively in the same direction. If they are separated by a distance of 0·2 m, calculate the force exerted by one on the other. What is the nature of force ? [*A.I.S.S.E., 1986*]

4. A current of 1 A flows in a wire of length 0·1 m in a magnetic field of 0·5 T. Calculate the force acting on the wire when it makes an angle of (*a*) 90° (*b*) 0° with respect to the magnetic field. [*A.I.S.S.E., 1985*]

5. A long straight wire carries a current of 5 A. A positron (it is a particle having same mass and charge as an electron but the charge is positive instead of negative) travels with a velocity 50 kms^{-1} parallel to the wire 0·25 m away from it. What is the force experienced by the positron ?

6. Calculate the force on a conductor of length 0·35 m carrying a current of 0·4 A and placed in a magnetic field of 12·5 weber m^2 (Tesla) if the angle between the directions of the current and the magnetic field is 30°.

7. A horizontal wire carries a current of 3 A. Find the magnitude and direction of the magnetic field, which can support the weight of the wire. The radius and density of the wire are 1 mm and 7·8 g/cc. respectively.

8. Two cocentric coils carry current 8 A and 10 A respectively in clockwise directions. Their respective radii are 10 cm and 12 cm and they have 40 and 60 number of turns respectively. Calculate the magnitude and direction of the resultant field at their centre.

9. Two cocentric circular coils of radii 20 cm and 25 cm carry currents 8 A and 10 A respectively and has number of turns 30 and 40 respectively. If the current in the two coils is flowing in opposite direction, what will be the resultant field at the centre ?

10. A magnetic field of 250 Gauss is required which is uniform over a region of 15 cm and area of cross-section 25 cm^2. The maximum current carrying capacity of the given solenoid is 16 A and number of turns per unit length is 1800 turns m^{-1}. Calculate particulars of solenoid for the required purpose.

11. A straight wire carrying a current of 14 A is bent into a semicircular arc of radius 2·2 cm. What is the direction and magnitude of the field at the centre of the arc ?

12. A straight wire carrying a current of 10·5 A is bent into semicircular arc of radius 3 cm (*a*) what is the direction and mag-

nitude of the field at the centre of the arc ? (*b*) Would your answer change if the arc is made up side down ?

13. Calculate the magnetic field at the centre of a square wire of side $\sqrt{2}$ cm and carrying a current of 10 A.

14. Calculate the magnetic field at the centre of a square made from a wire 60 cm long and carrying a current 9 A.

15. Calculate the magnetic field at the centre of a rectangle of side 8 cm $\times$ 5 cm and carrying a current of 8 A.

16. Two wires A and B have the same length equal to 50 cm and carry a current of $10\sqrt{2}$ A each. Wire A is bent into a square and wire B is bent into a circle, (*a*) which wire produces a greater field at the centre (*b*) calculate the magnitude of the field in each case at the centre of the wire.

17. A toroid has a non-ferromagnetic core of inner radius 15 cm and outer radius 16 cm around which 3000 turns of a wire are wound. If the current in the wire is 12 A, what is the magnetic field (*a*) inside the core of the toroid (*b*) outside the toroid (*c*) in the empty space surrounded by the toroid ?

18. A cyclotron's oscillator frequency is 6 MHz. What should be the operating magnetic field for accelerating the deutrons. (mass of a deutron $= 3 \cdot 3 \times 10^{-27}$ kg).

19. A closely wound coil has a diameter of 40 cm and carries a current of 5 A. How many turns does it have if the magnetic field at the centre of the coil is $7 \cdot 56 \times 10^{-4}$ T ?

20. A solenoid of length 60 cm and radius 3 cm is closely wound with 2 layers 150 turns of wire each. Calculate the magnetic field at the central axis of the solenoid if the current through the solenoid is 2A.

21. A vertical rectangular coil of side 6 cm and 4 cm has 120 turns and carries a current of 3A. Calculate the torque on the coil when it is placed in a uniform magnetic field of $0 \cdot 25$ T in horizontal direction with the plane of the coil (*a*) parallel to the field (*b*) perpendicular to the field.

22. Calculate the magnetic field at a point on the axis of the circular coil of radius 8 cm if it is carrying a current of 4 A. The point being 6 cm away from the centre of the coil.

23. Calculate the magnetic field in Q. No. 22 if $r = 3$ cm, $I = 7$A and $a = 4$ cm.

24. A cyclotron is accelerating the α-particles. The magnetic field applied on dees of cyclotron is $0 \cdot 314$ T. What should be the frequency of the oscillator connected to the dees ? (mass of α particles $= 6 \cdot 64 \times 10^{-27}$ kg).

25. What is the period of a deutron in a uniform magnetic field of 11/7 T (mass of the deutron$=3{\cdot}3\times10^{-27}$ kg).

26. A cyclotron oscillating frequency is 7MHZ. (*a*) What should be the operating magnetic field for accelerating protons ? (*b*) What is the K.E. in Mev of protons if the radius of dees is 50 cm. (mass of proton$=1{\cdot}67\times10^{-27}$ kg).

27. What is the K.E. of deutrons in a cyclotron whose oscillating frequency is 5MHZ. The radius of dees is 70 cms. (mass of deutron$=3{\cdot}31\times10^{-27}$ kg).

28. The magnetic field acting on a cyclotron is 2·5 T which is used to accelerate protons. What is the K.E. of protons if the radius of dees is 40 cm ? (mass of proton$=1{\cdot}67\times10^{-27}$ kg).

29. A proton is emitted by a source with a speed of 4×10^{6} ms^{-1} at an angle of 60° with the direction of magnetic field of 0·5 T. Show that the path of the proton is helix and find the radius of the helix (mass of proton$=1{\cdot}67\times10^{-27}$ kg).

30. A 4·05 KeV electron is projected into uniform magnetic field of 0·02 T with its velocity vector making an angle 40° with it. Show that the path of the electron is helix. Calculate the radius of the helix.

31. A positron emitted in the study of cosmic rays in Wilson cloud chamber is being accelerated through a potential difference of 18 KV. It enters in a uniform magnetic field of 0·04 T (*a*) in a direction perpendicular to the field (*b*) in a direction making an angle 30° with the field. Find out the trajectory of positron in each case.

32. On a smooth plane inclined at an angle of 30° with a horizontal, a thin current carrying conductor is placed parallel to the horizontal. If a uniform magnetic field of 3·5 T is acting in vertical direction, what should be the value of current in the conductor to make it stationary on the inclined plane. (mass per unit length of the conductor$=0{\cdot}2$ kg m^{-1}).

33. If in above Q.No. 32, angle θ is 45° and magnetic field is 0·5 T, what is the current through the conductor ? (mass per unit length of the conductor$=0{\cdot}12$ kg m^{-1}).

34. A short conductor of length 2·5 cm is placed parallel to a long conductor of length 2·5 m near its centre If the current carried by two conductors is 4 A and 5 A respectively in the opposite direction, what is the total force experienced by the long conductor when they are 1·0 cm apart.

35. A long conductor of length 3 m carries a current of 6 A. A short conductor of length 4 cm is placed parallel to long conductor carries a current 1·5 A. If the two conductors are

2·5 cm apart and carry current in the same direction, what is the force experienced by each conductor ?

36. Calculate the current sensitivity and voltage sensitivity of the following galvanometer :

$n=50$, $A=20$ cm^2, $B=0{\cdot}2$ T, $R=25$ Ω and torsion constant, $K=5\times10^6$ Nm rad^{-1}.

37. A moving coil galvanometer has following particulars :

$n=35$, $A=24$ cm^2, $B=0{\cdot}15$ T and $R=12$ Ω.

(*a*) How will you increase the current sensitivity by 20%.

(*b*) If in doing so the resistance of the coil gets changed to 18 Ω, is the voltage sensitivity of the modified meter greater or less than the original value ?

38. Compare the voltage sensitivity of the following moving coil galvanometers. Galvanometer A : $n=30$, $A=25$ cm^2, $B=0{\cdot}3$ T and $R=15$ Ω.

B : $n=40$, $A=2\times10^{-3}$ m^2, $B=0{\cdot}2$ T and $R=25$ Ω.

39. What is the ratio of current sensitivity of the above two galvanometers in Q. No. 38 ?

40. A wire of 8 π cm length with current carrying capacity 10 A is placed in a magnetic field of 400 G in north-south direction after making it a loop. (*a*) What is the shape and orientation of the loop which yields maximum turning effect ? (*b*) What is the magnitude of the maximum torque ?

41. A circular coil of 25 turns and radius 7 cm is placed in a uniform magnetic field of 0·25 T normal to the plane of the coil. If the current in the coil is 1·2 A, what is the (*a*) total torque on the coil, (*b*) total force on the coil, (*c*) the average force on each electron in the coil (area of cross-section of the wire in the coil$=1$ cm^2 and free electron density $=10^{29}$ m^{-3}).

42. A rectangular coil of 12 cm × 8 cm having 30 turns of wire is placed in a uniform magnetic field of 0·2 T with its plane (*a*) parallel to the field, (*b*) at an angle of 60° with the field. If the current in the coil is 1·5 A, calculate the deflecting torque in each case.

43. A rectangular coil of 5 cm × 3 cm having 20 turns of wire is placed in radial field of 0·4 T. What is the current through the coil if it gets deflected by an angle 30°. The restoring torque of suspension fibre is $5{\cdot}04\times10^{-2}$ Nm.

44. A circular coil of 40 turns and radius 7 cm carrying a current of 5·0 A is suspended vertically in a uniform horizontal magnetic field of 0·3 T. The magnetic field makes an angle of

30° with the normal to the coil. (*a*) the magnitude of the counter-torque that must be applied to prevent the coil from turning, (*b*) would your answer change if the coil is replaced by planer coil of the same area of some irregular shape ?

45. A long straight wire carries a current of 4 A. An electron travelling at distance of 2 cm from the wire has a velocity of 3×10^6 ms^{-1}. What force acts on the electron if its direction of motion is (*a*) parallel to wire, (*b*) at right angles to the wire away from it, (*c*) at right angles to the wire but at a constant distance from it ?

46. If in above Q. No. 45, I=2 A, r=2·5 cm and $v=4\times10^6$ ms^{-1}, calculate the force on electron in each case.

47. A solenoid 60 cm long and 4 cm in radius has 2 layers of windings of 1250 turns each. A wire of length 2 cm and mass 1·5 g lies inside the solenoid near its centre normal to its axis, both wire and solenoid's axis are in horizontal plane. If the current through the wire is 3 A, what value of current in the solenoid can support the weight of the wire ?

48. A solenoid 50 cm long and 3 cm in radius has 3 layers of windings of 700 turns each. A 4 cm long wire of mass 2 1 g is placed inside the solenoid near its centre and normal to its axis, both wire and axis of solenoid lie in same horizontal plane. If the 50 A current through the solenoid support the weight of the wire, what is value of current in the wire ?

49. A flat silver strip width 1·4 cm and thickness 0·6 mm carries a current of 200 A. A magnetic field of 4·0 T is applied perpendicular to the flat face of the strip. The Hall emf. developed across the width of the strip is measured to be 11·9 μV. Calculate the number density of free electrons in the metal.

50. A flat copper strip width 1 cm and thickness 0·4 mm carries a current of 150 A. A uniform magnetic field of 2·5 T is applied perpendicular to the flat face of the strip. If the number of free electrons per unit volume is 3×10^{29}, calculate the value of the Hall emf developed.

51. A solenoid has a mean diameter of 3·5 cm and length of 1 m. It has 5 layer of 1400 turns each. If the current through the solenoid is 2 A, calculate the flux density and magnetic flux at the centre.

52. What is the magnetic due to a circular coil of 200 turns, radius 0·1 m carrying a current 5 A (*a*) at the centre of the coil (*b*) at a point on the axis of coil at a distance of 0·20 m from the centre of the coil.

53. If in Q. No. 52 above n=250 turns, r=30 cm, I=3 A and

$x=40$ cm, calculate the magnetic field due to coil in both the cases (*a*) and (*b*).

54. Electrons at right angles to a uniform magnetic field moves in a circular orbit of radius 8 cm with a velocity 11×10^6 ms^{-1}. What is the magnitude of the magnetic field? ($m_e=9\times10^{-31}$ kg, $e=1{\cdot}6\times10^{-19}$C).

55. Electrons moving at right angles to a uniform magnetic field complete a circular orbit in 10^{-8} s. What is the magnitude of the magnetic field? ($m_e=9\times10^{-31}$ kg, $e=1{\cdot}6\times10^{-19}$ C).

56. Two long parallel wires carry currents of 3 A and 5 A respectively in opposite directions. If the separation between them is 10 cm, find the force exerted by one over the other.

57. Two long parallel wires carrying current attract each other with a force of $2{\cdot}4\times10^{-5}$ Nm^{-1}. If the separation between them is 20 cm and if the current in one wire is 4 A from right to left, what is the direction and magnitude of the current in the other wire?

58. The electron in the hydrogen circles round the proton with a speed of $2{\cdot}18\times10^6$ ms^{-1} in an orbit of radius $5{\cdot}3\times10^{-11}$ m. What magnetic field does it produce at the proton?

59. A rectangular coil of sides 40 cm and 8 cm carrying a current of 10 A is placed with its longer side parallel to a long straight conductor 2·0 cm apart carrying a current of 15 A. What is the net force on the coil?

OBJECTIVE TYPE QUESTIONS

60. Which of the following machines cannot be used for accelerating the charged particle :

(*a*) Van de Graff generator (*b*) cyclotron

(*c*) A.C. generator (*d*) Synchrotron.

61. In a cyclotron the frequency of the oscillator is given by :

(*a*) $\dfrac{Bq}{2\pi m}$ (*b*) $\dfrac{2\pi m}{Bq}$

(*c*) $\dfrac{Bm}{2\pi q}$ (*d*) $\dfrac{2\pi q}{Bm}$.

62. In a synchrotron the resonance does not occur between :

(*a*) mass and frequency

(*b*) magnetic field and frequency

(*c*) mass and magnetic field

(*d*) charge and velocity.

63. The force on a charged particle in motion due to a magnetic field is given by :

(a) $F = qv\, B$ (b) $F = qv\, B \sin\theta$

(c) $F = qv/B$ (d) $F = \frac{q}{vB \sin\theta}$.

64. The force on a charged particle in motion due to a magnetic field is maximum if the angle between the direction of motion and magnetic field is :

(a) 0 (b) 45°

(c) 90° (d) 180°.

65. Two thin long parallel wires separated by a distance b are carrying a current 'i' amp. each. The magnitude of force per unit length exerted by one wire on the other is :

(a) $\frac{\mu_0 i^2}{b^2}$ (b) $\frac{\mu_0 i^2}{2\pi b}$

(c) $\frac{\mu_0 i}{2\pi b}$ (d) $\frac{\mu_0 i}{2\pi b^2}$.

[*I.I.T. J.E.E. 1986*]

66. The force between two parallel infinitely long conductors carrying current in the same direction will be :

(a) attractive (b) repulsive

(c) can not be decided.

67. The path of a charged particle in a magnetic field, when its direction of motion is not at right angle to magnetic field, will be a :

(a) circle (b) helix

(c) parabola (d) straight line.

68. The path of a charged particle in a magnetic field, when its direction of motion is at right angle to magnetic field, will be a :

(a) circle (b) helix

(c) parabola (d) straight line.

69. The path of a charged particle moving in a uniform electrostatic field with initial velocity perpendicular to the field will be a :

(a) circle (b) helix

(c) parabola (d) straight line.

70. The path of a charged particle moving in a uniform electrostatic field with initial velocity parallel to the field will be a :

(a) circle (b) helix

(*c*) parabola (*d*) straight line.

71. A rectangular loop carrying a current i is situated near a long straight wire such that the wire is parallel to one of the sides of the loop and is in the plane of the loop. If a steady current I is established in the wire as shown in Fig. 5·11, the loop will :

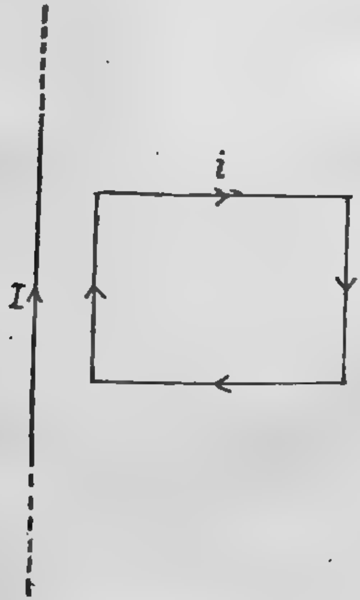


Fig. 5·11

(*a*) rotate about an axis parallel to the wire
(*b*) move away from the wire
(*c*) move towards the wire
(*d*) remain stationary. [*I.I.T. J.E.E. 1985*]

UNIT 6

Magnetism

IMPORTANT FORMULAE

1. (*a*) The frequency of oscillation of a bar magnet of magnetic moment '*m*' in a uniform magnetic field B is given by :

$$\nu = \frac{1}{2\pi}\sqrt{\frac{mB}{I}}$$

where I = moment of inertia of the magnet.

(*b*)

$$\frac{m_1}{m_2} = \frac{t_2^2 + t_1^2}{t_2^2 - t_1^2};$$

t_1 = time period when the two magnets with like poles together oscillate.

t_2 = time period when the two magnets with unlike poles together oscillate.

2. If a current of I ampere is circulating in a small plane loop enclosing an area of A metre², the magnetic moment associated with it is given by

$$m = I\,A$$

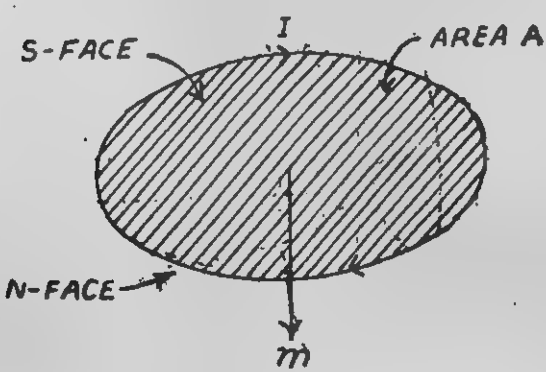


Fig 6·1.

3. (*a*) The torque on a magnetic moment m in an external field B is given by :

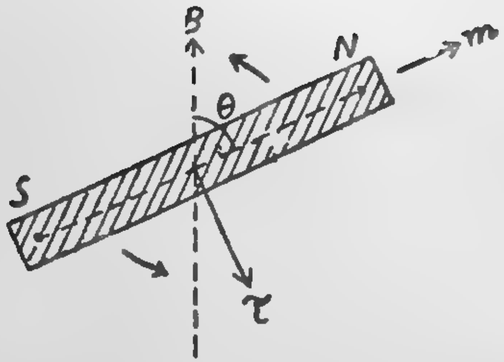


Fig. 6·2.

$$\vec{\tau}=\vec{m}\times\vec{B}=mB\sin\theta$$

(*b*) The amount of work done in turning the magnet by an angle from the direction of magnetic field,

$$W=mB\,(1-\cos\theta)$$

4. The magnetic moment of a solenoid,

$$m=nIA$$

where n=Total no. of turns in solenoid

5. Tangant law : $B=H\tan\theta$

or $$\frac{n\mu_0 I}{2r}=H\tan\theta$$

$\therefore$ $$I=\frac{2r\,H}{n\mu_0}\tan\theta$$

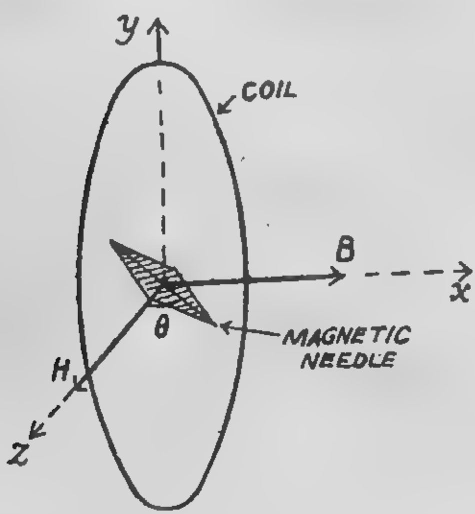


Fig. 6·3.

6. (*a*) In the magnetic meridian the angle which the resultant magnetic field 'B_0' makes with the horizontal component H is called angle of dip 'δ'.

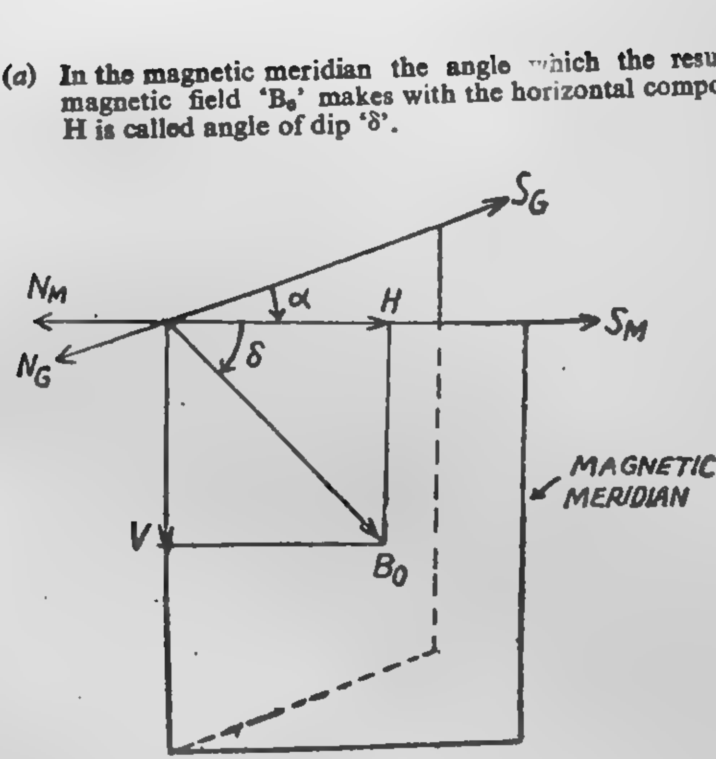


Fig. 6·4.

$$\therefore \qquad H = B_0 \cos \delta$$

and the vertical component,

$$V = B_0 \sin \delta.$$

(*b*) The angle between magnetic meridian and geographical meridian is called angle of declination (α).

7. For an electron/revolving in an orbit in an atom, total magnetic moment m is given by :

$$m = \frac{e}{2m_0}(L + 2\,S)$$

e = Charge on the electron

m_0 = dipole magnetic moment

L = angular momentum of electron

and S = spin angular momentum

8. Magnetization M is the total dipole magnetic moment per unit volume :

$$M = \frac{Nm}{V} \; ;$$

N = No. of atoms per unit volume

9. $M = \chi_m H$ where χ_m = Magnetic susceptibility
$B = \mu_0 (1 + \chi_m) H$ $\quad \mu_0$ = magnetic permeability of vacuum
$K_m = (1 + \chi_m)$ $\quad K_m$ = Relative magnetic permeability
and $\mu = \mu_0 \chi_m$ $\quad \mu$ = magnetic permeability.

10. The magnetic moment of a bar magnet,

m = pole strength $\times$ length of the magnet.

11. (*a*) The magnetic field on the normal bisector at a distance r from the centre of a bar magnet is given by,

$$B = \frac{\mu_0}{4\pi} \frac{m}{r^3},$$

provided r is much greater than the length of the magnet. ($r >>> l$).

(*b*) The magnetic field on the axis of a bar magnet at a point distance r away from its centre is given by

$$B = \frac{\mu_0}{2\pi} \frac{m}{r^3}, \text{ provided } r >>> l.$$

12. At null points, $B = H$

where (*a*) $$B = \frac{\mu_0 m}{2\pi r^3}$$

if N-pole of the magnet points toward north and S-pole towards south.

(*b*) $$B = \frac{\mu_0 m}{4\pi r^3}$$

if N-pole of the magnet points toward south and S-pole towards north.

SOLVED EXAMPLES

Example 1. *A 0·5 cm long bar magnet has each pole strength 8 Am. What is the magnetic field at a point distance 20 cm from the centre of the magnet on its axis ?*

Solution
$$B = \frac{\mu_0}{2\pi} \frac{m}{r^3}$$
$$= \frac{(4\pi \times 10^{-7})(8 \times \cdot 05)}{2\pi (0\cdot 20)^3}$$
$$= 10^{-5} \text{ T.}$$

Example 2. *A bar magnet of magnetic, moment 2·0 JT^{-1} lies aligned with the direction of a uniform magnetic field 0·25 T. (a) What is the amount of work done required to turn the magnet so as to align*

its magnetic moment, (i) opposite to the field direction (ii) normal to the field direction ? (b) What is the torque on the magnet in cases (i) and (ii).

Solution. (*a*) $W = mB(1 - \cos\theta)$

(*i*) $W = 2{\cdot}0 \times 0{\cdot}25\,(1 - \cos 180)$

$= 1\ J$

(*ii*) $W = 2{\cdot}0 \times 0{\cdot}25\,(1 - \cos 90)$

$= 0{\cdot}5\ J$

(*b*) $\tau = mB \sin\theta$

(*i*) $\tau = 2{\cdot}0 \times 0{\cdot}25 \sin 180$

$= 0$

(*ii*) $\tau = 2{\cdot}0 \times 0{\cdot}25 \sin 90$

$= 0{\cdot}5\ Nm$

in a direction that tries to align the magnetic moment along B.

Example 3. *A short magnet lies with its magnetic axis in magnetic meridian with its N-pole facing north. The neutral points are found on either side of the magnet at its perpendicular bisector at 12 cm. from the centre of the magnet. What is the magnetic moment of the magnet ? (Horizontal component of earth's magnetic field $= 0{\cdot}36$ Gauss).*

Solution. A null points

$$B = H$$

$$\frac{\mu_0}{4\pi}\frac{m}{r^3} = H$$

$$\therefore \quad m = \frac{4\pi H r^3}{\mu_0}$$

Now $H = 0{\cdot}36$ Gauss $= 0{\cdot}36 \times 10^{-4}$ Tesla,

$r = 12$ cm $= 0{\cdot}12$ m

$$\therefore \quad m = \frac{4\pi \times (0{\cdot}36 \times 10^{-4}) \times (0{\cdot}12)^3}{4\pi \times 10^{-7}} = 0{\cdot}622\ Am.^2$$

Example 4. *A closely wound solenoid of 2500 turns and area of cross-section $1{\cdot}5$ cm^2, carrying a current of 5 A is suspended through its centre allowing it to turn in a horizontal plane. (a) What is the magnetic moment of the solenoid ? (b) What are the force and torque on the solenoid if a uniform horizontal magnetic field of $6{\cdot}0 \times 10^{-2}$ T is applied at an angle of 60° with the axis of the solenoid.*

Solution. (*a*) Magnetic moment,

$$m = NIA$$

$$= 2500 \times 5 \times (1{\cdot}5 \times 10^{-4})$$
$$= 1{\cdot}875 \text{ Am}^2$$

(b) (i) The force in a uniform magnetic field on the solenoid
$$= 0$$

(ii) The torque on the solenoid,
$$\tau = mB \sin \theta$$
$$= 1{\cdot}875 \times (6{\cdot}0 \times 10^{-2}) \times \sin 60$$
$$= 0{\cdot}097 \text{ Nm.}$$

Example 5. *A compass needle is placed 40 cms east of a small magnet. The needle is deflected through 30°. Calculate the magnetic moment and the pole strength of the magnet if its length is 5 cm. and horizontal component of earth's magnetic field (H) is 0·36 Gauss.*

Solution. As the compass needle is placed east of the magnet, the needle is on the axis of the magnet and the horizontal component of earth's magnetic field (H) which is always along north-south direction will be perpendicular to the magnetic field of the magnet (B).

$$\therefore \quad B = H \tan \theta$$

but
$$B = \frac{\mu_0}{2\pi} \frac{m}{r^3}$$

$$\therefore \quad \frac{\mu_0}{2\pi} \frac{m}{r^3} = H \tan \theta$$

$$\therefore \quad m = \frac{2\pi \, Hr^3 \tan \theta}{\mu_0}$$

$$= \frac{2\pi \times (0{\cdot}36 \times 10^{-4}) \, (0{\cdot}40)^3 \tan 30}{4\pi \times 10^{-7}}$$

$$= 6{\cdot}65 \text{ Am}^{-2}$$

$$\therefore \quad \text{Pole strength} = \frac{m}{2l}$$

$$= \frac{6{\cdot}65}{0{\cdot}05}$$

$$= 133 \text{ Am.}$$

Example 6. *A telephone cable at a place has four long straight horizontal wires carrying a current of 2·0 A in the east to west direction each. The earth's magnetic field at the place is 0·4 G and the angle of dip is 30°. The magnetic declination is nearly zero. What are the resultant magnetic fields at points 8·0 cm below and above the cable ?*

Solution. The magnetic field due to all the four cables,

$$B=4\times\frac{\mu_0 I}{2\pi r}$$

$$=4\times\frac{4\pi\times10^{-7}\times2}{2\pi\times\cdot08}$$

$$=0\cdot20\times10^{-4}\ \mathrm{T}$$

$$=0\cdot20\ \mathrm{G}, \text{ along horizontal direction}$$

$\therefore$ Below the cable,

$$H=B_0\cos\delta-B$$

$$=0\cdot4\cos30-0\cdot20$$

$$=0\cdot4\times\frac{\sqrt{3}}{2}-0\cdot20$$

$$=0\cdot1462\ \mathrm{G}$$

$$V=B_0\sin\delta$$

$$=0\cdot4\sin30$$

$$=0\cdot4\times\frac{1}{2}$$

$$=0\cdot2\ \mathrm{G}$$

$\therefore$ Resultant magnetic field,

$$R=\sqrt{H^2+V^2}$$

$$=\sqrt{(0\cdot1462)^2+(0\cdot2)^2}$$

$$=\sqrt{0\cdot061374}$$

$$=0\cdot247\ \mathrm{G}$$

$$\cot\theta=\frac{H}{V}$$

$$=\frac{0\cdot1462}{0\cdot2}$$

$$=0\cdot731$$

$$\therefore\quad \theta=\cot^{-1}(0\cdot731)$$

$$=53^\circ\,46'$$

Above the cable,

$$H=B_0\cos\delta+B$$

$$=0\cdot5462\ \mathrm{G}$$

$$V=B_0\sin\delta$$

$$=0\cdot2\ \mathrm{G}$$

$$\therefore \quad R=\sqrt{H^2+V^2}$$
$$=\sqrt{(0.5462)^2+(0.2)^2}$$
$$=\sqrt{0.338334}$$
$$=0.581 \text{ G}$$

$$\cot\theta=\frac{H}{V}$$
$$=\frac{0.5462}{.2}$$
$$=2.731$$

$$\therefore \quad \theta=\cot^{-1}(2.731)$$
$$\theta=20^\circ\ 5'.$$

Example 7. *At a certain location, a compass needle points 15° east of the geographical north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 50° above the horizontal. Find out (a) angle of declination, (b) the direction and magnitude of the earth's field at the location (H=0·24 G).*

Solution. (*a*) Angle of declination,

α=Angle between geographical north-south and magnetic north-south directions.

$$=15^\circ$$

(*b*) We know $\quad H=B_e\cos\delta$

$$0.24=B_e\cos 50$$

$$\therefore \quad B_e=\frac{0.24}{0.6248}$$
$$=0.384 \text{ G}$$

So the earth's field is 0 384 G along a line in a vertical plane 15° east of the geographical meridian. This line is making an angle of 50° with the horizontal.

Example 8 *A short bar magnet of magnetic moment 4.5×10^{-2} Am² is placed with its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet on (a) its axis, (b) its normal bisector, is the resultant field inclined at 45° with the earth's field. (Earth's magnetic field at the place=0·36 G).*

Solution. Since the resultant field is inclined at an angle 45° with the earth's field, it means earth's field and field due to magnet are equal in magnitude.

$\therefore$ (*a*) Magnetic field due to a short bar magnet on its axis,

$$B = \frac{\mu_0 m}{2\pi r^3} = H \text{ (earth's field)}$$

$\therefore$ $$r^3 = \frac{\mu_0 m}{2\pi H}$$

$$= \frac{4\pi \times 10^{-7} \times 4{\cdot}5 \times 10^{-2}}{2\pi \times (0{\cdot}36 \times 10^{-4})}$$

$$r^3 = 250 \times 10^{-6}$$

$\therefore$ $$r = (250)^{1/3} \times 10^{-2} \text{ m}$$

Let $$x = 250^{1/3} \Rightarrow \log x = \frac{1}{3} \log 250$$

$\therefore$ $$\log x = \frac{1}{3} \times 2{\cdot}3979 = 0{\cdot}7993$$

$\therefore$ $$x = \text{Antilog } (0{\cdot}7993) = 6{\cdot}299$$

$\therefore$ $$r = 6{\cdot}279 \times 10^{-2} \text{ m}$$

$$\simeq 6{\cdot}3 \text{ cm.}$$

(*b*) Magnetic field due to a short bar magnet on its normal bisector,

$$B = \frac{\mu_0 m}{4\pi r^3} = H$$

$$r^3 = \frac{\mu_0 m}{4\pi H}$$

$$= \frac{4\pi \times 10^{-7} \times 4{\cdot}5 \times 10^{-2}}{4\pi (0{\cdot}36 \times 10^{-4})}$$

$$r^3 = 125 \times 10^{-6}$$

$\therefore$ $$r = 5 \times 10^{-2} \text{ m}$$

$$r = 5 \text{ cm.}$$

Example 9. *A compass needle free to turn in a horizontal plane is placed at the centre of a circular coil of 40 turns and radius 15 cm. The coil is in a vertical plane making an angle of 30° with the magnetic meridian. When the current in the coil is 0·3 A, the needle points west to east. (a) Determine the horizontal component of the earth's magnetic field at the location. (b) The current in the coil is reversed and the coil is rotated about its vertical axis by an angle 60° in the clockwise sense looking from above. Find the direction of the needle. Take the magnetic declination at the place to be zero.*

Solution. The needle may point along west to east direction only when (see Fig. 6·5),

$$H = B \cos (90 - \theta)$$

$$=\frac{\mu_0 In}{2r}\cos(90-\theta)$$

$$=\frac{4\pi\times10^{-7}\times0\cdot3\times40}{2\times0\cdot15}\cos(90-30)$$

$$=0\cdot25\times10^{-4}\text{ T}$$

$$=0\cdot25\text{ G.}$$

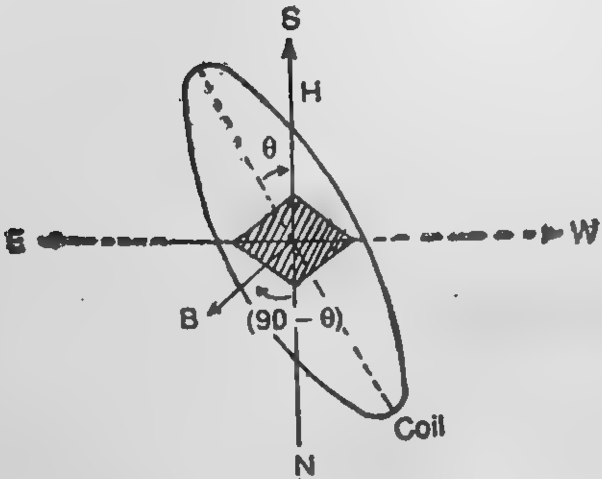


Fig. 6·5.

(*b*) The direction of the needle will be east to west *i.e.*, the needle will reverse its original direction.

Example 10. *A magnetic dipole is under the influence of two magnetic fields. The angle between the two fields is 40°. One of the fields has magnitude $1\cdot5\times10^{-2}$ T. What is the magnitude of the other field if the dipole comes to stable equilibrium at an angle of 10° with the first field.*

Solution. The dipole may come to equilibrium only when

$$B_2\sin(\theta-\alpha)=B_1\sin\alpha$$

$$\therefore\quad B_2\sin(40-10)=1\cdot5\times10^{-2}\sin10$$

$$B_2=\frac{1\cdot5\times10^{-2}\sin10}{\sin30}$$

$$=2\times(1\cdot5\times10^{-2})\times(0\cdot1736)$$

$$=5\cdot20\,8.\times10^{-3}\text{ T.}$$

Example 11. *A circular coil of 20 turns and radius 7 cm. carrying a current of 1·2 A rests with its plane normal to an external*

field of magnitude 4×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of 3·5 s^{-1}. What is the moment of inertia of the coil about its axis of rotation ?

Solution. We know that

$$\nu=\frac{1}{2\pi}\sqrt{\frac{mB}{I}}$$

and $$m=niA$$

$$\therefore \quad \nu=\frac{1}{2\pi}\sqrt{\frac{niAB}{I}}$$

$$3\cdot5=\frac{7}{2\times22}\sqrt{\frac{20\times1\cdot2\times\left(\frac{22}{7}\times0\cdot7^2\right)\times4\times10^{-2}}{I}}$$

$$\therefore \quad 22=\sqrt{\frac{1\cdot4784\times10^{-2}}{I}}$$

$$\therefore \quad I=\frac{1\cdot4784\times10^{-2}}{22\times22}$$

$$=3\cdot05\times10^{-5}\text{ kg m}^2.$$

Example 12. *Two bar magnets with their north poles pointing towards north made to make angular oscillations together in a horizontal plane. The period is found to be 3·2 sec. When the magnets are again swang together with one of them reversed, the period becomes 5·1 sec. Calculate the ratio of their magnetic moments.*

Solution.
$$\frac{m_1}{m_2}=\frac{t_2^2+t_1^2}{t_2^2-t_1^2}$$

$$=\frac{(5\cdot1)^2+(3\cdot2)^2}{(5\cdot1)^2-(3\cdot2)^2}$$

$$=\frac{26\cdot01+10\cdot24}{26\cdot01-10\cdot24}$$

$$=\frac{36\cdot25}{15\cdot77}$$

$$=2\cdot3.$$

Example 13. *A small magnet makes 30 oscillation in 4 minutes 30 second in earth's field. When a second magnet pointing its N-pole towards north is placed 40 cm due south of it in the direction of earth's field, it takes 2 minutes 15 seconds in making 30 oscillations. Calculate the magnetic moment of the second magnet if H=0·36 Gauss.*

Solution. In first case,

$$T=2\pi\sqrt{\frac{I}{mH}}$$

and in second case, $$T'=2\pi\sqrt{\frac{I}{m\,(B+H)}}$$

where $$B=\frac{\mu_0 m}{2\pi\, r^3}$$

$\therefore$ $$\frac{T^2}{T'^2}=\frac{(B+H)}{H}=\left(\frac{B}{H}+1\right)$$

$\therefore$ $$B=H\left[\left(\frac{T}{T'}\right)^2-1\right]$$

$$=0\cdot36\left[\left(\frac{270/30}{135/30}\right)^2-1\right]$$

$\therefore$ $$B=1\cdot08$$

$\therefore$ $$1\cdot08=\frac{(4\pi\times10^{-7})\,m}{2\pi\,(0\cdot40)^3}$$

or $$m=3\cdot456\times10^5\ \mathrm{Am}^2.$$

Example 14. *A monoenergetic electron beam of 18 KeV initially in the horizontal direction is subjected to a horizontal magnetic field of 0·30 Gauss normal to initial direction. Calculate the up or down deflection of the beam over a distance of 15 cm. ($m=9\times10^{-31}$ kg and $e=1\cdot6\times10^{-19}$ C).*

Solution. The electron beam will move in a circular path. If the radius of the circular path is R, then

$$\frac{mv^2}{R}=\rho vB$$

$\therefore$ $$R=\frac{mv}{\rho B}$$

But K.E. of the electrons beam,

$$E=\tfrac{1}{2}\,mv^2$$

$\therefore$ $$v=\sqrt{\frac{2E}{m}}$$

$\therefore$ $$R=\frac{m}{eB}\sqrt{\frac{2E}{m}}$$

$$=\frac{\sqrt{2mE}}{eB}$$

$$=\frac{\sqrt{2\times9\times10^{-31}\times(18\times10^{3}\times1.6\times10^{-19})}}{1.6\times10^{-19}\times(0.3\times10^{-4})}$$

$$=15 \text{ m.}$$

$$\sin\theta=\frac{0.15}{15}=.01$$

$$\therefore \quad \cos\theta=\sqrt{1-\sin^2\theta}$$

$$=\sqrt{1-(.01)^2}$$

$$=\sqrt{0.9999}$$

$$=0.9999$$

$\therefore$ up or down deflection,

$$y=R\,(1-\cos\theta)$$

$$=15\,(1-0.9999)$$

$$=15\times0.0001$$

$$=0.0015 \text{ m}$$

$$=1.5 \text{ mm.}$$

Example 15. *A Rowland ring of mean radius 12 cm has 3200 turns of wire on a ferromagnetic core of relative permeability 750. What is the magnetic field in the core for a magnetizing current of 1.5 A?*

Solution. $$B=\frac{\mu\, nI}{2\pi r}$$

$$=\mu_0\,\chi_m\,\frac{n.I}{2\pi r}$$

where χ_m = Relative permeability.

$$=(4\pi\times10^{-7})\ (750)\ \frac{3200\times1.5}{2\pi\times0.12}$$

$$=6\text{T}.$$

Example 16. *A toroidal solenoid 14 cm. in mean radius has an area of cross-section 5 cm² has 880 turns and the core is of soft iron. The magnetic flux in the core for a current of 2A is 6.4×10^{-4} weber. What is the perm ability and the relative permeability of the soft iron core.*

Solution. $\phi=6.4\times10^{-6}$ wb, $A=5\text{ cm}^2=5\times10^{-4}\text{ m}^2$

$$B=\phi/A$$

$$=\frac{6.4\times10^{-4}}{5\times10^{-4}}$$

$$=1.28 \text{ wb/m}^2$$

Now $B=\mu\, n\, I$

where n (number of turns per unit length)

$$\therefore \quad n=\frac{880}{2\pi\times(0{\cdot}14)}$$

$$=1000$$

$$\therefore \quad 1{\cdot}28=\mu\times1000\times2$$

$$\mu=6{\cdot}4\times10^{-4}\ \mathrm{T}\ m\mathrm{A}^{-1}$$

Now $\mu_0=4\pi\times10^{-7}\ \mathrm{T\ mA^{-1}}$

$\therefore$ Relative permeability,

$$\chi_m=\frac{\mu}{\mu_0}$$

$$=\frac{6{\cdot}4\times10^{-4}\times7}{4\times22\times10^{-7}}$$

$$=509.$$

EXERCISE 6

1. At what angle with the magnetic meridian will an ordinary magnetic needle rest if it is subjected to a magnetic field perpendicular to the magnetic meridian and of strength double that of earth's field ?

2. A magnetic needle pivoted through its centre of mass and free to rotate in a plane containing a uniform magnetic field of 160 G is displaced slightly from its stable equilibrium. The frequency of its angular oscillations of small amplitudes is measured to be $1{\cdot}2\ s^{-1}$. If the moment of inertia of the needle about its axis of rotation is $4\ 9\times10^{-6}$ kg m^2, calculate the magnetic moment of the needle.

3. If in above Q. No. 2, $B=250$ G, $I=9\times10^{-6}$ kgm^2 & $m=0{\cdot}4$ Am^2, calculate the frequency of angular oscillations.

4. A short magnet placed 50 cm to the west of a compass needle deflects it through 45°. Calculate the value of magnetic moment of the magnet if the value of earth's horizontal field is 0·36 G.

5. Compare the magnetic moments of two magnets which makes 12 and 15 swings in one minute at a place. The dimensions and masses of the magnets are same.

6. A magnetic needle makes one complete oscillation in 4S in Delhi where the value of the horizontal component of earth's magnetic field is 0·31 G. What will be the value of this component at a place where the same needle makes one oscillation in 3 s ?

7. Two bar magnets are bound together side by side and are suspended so as to make 15 oscillations in 3 minutes when like poles are together and in 4 minutes when the direction of one of them is reversed. Compare the magnetic moments of the two magnets.

8. A magnet whose moment of inertia is 4.9×10^{-5} kg m^2 oscillates in a uniform field of 0·36 G with a period of 11/12 s. Find the magnetic moment of the magnet.

9. A small needle makes 12 oscillations per minute in the earth's uniform magnetic field at a place. When a second magnet pointing its N-pole towards north is placed 50 cm due south of it in the direction of earth's field it takes 36 s in making 12 oscillations. Calculate the magnetic moment of the magnet if H=0·36 G.

10. If in Q. No. 9 above the magnet is placed due north of the needle keeping all other factors constant such that (*a*) the north pole of the magnet is pointing toward south and south pole pointing towards north (*b*) the north pole pointing towards north and south pole pointing towards south. Calculate in each case the time period of the oscillating needle.

11. A short bar magnet placed with its axis at 60° with a uniform external magnetic field of 0·25 T experiences a torque of magnitude 0·0433 J (*a*) What is the magnetic moment of the magnet ? (*b*) If the bar magnet is free to rotate which orientation would correspond to its (*i*) stable (*ii*) unstable equilibrium (*c*) What is its potential energy in cases (*i*) & (*ii*).

12. A bar magnet of magnetic moment 2·5 JT^{-1} lies aligned with the direction of a uniform field of 0·15 T (*a*) What is the amount of work done so as to align its magnetic moment (*i*) normal to the field direction (*ii*) opposite to the field direction (*b*) What is the torque on the magnet in each case ?

13. If a torque acting on a magnet placed at an angle of 30° with a uniform field of 0·12 T is 3N*m* (*a*) What is the magnetic moment of the magnet ? (*b*) What is the work done by the field to bring it in the direction of the field ?

14. A closely wound solenoid of 1500 turns and area of cross section 1.5×10^{-4}m^2 carries a current of 3A. It is placed with its horizontal axis at 60° with the direction of a uniform horizontal field of 0·24 T. (*a*) What is the torque experienced by the solenoid due to the field ? (*b*) If the solenoid is free to rotate about the vertical direction, when will it be in stable and unstable equilibrium ? What is the amount of work done to displace the solenoid from stable to unstable equilibrium ?

15. A closely wound solenoid having 2000 turns and area of cross-section $2 \cdot 5$ cm^2 carry a current of 4A. It is suspended through its centre allowing it to turn in a horizontal plane (*a*) What is the magnetic moment of the solenoid (*b*) What is the force and torque on the solenoid if a uniform horizontal field of $5 \cdot 0 \times 10^{-2}$ T is applied at an angle of 30° with the axis of the solenoid ?

16. A magnetic needle free to rotate about the vertical direction points 5° west of the geographical north. Another magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north pole pointing down at 19° with the horizontal. The magnitude of the horizontal component of the earth's magnetic field is 0·36 G. (*a*) What is the declination and angle of dip at the place (*b*) What is the magnitude and direction of the earth's magnetic field at the place ?

17. The value of H at a place is 0·24 G and the angle of dip is 30°. What is the total intensity at that place ?

18. Find the total intensity at a place where dip is 45° and value of H is 0·25 G.

19. If the resultant earth's magnetic field at a place is 0·4 G and angle of dip is 30°, What is the horizontal and vertical component's of the field ?

20. If the horizontal and vertical component of earth's field at a place 0·3 H and 0·4G. What is the resultant field at that place ?

21. The horizontal component of earth's magnetic field 0 4 G at a place is 0·36 G. What is the angle of dip ?

22. The vertical component of earth's magnetic field at a place is 0·21 G. If the resultant earth's field at the place is 0·42 G, what is the angle of dip ?

23. A short magnet lies with its N-pole pointing toward north and S-pole pointing towards south. The null points are found on perpendicular bisector at 10 cm. from the centre of the magnet What is the magnetic moment of the magnet ?
(H=0·32G)

24. A short bar magnet is placed in a horizontal plane with its axis in the magnetic meridian. Null points are found on its equatorial line (*i.e*, its normal bisector) at 15 cm. from the centre on the magnet. The earth's magnetic field at the place is 0 36 G and angle of dip is zero (*a*) What is the magnetic moment at null points on the axis of the magnet ? (*b*) Locate the null points when the bar magnet is turned around by 180°. (*c*) What is the total magnetic field at points on axis at 15 cm. away from the centre.

25. A short magnet lies with its N-pole pointing towards south.

and N-pole pointing towards north. The null points are found on the axis of the magnet at 12 cm. from the centre of the magnet. What is the magnetic moment of the magnet. The horizontal component of the earth's magnetic field is 0·28 G ?

26. A circular coil of 50 turns and radius 10·0 cm carrying a current of 3·5 A rests with its plane normal to an external field of magnitude 6×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction when the coil is turned slightly and released with a frequency of 4·2 s^{-1}. What is the moment of inertia of the coil about its axis of rotation ?

27. If in above Q. No. 26 $n=100$ turns, $r=5$ cm, $i=2$ A, $B=3{\cdot}5\times 10^{-2}$ T and time period of oscillations of coil is 0·22 S, what is the moment of inertia of the coil about its axis of rotation ?

28. A compass needle is placed 25 cms. east of a small magnet. The needle is deflected through 45° Calculate the magnetic moment and the pole strength of the magnet if its length is 4 cm and if horizontal component of earth's magnetic field is 0·30 G.

29. A short bar magnet of magnetic moment 2×10^{-2} Am^2 is placed with its axis perpendicular to the earths field direction. At what distance from the centre of the magnet on (*a*) its axis (*b*) its normal besector, is the resultant field inclined at 45° with the earth's field of 0·35 G.

30. If in Q. No. 29 above, $m=13\times 10^{-2}$ Am^2 and H=0·26 G. Calculate the values as in (*a*) and (*b*).

31. At a certain place, a compass needle points 12° east of the geographical north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60° the horizontal. (*a*) What is the angle of dip and declination at the place, (*b*) the direction and magnitude of the earth's field at the place. (H=0·32 G).

32. A telephone cable at a place has 5 long straight horizontal wires carrying a current of 2A west to east direction each. The earth's magnetic field at the place 0·5 G and the angle of dip is 60°. The magnetic declination is nearly zero. What are resultant fields at points 20 cm. (*a*) below the cable and (*b*) above the cable.

33. A compass needle free to turn in a horizontal plane in placed at the centre of a circular coil of 70 turns and radius 18 cm. The coil is in a vertical plane making an angle of (*a*) 45°, (*b*) 60° with the magnetic meridian when the current in the coil is 0·9 A the needle points east-west. Determine the

horizontal component of the earth's magnetic field at the place.

34. What will be the direction of the needle if the current in the coil in Q. No. 33 above is reversed and the coil is rotated about its vertical axis by an angle (*a*) 45° clockwise and (*b*) 30° anticlockwise respectively looking from the above ?

35. A magnetic dipole is under the influence of two magnetic fields. The angle between the two field is (*a*) 45°. (*b*) 60°. One of the fields has magnitude 2×10^{-2} T. What is the magnitude of the other field if the dipole comes to stable equilibrium at an angle of 15° with the first field ?

36. A monoenergetic electron beam of 2 KeV initially in the horizontal direction is subjected to a horizontal magnetic field of 0·20 G normal to its direction Calculation the up or down deflection of the beam over a distance of (*a*) 22·5 cm, (*b*) 1.5 m ($m_e=9\times10^{-31}$ kg and $e=1{\cdot}6\times10^{-19}$ C).

37. A Rowland ring of mean radius 16 cm has 2500 turns of wire on a ferromagnetic core of relative permeability 500. What is the magnetic field in the core for a magnetising current of 2 A ?

38. If in Q. No. 37 above, $r=25$ cm, $n=2800$ turns, $\chi_m=700$ and I=3A, what is the magnetic field in the core ?

39. A toroidal solenoid 10·5 cm in mean radius has an area of cross-section 4 cm² has 660 turns and the core is of soft iron. The magnetic flux in the core for a current of 2·5 A is 6×10^{-4} weber. What is the permeability and the relative permeability of the soft iron core ?

40. The core of a toroid having 2200 turns has inner and outer radii 7 m and 8 cm respectively. The magnetic field in the core for a current of 0·5 A is 2·0 T. What is the relative permeability of the core ?

41. The core of a toroid having relative permeability 600 has 550 turns and mean radius 7 cm. What current must be set up in the windings to attain a magnetic field 3·14 T ?

OBJECTIVE TYPE QUESTIONS

42. At null points :

(*a*) B=H (*b*) B ∥ H

(*c*) B⊥H (*d*) B=−H.

43. N-pole of a magnet pointing towards north and S-pole pointting towards south gives the null points at

(a) magnet's axis (b) magnetic equator
(c) magnetic centre.

44. N-pole of a magnet pointing towards south and S-pole pointing towards north gives the null points at

(a) magnetic axis (b) magnetic equator
(c) magnet's centre.

45. The magnetic field due to a short magnet at a point on its axis is found to be

(a) $\frac{\mu_0 m}{2\pi r^2}$ (b) $\frac{\mu_0 m}{4\pi r^2}$

(c) $\frac{\mu_0 m}{2\pi r^3}$ (d) $\frac{\mu_0 m}{4\pi r^3}$.

46. The magnetic field due to a short magnet at a point on its perpendicular bisector (equator) is found to be :

(a) $\frac{\mu_0 m}{2\pi r^3}$ (b) $\frac{\mu_0 m}{4\pi r^2}$

(c) $\frac{\mu_0 m}{2\pi r^3}$ (d) $\frac{\mu_0 m}{4\pi r^3}$.

47. Tangent law states :

(a) $B = H \tan \theta$ (b) $H = B \tan \theta$
(c) $B = H \sin \theta$ (d) $B = H \cos \theta$.

48. The magnetic permeability of vacuum is :

(a) $\pi \times 10^{-7}$ (b) $2\pi \times 10^{-7}$
(c) $3\pi \times 10^{-7}$ (d) $4\pi \times 10^{-7}$.

49. The relative magnetic permeability is

(a) $\frac{\mu_0}{\mu}$ (b) $\frac{\mu}{\mu_0}$

(c) $\mu \times \mu_0$ (d) $(\mu + \mu_0)$.

50. The ratio of magnetic field due to a short bar magnet at a point on the same distance on magnetic axis and magnetic equator from the centre of the magnet is :

(a) 4 : 1 (b) 3 : 1
(c) 2 : 1 (d) 1 : 2.

51. The magnetic susceptibility is defined as :

(a) $\frac{M}{H}$ (b) $\frac{H}{M}$

(c) MH (d) (M+H)

52. The relative magnetic permeability of a soft iron core is 700. The permeability of iron will be :
(a) $4\pi \times 10^{-7}$ (b) $4\pi \times 10^{-5}$
(c) 88×10^{-7} (d) 88×10^{-5}.

53. If a current flowing through a loop of wire is doubled, its magnetic moment will become :
(a) half (b) double
(c) one-fourth (d) four times.

54. On making the moment of inertia of a magnet four times, its frequency of oscillation will become :
(a) half (b) double
(c) one-fourth (d) four times.

55. Magnetization is
(a) mV (b) m/V
(c) Nm/V (d) V/Nm.

□ □

UNIT 7

Electromagnetic Induction

IMPORTANT FORMULAE

1. Magnetic flux,

(a) $\phi = BA \cos\theta$

where B = Intensity of magnetic field

A = Area of loop of wire

θ = Angle which magnetic field makes with the normal on the loop.

(b) If 'α' is the angle which magnetic field makes with the plane of the loop, then

$$\theta = (90 - \alpha)$$

$$\therefore \quad \phi = BA \cos(90 - \alpha)$$

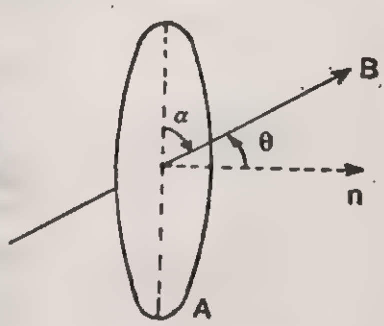


Fig. 7·1.

2. The induced e.m.f.,

$$E = -\frac{N(\phi_2 - \phi_1)}{t}$$ where N = No. of turns in the coil

3. Self induced *e.m.f.* across a coil is given by

(a) $$E = -L\left(\frac{I_2 - I_1}{t}\right)$$

where L = (Self) inductance of the coil

(b) $$N\phi = Li$$

(c) Inductance of a solenoid

$$L = \mu_0 n^2 l A$$ where n = No. of turns per unit length

l = length of solenoid

A = the area of cross-section.

4. Mutual inductance between two coils,

$$M = \mu_0 n_1 n_2 l A$$

$$\phi = Mi$$

5. Transformer—

(a) $$\frac{V_2}{V_1}=\frac{N_2}{N_1}$$

(b) $V_1I_1=V_2I_2$ (for an ideal transformer)

(c) The efficiency of a transformer,

$$\eta=\frac{P_2}{P_1}\times 100=\frac{V_2I_2}{V_1I_1}\times 100$$

(d) Average power dissipated at the load across secondary,

$$P=\frac{\pi n^2A^2}{R\omega}$$

where $n=\frac{N_2}{N_1}$ (turns ratio)

A=Peak value of input voltage

ω (Angular frequency)$=2\pi f$

and R=impedance (resistance) of the load.

6. Induced *e.m.f.* due to the motion of a conductor of length l with a uniform velocity v in a uniform magnetic field B, three being mutually perpendicular to each other, is given by

$$E=Blv$$

7. If a coil of area A, having number of turns N, rotates with uniform angular velocity ω in a uniform magnetic field B, the induced *e.m.f.*

$$E=E_0 \sin \omega t$$

where $E_0=\omega NBA$

SOLVED EXAMPLES

Example 1. *Calculate the magnetic flux linked with a rectangular coil of area 6 cm×8 cm when it is perpendicular to a magnetic field 0·5 Wb m⁻²*

Solution. $B=0{\cdot}5$ Wb m^{-2}, $A=0{\cdot}06$ m$\times 0{\cdot}08$ m$=0{\cdot}0048$ m^2

and $\theta=0$

$$\phi=B.A.\cos\theta$$
$$=0{\cdot}5\times 0{\cdot}0048\times\cos 0$$
$$=0{\cdot}0024 \text{ weber}$$

Example 2. *A rectangular coil of 5 cm×10 cm is perpendicular to a magnetic field 10^{-2} Wb m^{-2} and has 100 turns. What is the magnetic flux linked with the coil ? If the field drops to zero value in 40 millisecond, calculate the induced emf.*

Solution. $B=10^{-2}$ Wb m^{-2} ; $A=0{\cdot}05$ m$\times 0{\cdot}10$ m$=0{\cdot}005$ m^2
$N=100$; $t=40$ *ms*$=0{\cdot}04$ s and $\theta=0°$, $B_2=0$

$\therefore$ The magnetic flux linked with the coil,

$$\phi_1=NBA\cos\theta \quad \& \quad \phi_2=NB_2A\cos\theta$$
$$=0$$
$$=100\times 10^{-2}\times 0{\cdot}005\times\cos 0$$
$$=0{\cdot}005 \text{ Wb}=5\times 10^{-3} \text{ Wb}$$

Now
$$E=-\frac{(\phi_2-\phi_1)}{t}$$
$$=-\frac{(0-5\times 10^{-3})}{0{\cdot}04}=0{\cdot}125 \text{ Volt.}$$

Example 3. *If a coil of area 0·15 m² with 50 turns is perpendicular to a magnetic field which changes from 5×10^{-3} Wb m^{-2} to 2×10^{-3} Wb m^{-2} in a time interval 30 ms, calculate the induced emf.*

Solution. $A=0{\cdot}15$ m^2; $N=50$; $B_1=5\times 10^{-3}$ Wb m^{-2}; $B_2=2\times 10^{-3}$ Wbm^{-2}; $\theta=0°$ and $t=30$ ms$=30\times 10^{-3}$ s.

$$\phi_2-\phi_1=B_2A\cos\theta-B_1A\cos\theta$$
$$=(B_2-B_1)A\cos\theta$$
$$=(2\times 10^{-3}-5\times 10^{-3})\,0{\cdot}15\cos 0.$$
$$=-4{\cdot}5\times 10^{-4} \text{ Wb}$$

$\therefore$
$$E=-N\frac{(\phi_2-\phi_1)}{t}$$
$$=\frac{-50\times -4{\cdot}5\times 10^{-4}}{30\times 10^{-3}}=0{\cdot}75 \text{ Volt.}$$

Example 4. *A coil of wire enclosing an area 100 cm² is placed with its plane making an angle of 70° with a magnetic field B of strength 10^{-2} weber m^{-2}. What is the flux through the coil? B is reduced to zero in 10^{-3} s. What e.m.f. is induced in the coil?*

[*D.S.S.E. 1989*]

Solution. $A=100$ cm$^2=100\times 10^{-4}=10^{-2}$m^2
$\theta=(90-70)=20°$, $B_1=10^{-1}$Wbm^{-2} & $B_2=0$

$\therefore$
$$\phi_1=B_1A\cos\theta$$
$$=10^{-1}\times 10^{-2}\times\cos 20$$
$$=0{\cdot}9397\times 10^{-3} \text{ Wb.}$$
$$\phi_2=B_2A\cos\theta$$
$$=0$$

$$\therefore \qquad E = -\left(\frac{\phi_2 - \phi_1}{t}\right) = -\left(\frac{0 - .9397 \times 10^{-3}}{10^{-3}}\right)$$

$$= 0.9397 \text{ Volt.}$$

Example 5. *A coil with 80 turns and area 0.1 m² being perpendicular to the field is reversed in a time of 50 ms. Calculate the induced emf if the magnetic field acting on the coil is 4×10^{-3} Wb m^{-2}.*

Solution. $N = 80$; $t = 50 \text{ ms} = 0.050 \text{ s}$; $A = 0.1 \text{ m}^2$; $\theta_1 = 0$

$\theta_2 = 180°$ and $B = 4 \times 10^{-3} \text{ Wb m}^{-2}$.

Here
$$\phi_2 - \phi_1 = BA \cos\theta_2 - BA \cos\theta_1$$
$$= BA(\cos\theta_2 - \cos\theta_1)$$
$$= 4 \times 10^{-3} \times 0.1 (\cos 180 - \cos 0)$$
$$= -8 \times 10^{-4} \text{ Wb}$$

Now
$$E = -N\frac{(\phi_2 - \phi_1)}{t}$$
$$= \frac{80 \times 8 \times 10^{-4}}{0.05} = 1.28 \text{ Volt.}$$

Example 6. *A coil with 40 turns is pulled in 0.03 s from the space between the poles of a magnet where its area includes 4×10^{-3} Wb to a place where its area includes 1×10^{-3} Wb. Calculate the induced emf.*

Solution. $N = 40$; $t = 0.03$; $\phi_1 = 4 \times 10^{-3}$ Wb and

$\phi_2 = 1 \times 10^{-3}$ Wb

$$\therefore \qquad E = -N\frac{(\phi_2 - \phi_1)}{t}$$
$$= -40\frac{(1 \times 10^{-3} - 4 \times 10^{-3})}{0.03} = 4 \text{ Volt.}$$

Example 7. *A railway engine is travelling on the level rails with a uniform speed of 54 Km hr^{-1}. Calculate the emf induced between the ends of an axle, 1.50 m. long of the engine, The vertical component of earth's field is 4×10^{-5} Wbm^{-2}.*

Solution. $v = 54 \text{ km hr}^{-1} = 54 \times \frac{5}{18} = 15 \text{ ms}^{-1}$; $l = 1.50$ m

and $B = 4 \times 10^{-5} \text{ Wb m}^{-2}$

$$E = BlV$$
$$= 4 \times 10^{-5} \times 1.50 \times 15 = 9.0 \times 10^{-4} \text{ Volt.}$$

Example 8. *The current in an electromagnet changes from 6A to 2 A in 10 ms. If the induced emf across the coil is 100 V, what is self induction of the coil.*

Solution.

$$E=-L\frac{(I_2-I_1)}{t}$$

$$\therefore \quad L=\frac{E\times t}{(I_1-I_2)}$$

$$=\frac{100\times 0{\cdot}01}{(6-2)}=0{\cdot}25 \text{ Henry}.$$

Example 9. *Calculale the induced emf in the secondary when the current in the primary changes from 5 A to 3 A in 2 s. The mutual inductance between primary and secondary of transformer is 0·16 henry.*

Solution.

$$E=-M\frac{(I_2-I_1)}{t}$$

$$=-0{\cdot}16\frac{(3-5)}{2}$$

$$=0{\cdot}16 \text{ Volt.}$$

Example 10. *A rectangular coil of dimension 30 cm × 10 cm having 100 turns rotates about an axis perpendicular to the uniform magnetic field of 0·04 Wb m*$^{-2}$. *If the coil makes 2000 revolutions per minute, calculate the intantaneous value of induced emf when the angle which the plane of the coil makes with the field is (a) 0° (b) 30° (c) 45° (d) 90°. If the coil forms a closed loop of resistance 25 Ω how much power is dissipated as heat ?*

Solution. Peak value of emf generated,

$$E_0=NBA\omega$$

$$=100\times 0{\cdot}04\times(0{\cdot}3\times 0{\cdot}1)\times\left(2\pi\times\frac{2000}{60}\right)$$

$$=25{\cdot}143 \text{ Volt.}$$

(*a*) θ is the angle which the normal to the plane of the coil makes with the field

$$\therefore \quad \theta=(90-0)=90^\circ$$

$$\therefore \quad E=E_0 \sin\theta$$

$$=25{\cdot}143 \sin 90=25{\cdot}143 \text{ Volt.}$$

(*b*)

$$\theta=(90-30)=60^\circ$$

$$\therefore \quad E=25{\cdot}143 \sin 60=21{\cdot}77 \text{ Volt.}$$

(*c*)

$$\theta=(90-45)=45^\circ$$

$$\therefore \quad E=25{\cdot}143 \sin 45=17{\cdot}779 \text{ Volt.}$$

(*d*) $$\theta=(90-90)=0$$

$$E=25{\cdot}143 \sin 0=0$$

$$I_0=\frac{E_0}{R}=\frac{25{\cdot}143}{25}=1{\cdot}00572$$

$\therefore$ Power dissipated as heat$=E_0I_0=25{\cdot}143\times1{\cdot}00572$

$$=25{\cdot}287 \text{ watt.}$$

Example 11. *Find the dimension of magnetic flux in terms of M, L, T & I.*

Solution. $$\phi=BA\cos\theta$$

and $$F=qvB\sin\alpha \Rightarrow B=\frac{F}{qv\sin\alpha}$$

Since cos θ and sin α are dimensionless factors,

$$\phi=\left(\frac{F}{qv}\right)A$$

$$=\frac{MLT^{-2}}{(IT)(LT^{-1})}L^2$$

$$=ML^2T^{-2}I^{-1}$$

Example 12. *A step up transformer is used to operate a device with an impedance of 440 ohms. The voltage is stepped up from 100 volt to 220 volt. Calculate the currents in primary and secondary. Suppose the transformer to be ideal.*

Solution. $$I_2=\frac{V_2}{Z}$$

$$=\frac{220}{440}=0{\cdot}5 \text{ A}$$

For ideal transformer,

$$V_1I_1=V_2I_2$$

$$I_1=\frac{V_2I_2}{V_1}$$

$$=\frac{220\times{\cdot}5}{100}=1{\cdot}10 \text{ A.}$$

Example 13. *A step down transformer is used on a 220 V line to deliver 25 mA at 11 V. Calculate the curreut drawn from the line. Suppose the transformer is ideal one.*

Solution. $V_1=220$ V; $I_2=25$ mA$=25\times10^{-3}$ A and $V_2=11$ V

For an ideal transformer,

$$V_1I_1=V_2I_2$$

$$\therefore \qquad I_1=\frac{V_2I_2}{V_1}$$

$$=\frac{11\times25\times10^{-3}}{220}$$

$$=1{\cdot}25\times10^{-3}\text{ A}=1{\cdot}25\text{ mA.}$$

Example 14. *A step up transformer connected to an A.C. mains of 220 V, 20 kw steps up the voltage to 880 V. If the transformer is operating a device of resistance 55 w, Find (a) turns ratio of the transformer (b) current in the secondary, (c) efficiency of the transformer, (d) loss of power in the transformer, (e) What is the average power dissipate at the load across secondary if input voltage is V=310 sin 314 t*

Solution. (*a*)

$$\frac{n_2}{n_1}=\frac{V_2}{V_1}$$

$$=\frac{880}{220}$$

$$=4$$

(*b*)

$$I_2=\frac{V_2}{R}$$

$$=\frac{880}{55}$$

$$=16\text{ A}$$

(*c*)

$$\eta=\frac{V_2I_2}{V_1I_1}\times100=\frac{V_2I_2}{P_1}\times100$$

$$=\frac{880\times16}{20000}\times100$$

$$=70{\cdot}4\%$$

(*d*) Power loss $=(P_1-P_2)$

$$=(20{,}000-880\times16)$$

$$=5920\text{ Watt.}$$

(*e*)

$$P=\frac{\pi n^2A^2}{Rw}$$

$$=\frac{3{\cdot}14\times4^2\times310^2}{55\times314}=279{\cdot}5\text{ Watt.}$$

Example 15. *A wheel with 8 metallic spokes each 0·40 m long is rotated with a speed of 150 rev/min in a plane normal to the earth's magnetic field at the place. If the magnitude of the field is 0·15 Tesla, what is the induced emf between the axle and the rim of the wheel ?*

Solution. $$|E| = \frac{d\phi}{dt}$$

$$= \frac{d\,(BA\cos 0)}{dt}$$

$$= B\,\frac{dA}{dt}$$

$$= B\,(N\pi r^2) \quad \text{where } N = \text{No. of revolution per sec.}$$

$$= 0{\cdot}15 \times \frac{150}{60} \times 3{\cdot}14 \times (0{\cdot}40)^2$$

$$|E| = 0{\cdot}1884 \text{ Volt.}$$

Example 16. *A square loop of side 8 cm moves with a velocity of 5 cms^{-1} in a non uniform magnetic field of gradient 2×10^{-3} T cm^{-1} with its plane normal to the field. The magnetic field is not only decreasing with distance along the direction of motion of the loop but also with time at the rate of 4×10^{-3} Ts^{-1}. If the resistance of the loop is 2·5 mΩ, what is the magnitude of the induced current in the loop ?*

Solution. Rate of change of flux due to change in field with distance,

$$\frac{d\phi_1}{dt} = B_1 v A$$

$$= (2\times 10^{-3})\,(5)\,(0{\cdot}08)^2$$

$$= 6{\cdot}4\times 10^{-5} \text{ Wb s}^{-1}$$

Rate of change of flux due to change in magnetic field with time,

$$\frac{d\phi_2}{dt} = B_2 A$$

$$= (4\times 10^{-3})(0{\cdot}08)^2$$

$$= 2{\cdot}56\times 10^{-5} \text{ Wbs}^{-1}.$$

Since both rate of change of flux causes induced emf in the same direction so both effects are added and hence induced emf in loop,

$$E = \left(\frac{d\phi_1}{dt} + \frac{d\phi_2}{dt}\right)$$

$$= (6{\cdot}4\times 10^{-5} + 2{\cdot}56\times 10^{-5})$$

$$E = 8{\cdot}96\times 10^{-5} \text{ Volt}$$

∴ The induced current in the loop,

$$I = \frac{E}{R}$$

$$=\frac{8{\cdot}96\times10^{-5}}{2{\cdot}5\times10^{-3}}$$

$$=3{\cdot}584\times10^{-2}\text{ A}$$

Example 17. *A small flat search coil of area 5 cm² with 140 closely wound turns is placed between the poles of a powerful loud-speaker magnet and then quickly snatched out of the field region. The total charge flown in the coil is 10·5 mC. If the resistance of the coil is 0·6 Ω, what is the field strength of the magnet ?*

Solution. $\phi_2=0$

$$\therefore \quad E=-N\left(\frac{\phi_2-\phi_1}{t}\right) \text{ becomes}$$

$$E=\frac{N\,\phi_1}{t}$$

$$\therefore \quad \phi_1=E\,\frac{t}{N}$$

$$=IR\,\frac{t}{N}$$

$$=\frac{QR}{N} \qquad [\because \; It=Q]$$

$$=\frac{(10{\cdot}5\times10^{-3})\,(0{\cdot}6)}{140}$$

$$=4{\cdot}5\times10^{-5}\text{ Wb}$$

$$\therefore \quad B=\frac{\phi_1}{A}$$

$$=\frac{4{\cdot}5\times10^{-5}\,Wb}{5\times10^{-4}\text{ m}^2}$$

$$B=0{\cdot}09\;Wb\text{ m}^{-2}$$

Example 18. *An air cored solenoid with length 25 cm and area of cross-section 18 cm² and number of turns 400 carries a current of 3·5 A. The current is suddenly switched off in a time of 1 ms. How much is the average back e.m.f. induced across the ends of the open switch in the circuit ?*

Solution. Since final current in the solenoid is zero,

$$\phi_2=0$$

$$\therefore \quad E=-N\left(\frac{\phi_2-\phi_1}{t}\right)$$

$$=\frac{N\,\phi_1}{t}$$

$$=\frac{NBA}{t}$$

But $$B=\mu_0 nI$$

$$=4\pi\times10^{-7}\left(\frac{400}{0{\cdot}25}\right)\times3{\cdot}5$$

$$=7{\cdot}04\times10^{-3}\ T$$

$\therefore$ $$E=\frac{400\times(7{\cdot}04\times10^{-3})\times(18\times10^{-4})}{(1\times10^{-3})}$$

$$E=5{\cdot}0688\ \text{Volts.}$$

Example 19. *A toroidal solenoid with an air core has an average radius of 12 cm and area of cross-section 10 cm² and 1400 turns. (a) What is the self inductance of the toroid (b) If a second coil of 320 turns is wound closely on the toroid and the current in the primary coil is increased from 0 to 1·5 A in 0·03 s, what the emf induced in the secondary.*

Solution.

(*a*) $$B=\frac{\mu_0 NI}{2\pi r}$$

and $$LI=N\phi$$

$\therefore$ $$L=\frac{N\,(BA)}{I}$$

$$=\frac{N\left(\frac{\mu_0 NI}{2\pi r}\right)A}{I}$$

$$L=\frac{\mu_0 N^2 A}{2\pi r}$$

$$=\frac{4\pi\times10^{-7}\times(1400)^2\,(10\times10^{-4})}{2\pi\,(0{\cdot}12)}$$

$$=3{\cdot}26\times10^{-3}\ H$$

$$=3{\cdot}26\ mH$$

(*b*) **The magnitude of the induced emf in the secondary,**

$$|E|=N\frac{d\phi}{dt}=N_2\frac{d}{dt}\,(BA)$$

$$=N_2\frac{d}{dt}\left(\frac{\mu_0 N_1 I}{2\pi r}A\right)$$

$$=\frac{\mu_0 N_1 N_2 A}{2\pi r}\frac{dI}{dt}$$

$$=\frac{4\pi\times10^{-7}\times1400\times320\,(10\times10^{-4})}{2\pi\,(0{\cdot}12)}\times\frac{(1{\cdot}5-0)}{0{\cdot}03}$$

$$=37{\cdot}256\text{ mV.}$$

Example 20. *A short solenoid of length 5 cm and radius 1·2 cm and number of turns 80 lying inside on the axis of a long solenoid 60 cm in length and having number of turns 1600. (a) What is the flux through the long solenoid if a current of 4·0 A flows through the short solenoid ? (b) What is the mutual inductance of the two solenoids ?*

Solution. (*a*) The total magnetic flux linked with the long solenoid is

$$\phi_1=MI_2$$

$$=(1{\cdot}45\times10^{-3})\times4$$

$$=5{\cdot}8\times10^{-3}\text{ Wb.}$$

(*c*) In this question the mutual inductance between the two solenoids will be given by

$$M=\frac{\mu_0\,N_1\,N_2\,(\pi R_2^2)}{l_1}$$

where 1 represents the long solenoid and 2 the short solenoid.

$$\therefore\qquad M=\frac{4\pi\times10^{-7}\times1600\times80\times\pi\times(0{\cdot}012)^2}{0{\cdot}05}$$

$$=1{\cdot}45\times10^{-3}\text{ H}$$

$$=1{\cdot}45\text{ mH}$$

Example 21. *A square metal wire loop of side 10 cm and resistance 1 ohm is moved with a constant velocity v_0 in a uniform*

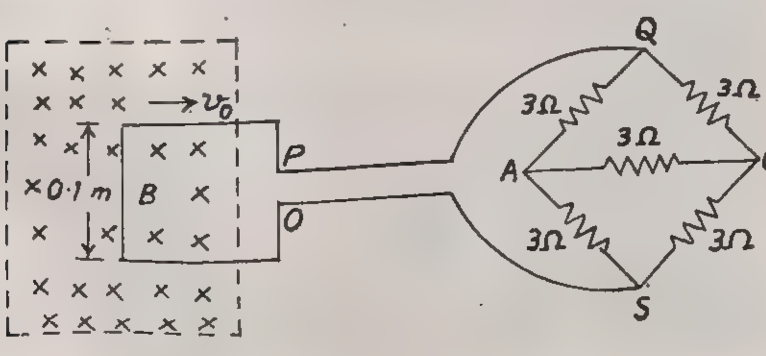


Fig. 7·2.

magnetic field of induction $B=2$ Wb m^{-2} as shown in Fig. 7·2. The magnetic lines are perpendicular to the plane of the loop (directed into the paper). The loop is connected to a network of resistors each of value 3 Ω. The resistance of the loop wires OS and PQ are negligible. What should be the speed of the loop so as to have a steady current of 1 mA in the loop ? Give the direction of the current in the loop.

[*I.I.T. J.E.E. 1983*]

Solution. The resistance of each arm QCS and QAS is $(3+3)=6\ \Omega$ which are connected in parallel with each other. So the resistance of the network QCSA,

$$\frac{1}{R}=\frac{1}{6}+\frac{1}{6}$$

$\therefore$ $$R=3\ \Omega.$$

$\therefore$ The total resistance of the squase loop and the network QCSA,

$$R'=3+1$$

or $$R'=4\ \Omega$$

$$I=1\text{ mA}=10^{-3}\text{ A}$$

$\therefore$ $$V=R'\,I$$

$$V=4\times10^{-3}\text{ Volts}$$

Now $$V=Bl\,v_0$$

$\therefore$ $$v_0=\frac{V}{Bl}$$

$$=\frac{4\times10^{-3}}{2\times0{\cdot}1}$$

$$=2\times10^{-2}\text{ ms}^{-1}=2\text{ cm s}^{-1}$$

The direction of current will be PQSO according to Fleming's right hand rule.

EXERCISE 7

1. Calculate the magnetic flux linked with a coil of area 42 cm^2 which is perpendicular to a uniform magnetic field 2×10^{-3} Wb m^{-2}.
2. A rectangular coil of dimension 9 cm × 6 cm is inclined at an angle of 30° to the uniform magnetic field 0·02 Wb m^{-2}. Calculate the magnetic flux passing through the coil.
3. A circular coil of radius of 15 cm is perpendicular to a uniform magnetic field 7×10^{-2} Wb m^{-2} and has 50 turns. What is the magnetic flux linked with the coil ? If the field drops to zero value in 30 ms, calculate the induced emf.
4. A conductor whose active length in a magnetic field of 0·05 Wb m^{-2} is 40 cm moves at a velocity of 5m s^{-1} perpendicular to the field. Calculate the induced emf across the conductor.

5. An aeroplane with wing span 40 m is moving with uniform velocity 360 km hr^{-1} in earth's magnetic field in horizontal direction. If the vertical component of the earth field is $5 \cdot 4 \times 10^{-5}$ Wb m^{-2}, calculate the induced emf between the tips of wings.

6. A bicycle generator generates 3 volt when the bicycle is travelling at a speed of 9·0 km hr^{-1}. What emf is generated when the bicycle is travelling at 15 km hr^{-1} ?

7. A bar 10 cm long is perpendicular to a uniform magnetic field of 0·04 Wb m^{-2}. Calculate the speed with which the bar should move through the field to generate the emf of 0·015 volt between its ends.

8. A circular coil of 400 turns and radius 10 cm is kept horizontally on a table. Calculate the induced emf when it is turned over in 0·2 second. Vertical component of earth's magnetic field is $4 \cdot 2 \times 10^{-5}$ Wb m^{-2}.

9. A rectangular coil of dimension 15 cm $\times$ 30 cm having 500 turns rotates in a field of 5×10^{-3} Wb m^{-2} at a speed of 1000 revolutions per minute. Calculate the peak value of induced emf and instantaneous value of emf in the coil when its plane make an angle of 45° with the direction of field.

10. A rectangular coil 50 cm $\times$ 30 cm with 100 turns is perpendicular to a magetic field which changes from 6×10^{-2} Wb m^{-2} to 2×10^{-2} Wb m^{-2} in 0·02 s. Calculate the induced emf.

11. A rectangular coil of dimension 0·35 m $\times$ 0·15 m having 200 turns rotates about an axis perpendicular to uniform magnetic field 3×10^{-2} Wb m^{-2} If the coil makes 3000 revolutions per minute, calculate the instantaneous value of induced emf when the angle which the plane of the coil makes with the field is (*a*) 0° (*b*) 30° (*c*) 60° and (*d*) 90°.

12. Calculate the value of induced emf in a coil of inductance 0·03 H when current passing through it changes at the rate of 150 A per second. *[DSSE 1990]*

13. When a current in an electromagnet changes from 7A to 3A in 0·02 s, an emf of 200 volt is generated in the coil. What is the self inductance of the coil ?

14. If the induced emf in secondary of a transformer is 5 volt due to the change in primary current from 0·2 A to 0·5 A in 0·04s, calculate the mutual inductance of the two coils.

15. In order to step up voltage from a 200 volt line, a step up transformer having 50 turns in primary is used to operate a device whose impedance is 400 Ω If the secondary draws 0·4 A current, calculate the number of turns in the secondary, supposing transformer to be 100% efficient.

16. A step down transformer is used on a 66 KV line to deliver a current of 200 A at 220 V across the secondary. Calculate the current drawn from the line, supposing the transformer to be ideal one.

17. How much current is drawn by the secondary and the primary coil of a transformer which steps up Voltage from 200 V to 1 KV to operate a device with an impedance of 400 Ω.

18. A step down transformer is used to operate a device whose impedance is 440 Ω. The voltage is stepped down from 220 V to 22 volt. Calculate the current in primary.

19. The primary of a transformer having turns ratio 5 : 2 is connected to 220 V a.c. supply. If it is used to operate a device whose impedance is 400 ohms, calculate the current in secondary and primary.

20. A step up transformer connected to an A.C. mains of 200 V and 2 KW steps up voltage to 1200 V. If the transformer is operating a device of resistance 800 Ω, find (*a*) turns ratio of transformer (*b*) current in the primary and secondary (*c*) efficiency of the transformer (*d*) loss of power in the transformer (*e*) What is the power dissipated at the load across secondary if input voltage is $V=310 \sin 314\, t$.

21. If in Q.No. 20 above, we have a step down transformer with $V_1=240$ V, $P_1=600$ W and $V_2=12$ V, $R_2=0{\cdot}6$ Ω, find all the quantities (*a*) to (*e*) if input voltage is $V=300 \sin 157\, t$.

22. If the voltage of 220 V is applied on the primary of a transformer its secondary produces 96 V. The primary has 550 turns. If the leakage of magnetic flux amounts to 4%, what is the number of turns in the secondary.

23. In order to step down voltage from 11 KV line to 220 V, a step down transformer having 2000 turns in primary is used. Find the number of turns in the secondary and the current drawn from the line if the current flowing in secondary is 100 A.

24. A transformer has an efficiency of 96%. It works at 200 V and 5 KW line. If the secondary voltage is 240 V, calculate the currents in primary and secondary.

25. A battery eliminator's transformer draws a current of 0·1 A at 240 V from mains and steps down it to 6 V. If the efficiency of the transformer is 98% (*a*) what is the current in the secondary (*b*) if the number of turns in primary is 320, what is the number of turns in secondary ?

26. A long solenoid of 35 turns/cm has a small loop of area 6·5 sq. cm. placed inside with the normal of the loop parallel

to axis. Calculate the voltage across the loop if the current in in the solenoid is changed at a steady rate from 1A to 2A in 0·2 s.

27. A short solenoid of length 4·4 cm and radius 0·7 cm and the number of turns 50 lying inside on the axis of a long solenoid 75 cm in length and having 1500 turns (*a*) what is the flux through the long solenoid if a current of 5·0 A flows through the short solenoid ? (*b*) what is the mutual inductance of the two solenoid ?

28. A toroidal solenoid with an air core has an average radius of 12 cm and area of cross section 8 cm^2 and 1200 turns (*a*) What is the self inductance of the toroid (*b*) If a second coil of 400 turns is wound closely on the toroid and the current in the primary coil is increased 1A to 2A in 0·02 s, what is the e.m.f. induced in the secondary ?

29. If in Q. No. 28 above $r=15$ cm, $A=12$ cm^2, $N_1=1500$ (*a*) What is the self inductance of the coil (*b*) If $N_2=600$, $I_1=2A$, $I_2=2·5$ A and $t=0·01$ s, what is the e.m.f. induced in the secondary.

30. An air cored solenoid with length 30 cm and area of cross section 16 cm^2 and number of turns 700 carries a current of 3A. The current is suddenly switched off in a time of 0·4 mS. How much is the average back e.m.f. induced across the ends of the open switch in the circuit ?

31. A small flat search coil of area 10 cm^2 with 120 closely wound turns is placed between the poles of a powerful magnet and then quickly snatched out of the field region. The total charge flown in the coil is 8 mc. What is the field strength of the magnet if the resistance of the coil is 1·5 Ω ?

32. A rectangular loop of 15 cm × 7 cm move with a velocity of 4 cm s^{-1} in a non uniform magnetic field of gradient $1·5\times10^{-3}$ T cm^{-1} with its plane normal to the field. The magnetic field is not only increasing with distance along the direction of motion of the loop but also with time at the rate of 2×10^{-3} T s^{-1}. If the resistance of the loop is 4 m Ω, what is the magnitude of the induced current in the loop ?

33. If in Q. No. 32 above $A=10$ cm × 8 cm, $v=6$ cm s^{-1}, $B_1=4\times10^{-4}$ T cm^{-1}, $B_2=3\times10^{-3}$ T s^{-1} and $R=5$ m Ω, what is the magnitude of the induced current in the loop.

34. A wheel with 10 metallic spokes each 0·20 m long is rotated with a speed of 210 rev /min in a plane normal to the earth's magnetic field at the place. If the magnitude of the field is 2T, what is the induced e m.f. between the axle and the rim of the wheel ?

35. A conducting rod of 50 cm length moves with a frequency of 900 rev./min, with one end at the centre and the other end at the circumference of a circular metallic ring of radius 50 cm,

about an axis passing through the centre of the coil perpendicular to the plane of the coil. A constant magnetic field parallel to the axis is present everywhere. What is the e.m.f. developed between the centre and the metallic ring ? (B=0·25 T)

OBJECTIVE TYPE QUESTIONS

36. A moveable wire XY sliding to the right in the presence of a uniform magnetic field induces an anticlockwise current as shown in Fig. 64 below. What amongst the following is a possible direction of magnetic induction in region P ?

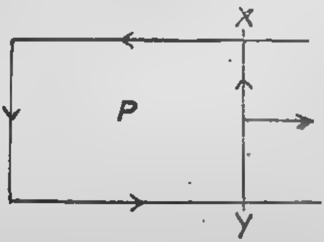


Fig. 7·3.

(*a*) Downward into the paper
(*b*) To the right
(*c*) To the left
(*d*) Upwards perpendicular to the paper.

37. A rectangular coil ABCD lying flat on the table moves towards a current carrying conductor PQ which is also lying on the table and parallel to side AD along the direction as shown in Fig. 7 4. The direction of the induced current in the coil is

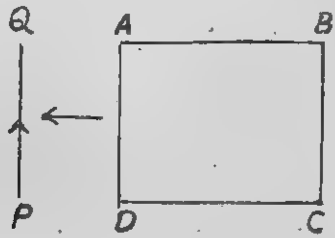


Fig. 7·4.

(*a*) Clockwise direction (*b*) Anticlockwise direction
(*c*) Current will not induce (*d*) Undecided.

38. A circular coil and a bar magnet are moving away from each other with a constant velocity, the plane of the coil is being perpendicular to the axis of the magnet. Then there will be an e.m.f. in the coil which is

(*a*) increasing with time (*b*) decreasing with time
(*c*) steady e.m.f. (*d*) no e.m.f.

39. A magnet is allowed to fall through a metal ring, then during the fall its acceleration '*a*' is
(*a*) $a < g$ (*b*) $a > g$
(*c*) $a = g$ (*d*) $a = 0$.

40. Whenever the flux linked with a circuit changes, there is an induced e.m.f. in the circuit. This e.m.f. would last in the circuit
(*a*) for a very short time
(*b*) for a very long time
(*c*) for ever
(*d*) as long as the flux in the circuit changes.

41. When a magnet is plunged into a coil, the e.m.f. induced in the coil does not depend upon
(*a*) No. of terms in the coil
(*b*) the speed with which the magnet is moved
(*c*) the resistance of the coil
(*d*) the magnetic field of the magnet.

42. Two circular loops of wire are placed in the proximity of one another and one of them is carrying a current in clockwise direction. If the current in this coil is suddenly stopped. Then the induced current in the other loop will be
(*a*) Anticlockwise (*b*) Clockwise
(*c*) no induced current (*d*) undecided.

43. If the number of turns in a coil is doubled, then its self-inductance will become
(*a*) double (*b*) halved
(*c*) four times (*d*) unchanged.

44. A coil and a bulb are connected in series with a 3 volt d.c. battery. A soft iron core is then inserted in the coil. Then the intensity of the bulb
(*a*) remains the same (*b*) increases
(*c*) decreases (*d*) undecided.

45. A bulb and an inductor is connected in series with a steady voltage battery in a circuit. When the key in the circuit is made off, the intensity of the light of the bulb will become
(*a*) zero immediately (*b*) zero gradually
(*c*) very large before becoming zero
(*d*) undecided.

□ □

Alternating Current Circuits

IMPORTANT FORMULAE

1. Alternating current,

$$I = I_0 \sin \omega t \quad \text{or} \quad I = I_0 \cos \omega t$$

2. Effective values of alternating emf and alternating current,

$$E_{eff} = \frac{E_0}{\sqrt{2}} \quad \text{and} \quad I_{eff} = \frac{I_0}{\sqrt{2}}$$

3. In an *a.c*, circuit,

(*a*) if it contains only resistance 'R',

$$I_{eff} = \frac{E_{eff}}{R} \;;$$

(*b*) if it contains pure inductance 'L',

$$I_{eff} = \frac{E_{eff}}{X_l}$$

where X_l (Inductive reactance) $= \omega L = 2\pi\nu L$ and emf applied to a purely inductive circuit leads the current in circuit in phase by $\pi/2$.

(*c*) if it contains only capacitance C,

$$I_{eff} = \frac{E_{eff}}{X_c}$$

where X_c (Capacitive reactance) $= \dfrac{1}{\omega C} = \dfrac{1}{2\pi\nu C}$

and emf applied to a purely capacitive circuit lags behind the current in the circuit in phase by $\pi/2$.

(*d*) if it contains inductance, capacitance and resistance *i.e.*, for LCR circuit,

$$I_{eff} = \frac{E_{eff}}{Z}$$

Where Z (Impedance) $= \sqrt{R^2 + (X_l - X_c)^2}$ and phase angle, ϕ, is given by

$$\tan \phi = \frac{X_l - X_c}{R}$$

4. Resonant frequency of a LCR circuit,

$$f_r=\frac{1}{2\pi}\,\frac{1}{\sqrt{LC}}$$

5. The average power of an *a.c.* circuit,

$$P_{av}=E_{eff}\,I_{eff}\cos\theta$$

$$\text{where } \cos\theta=\frac{R}{\sqrt{R^2+\left(L\omega-\frac{1}{C\omega}\right)^2}} \text{ for a}$$

Circuit containing L, C and R and is called power factor.

6. In a *d.c.* circuit consisting of C & R,

(*a*) during charging of a capacitor,

$$Q=Q_0\left(1-e^{-\frac{t}{RC}}\right)$$

where $Q_0=CV$ & e (exponential constant$=2{\cdot}718$)

and $$I=I_0\,e^{-\frac{t}{RC}}$$

(*b*) during discharging of a capacitor.

$$Q=Q_0e^{-\frac{t}{RC}}$$

and $$I=-I_0e^{-\frac{t}{RC}}$$

7. In *d.c.* circuit consisting of L & R,

(*a*) during growth of current, $I=I_0\left(1-e^{-\frac{R}{L}t}\right)$

(*b*) during decay of current, $I=I_0\,e^{-\frac{R}{L}t}$

SOLVED EXAMPLES

Example 1. *What will be instantaneous voltage across a resistor after the time $\frac{1}{300}$ S. When it is connected to 220 Volt, 50 Hz a.c.*

Solution. $E_{eff}=220$ Volt, $\nu=50$ Hz, $t=\frac{1}{300}$ s

$$\therefore \quad E_0=\sqrt{2}.E_{eff}$$
$$=1{\cdot}414\times220=311{\cdot}08 \text{ volt}$$

$$\therefore \quad E=E_0\sin 2\pi\nu t$$

$$=311{\cdot}08 \sin 2\pi \times 50 \times \frac{1}{300}$$

$$=311{\cdot}08 \times \frac{\sqrt{3}}{2} = 269{\cdot}4 \text{ volt.}$$

Example 2. *What is the reactance of a capacitor of 5 μf at (a) 50 Hz, (b) 10^6 Hz.* [*AISSE 1980*]

Solution. $C = 5\mu f = 5 \times 10^{-6} f$

(*a*)
$$X_c = \frac{1}{2\pi\nu C}$$

$$= \frac{1}{2\pi \times 50 \times 5 \times 10^{-6}}$$

$$= 636{\cdot}4\ \Omega$$

(*b*)
$$X_c = \frac{1}{2\pi \times 10^6 \times 5 \times 10^{-6}}$$

$$= 0{\cdot}0318\ \Omega$$

Example 3. *The reactance of a coil is 31·4 ohm at 50 Hz. Calculate the inductance of the coil. What will be its reactance at 4 KHz ?*

Solution. $X_L = 31{\cdot}4$ ohm ; $\nu_1 = 50$ Hz ; $\nu_2 = 4$KHz
$= 4 \times 10^3$ Hz

$$X_L = 2\pi\nu_1 L$$

$$\therefore \quad L = \frac{X_l}{2\pi\nu_1}$$

$$= \frac{31{\cdot}4}{2 \times 3{\cdot}14 \times 50} = 0{\cdot}1 \text{ Henry}$$

New reactance,

$$X^1_L = 2\pi\nu_2 L$$

$$= 2 \times 3{\cdot}14 \times (4 \times 10^3) \times 0{\cdot}1$$

$$= 2512 \text{ Ohm.}$$

Example 4. *A coil has a resistance of 20 ohm and inductance of 0·07 Henry. Calculate the inductive reactance and impedance of the coil at 50 Hz.*

Solution. $R = 20\ \Omega$; $L = 0{\cdot}07$ H ; $\nu = 50$ Hz.

Inductive reactance,

$$X_L = 2\pi\nu L$$

$$= 2 \times \frac{22}{7} \times 50 \times 0{\cdot}07 = 22\Omega$$

Impedance,

$$Z = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{20^2 + 22^2} = 29{\cdot}73 \text{ Ohm}$$

Example 5. *When a coil is connected to a 100 Valt d.c. supply, the current is 1·0 A. But if the same coil is connected to 100 Volt a.c. supply at 50 Hz., the current is only 0·5 A. Calculate the resistance, inductance and impedance of the coil.*

Solution. For *d.c.* supply,

$$\frac{V}{I}=R$$

$\therefore$ Resistance, $R=\dfrac{100}{1{\cdot}0}=100\ \Omega$

For *a.c.* supply,

$$\frac{V_{eff}}{I_{eff}}=Z$$

$\therefore$ **Impedance**, $Z=\dfrac{100}{0{\cdot}5}=200\ \Omega$

Now $\qquad Z^2=R^2+X_L^2$

$\therefore$ Reactance, $X_L=\sqrt{Z^2-R^2}$

$$=\sqrt{200^2-100^2}$$

$$=100\sqrt{2^2-1^2}$$

$$=173{\cdot}2\ \Omega$$

Inductance, $\qquad L=\dfrac{X_L}{2\pi\nu}$

$$=\frac{173{\cdot}2}{2\times3{\cdot}14\times50}=0{\cdot}55 \text{ Henry.}$$

Example 6. *What will be the reading of an ammeter in an a.c. circuit containing a capacitance of 40 μf and a resistor of 10 Ohms in series. The electric source in the circuit is marked 230 V, 50 Hz. Neglect the resistance of the ammeter.*

Solution. $C=40\ \mu f=40\times10^{-6} f$; $R=10\ \Omega$; $V_{eff}=230$ V and $\nu=50$ Hz.

$$X_c=\frac{1}{2\pi\nu C}$$

$$=\frac{1}{2\times3{\cdot}14\times50\times(4\times10^{-5})}=79{\cdot}6\ \Omega$$

$\therefore \qquad Z=\sqrt{R^2+X_c^2}$

$$=\sqrt{10^2+79{\cdot}6^2}=80{\cdot}23\ \Omega$$

The reading of ammeter,

$$I_{eff}=\frac{V_{eff}}{Z}$$

$$=\frac{230}{80{\cdot}23}=2.87 \text{ A.}$$

Example 7. *An a.c. circuit contains $L=100$ mH, $C=10$ μf and $R=30\ \Omega$ all in series and the instantaneous emf in the circuit may be given by $E=310 \sin 314\, t$. Calculate the following :*

(*a*) *the frequency of the applied emf.*

(*b*) *the net reactance in the circuit.*

(*c*) *the impedance of the circut.*

(*d*) *the effective emf and current in the circuit.*

(*e*) *the phase angle of the current with applied emf.*

(*f*) *the voltages across inductor, capacitor and resistor.*

(*g*) *construct a vector diagram for these voltages.*

(*h*) *the equation for instantaneous current in the circuit.*

(*i*) *the additional inductance required to make the circuit as resonant circuit.*

Solution. $L=100\text{ mH}=0{\cdot}1\text{ H}$; $C=10\ \mu f=10\times10^{-6} f$

$R=30$ ohms

Given ; $E=310 \sin 314\, t$

But $E=E_0 \sin \omega t$

$\therefore$ $E_0=310$ Volt ; and $\omega=314$

(*a*) $\therefore$ $\nu=\dfrac{\omega}{2\pi}$

$$=\frac{314}{2\times3{\cdot}14}=50\text{ Hz.}$$

(*b*) $X_L=\omega L$

$$=314\times0{\cdot}1=31{\cdot}4\ \Omega$$

$$X_C=\frac{1}{\omega C}$$

$$=\frac{1}{314\times10^{-5}}=318{\cdot}5\ \Omega$$

Net reactance $=X_C-X_L$

$$=318{\cdot}5-31{\cdot}4$$

$$=287{\cdot}1\ \Omega \text{ capacitive (as } X_C>X_L)$$

(*c*) $Z=\sqrt{R^2+(X_C-X_L)^2}$

$$=\sqrt{30^2+287{\cdot}1^2}=288{\cdot}6\text{ Ohm}$$

(*d*) $E_{eff}=\dfrac{E_0}{\sqrt{2}}$

$$=0{\cdot}707\times310=219{\cdot}2\text{ Volt}$$

$$I_{eff}=\frac{E_{eff}}{Z}$$

$$=\frac{219{\cdot}2}{288{\cdot}6}=0{\cdot}76\text{ A}$$

(e)
$$\phi = \tan^{-1}\left(\frac{X_c - X_L}{R}\right)$$
$$= \tan^{-1}\left(\frac{287 \cdot 1}{30}\right)$$
$$= \tan^{-1}(9 \cdot 57) = 84^\circ$$

It means the applied emf in the circuit lags (since $X_c > X_L$) behind the current in phase by 84°.

(f)
$$V_L = X_L . I_{eff}$$
$$= 31 \cdot 4 \times 0 \cdot 76 = 23 \cdot 86 \text{ Volt}$$
$$V_c = X_{c.eff}$$
$$= 318 \cdot 5 \times 0 \cdot 76 = 242 \cdot 06 \text{ Volt}$$
$$V_R = R . I_{eff}$$
$$= 30 \times 0 \cdot 76 = 22 \cdot 8 \text{ Volt}$$

(g) Let 20 Volt = 1 cm

Then
$$V_L = \frac{23.86}{20} \simeq 1 \cdot 2 \text{ cm (OB)}$$
$$V_c = \frac{242 \cdot 06}{20} \simeq 12 \cdot 1 \text{ cm } (\infty)$$

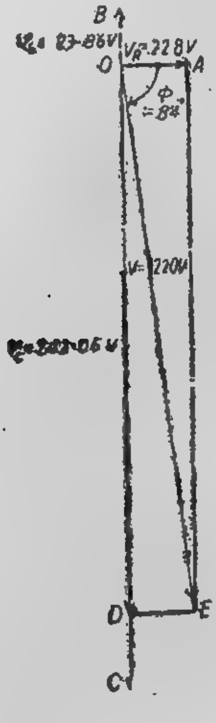


Fig. 8·1.

and $$V_R = \frac{22.8}{20} \simeq 1.1 \text{ cm (OA)}$$

$\therefore$ $$OD = OC - OB = 12.1 - 1.2 = 10.9.$$

Construct the vector diagram is shown in Fig. 8.1.

Measure OE. It is found to be 11 cm

$\therefore$ the resultant voltage $= 11 \times 20 = 220$V

(*h*) $$I_0 = \frac{E_0}{Z} = \frac{310}{288.6} = 1.07 \text{ A}$$

and $$\phi = 84° = \frac{84 \times \pi}{180} = 1.466 \text{ rad}$$

$$I = I_0 \sin(\omega t + \phi)$$
$$= 1.07 \sin(314t + 1.466)$$

(*i*) For resonant circuit,

$$X_L = X_c$$

$\therefore$ $$\omega L = X_c$$

or $$L = \frac{X_c}{\omega}$$

$$= \frac{318.5}{314} = 1.014 \text{ H}$$

$\therefore$ Additional inductance $= 1.014 - 0.1 = 0.914$ H.

Example 8. *Prove that when a voltage $V = V_0 \sin 2\pi\nu t$ is applied across a resistor, the average power dissipated per cycle is given by*

$$P_{av} = \frac{V^2_{eff}}{R} \textit{ where } V_{eff} = \frac{V_0}{\sqrt{2}}.$$

Solution. Let instantaneous voltage across the resistor be

$$V = V_0 \sin 2\pi\nu t$$

$\therefore$ Instantaneous current through the resistor,

$$I = \frac{V_0}{R} \sin 2\pi\nu t$$

$$= I_0 \sin 2\pi\nu t \quad \text{where } I_0 = \frac{V_0}{R}$$

$\therefore$ Instantaneous power consumed,

$$= VI$$
$$= V_0 \sin 2\pi\nu t \times I_0 \sin 2\pi\nu t$$
$$= V_0 I_0 \sin^2 2\pi\nu t$$
$$= V_0 I_0 \tfrac{1}{2}(1 - \cos 4\pi\nu t)$$

Now the average value of $\cos 4\pi\nu t$ will be zero as it is positive as long a time as it is negative for any cycle.

$\therefore$ Average power consumed per cycle,

$$=\frac{1}{2} V_0 I_0$$

$$=\frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} = \frac{V_0}{\sqrt{2}} \times \frac{V_0/R}{\sqrt{2}}$$

or $$P_{av} = V_{eff} \times I_{eff} = V_{eff} \times \frac{V_{eff}}{R} \text{ where } V_{eff} = \frac{V_0}{\sqrt{2}}$$

$\therefore$ $$P_{av} = \frac{V^2_{eff}}{R}$$

Example 9. *The L-C tuning circuit of a radio receiver has L=0·020 henry. Calculate the value of the capacitance required to tune the radio at a frequency 800 KHz.*

Solution. $L=0{\cdot}02$ H and $\nu=800$ KHz$=800\times10^3$ Hz

For L–C circuit to reasonate,

$$X_L = X_c$$

$$2\pi\nu L = \frac{1}{2\pi\nu C}$$

$\therefore$ $$C = \frac{1}{4\pi^2\nu^2 L}$$

$$= \frac{1}{4\times(3{\cdot}14)^2\times(8\times10^5)^2\times0{\cdot}02}$$

$$=1{\cdot}98\times10^{-12}\,f = 1{\cdot}98\,pf.$$

Example 10. *An AC circuit with an R in series with a parallel combination of L and C with $V=V_0\cos wt$ is shown in Fig. 8.2 below. (a) Calculate the impedance of the circuit and the current in the circuit if $V_0=300$ V, $\omega=250\ s^{-1}$, $R=80\ \Omega$, $L=2$ H & $C=20\ \mu f$. (b) what is current amplitude in L and C arms of circuit (c) what is average power transferred to the inductor, capacitors and resistor (d) what is the total power input to the circuit (e) what is the power factor of the circuit.*

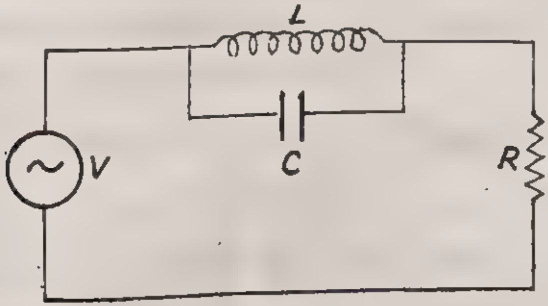


Fig. 8·2.

Solution. We solve such problems by using the complex numbers. If Z_{eq} is the impedance of the parellel combination of L and C,

$$\frac{1}{Z_{eq.}}=\frac{1}{Z_L}+\frac{1}{Z_C}$$

$$=\frac{1}{j\omega L}+\frac{1}{1/j\omega C}$$

$$=\frac{1}{j}\left[\frac{1}{\omega L}+j^2.\omega C\right]$$

$$=\frac{1}{j}\left[\frac{1}{300\times 2}-300\times 20\times 10^{-6}\right]\left[\because\ j^2=-1\right]$$

$$=\frac{1}{j}\left[\frac{1}{600}-\frac{3}{500}\right]$$

$$=\frac{1}{j}\ \frac{13}{3000}$$

$\therefore\quad Z_{eq}\simeq j\,231$

$\therefore\quad$ Total impedance, $Z=R+Z_{eq}$

$$=(80+j\,231)$$

The complex current is then

$$I=\frac{300\times e^{(j\,250\,t)}}{(80+j231)}$$

The physical current i is the real part of current I written as $Re(I)$.

$$\therefore\quad i=Re\,(I)$$

$$=Re\,\frac{300\,(\cos 250\,t+j\sin 250\,t)}{(80+j\,231)}$$

$$=Re\,\frac{300\,(\cos 250\,t+j\sin 250\,t)}{(80+j\,231)}\times\frac{(80-j\,231)}{(80-j\,231)}$$

$$=Re\,\frac{300}{80^2+231^2}\,[(80\cos 250\,t+231\sin 250\,t)$$
$$+j\,(80\sin 250\,t-231\cos 40\,t)]$$

or $$i=\frac{300}{80^2+231^2}\,(80\cos 250\,t+231\sin 250\,t)$$

Let $\quad A\cos\phi=80$

and $\quad A\sin\phi=231$

$$\therefore\quad A^2\cos^2\phi+A^2\sin^2\phi=80^2+231^2$$

$$A^2=(6400+53361)$$

$$A=\sqrt{59761}=244{\cdot}46\ \Omega$$

and $\tan\phi = \frac{231}{80}$

$= 2.8875$

$\therefore$ $\phi = 70°54' = 1.238$ rad

$\therefore$ $i = \frac{300}{A^2} A\,(\cos\phi \cos 250\,t + \sin\phi \sin 250\,t)$

$= \frac{300}{A} \cos(250\,t - \phi)$

$= \frac{300}{244.46} \cos(250\,t - 1.238)$

or $\boxed{i = 1.23 \cos(250\,t - 1.238)}$

$\therefore$ $I_{eff} = \frac{I_0}{\sqrt{2}} = \frac{1.23}{1.414} = 0.87$ A

(b) $$I_L° = \frac{I_0\left(\frac{1}{\omega C}\right)}{\left(\omega L - \frac{1}{\omega C}\right)} = \frac{I_0}{(\omega^2 LC - 1)}$$

$$= \frac{1.23}{(300^2 \times 2 \times 20 \times 10^{-6} - 1)}$$

$$= \frac{1.23}{2.6} = 0.473 \text{ A}$$

$$I_C° = \frac{I_0\,\omega L}{\left(\omega L - \frac{1}{\omega C}\right)} = \frac{I_0 \omega^2 LC}{(\omega^2 LC - 1)}$$

$$= \frac{1.23 \times 3.6}{2.6} = 1.703 \text{ A.}$$

(c) Average power dissipated, $P = V_{eff}\, I_{eff} \cos\theta$

For inductor and capacitor, $\theta = \frac{\pi}{2}$, $P = 0$

For resistor $\theta = 0$, $P = \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} = \frac{I_0^2}{2} R$

$$= \frac{1.23^2 \times 80}{2} = 60.5 \text{ W}$$

(d) Total power input to the circuit $= 60.5$ W

(e) Power factor $= \frac{P}{V_{eff}\, I_{eff}} = \frac{2P}{V_0\, I_0} = \frac{2 \times 60.5}{300 \times 1.23} = 0.328.$

Example 11. *Two capacitors 5 μf and 8 μf in series are connected through a resistance of 13 k Ω to a 26 V battery of negligible internal resistance. After a time of about 6 s, the battery is disconnected and the capacitors are allowed to discharge through the resistance. Determine the voltage across each capacitor after a time lapse 40 ms.*

Solution.
$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}$$
$$=\frac{1}{5}+\frac{1}{8}$$
$$\therefore \quad C=\frac{40}{13}\ \mu f$$
$$C=\frac{40}{13}\times 10^{-6} f$$
$$R=13\ k\ \Omega$$
$$=13\times 10^3\ \Omega$$

$\therefore$ Time constant,
$$RC=13\times 10^3\times\frac{40}{13}\times 10^{-6}$$
$$=40\times 10^{-3}\ s$$
$$=40\ ms$$

Now during charging,
$$Q=Q_0\,(1-e^{-\frac{t}{RC}})$$
$$\frac{t}{RC}=\frac{6}{40\times 10^{-3}}$$
or
$$\frac{t}{RC}>>>1$$

i.e., Q is nearly Q_0 and capacitors are fully charged.
$$\therefore \quad Q_0=CV$$
$$=\frac{40}{13}\times 10^{-6}\times 26$$
$$=80\ \mu C.$$

During discharging for $t=40$ ms
$$Q=Q_0\,e^{-\frac{t}{RC}} \quad \text{becomes.}$$
$$Q=80\,e^{-\frac{40}{40}}$$
$$=\frac{80}{e}$$

$$= \frac{80}{2 \cdot 718}$$

$$= 29 \cdot 4 \ \mu C$$

$$\therefore \quad V_1 = \frac{Q}{C_1} = \frac{29 \cdot 4}{5} = 5 \cdot 88 \text{ V}$$

and $$V_2 = \frac{Q}{C_2} = \frac{29 \cdot 4}{8} = 3 \cdot 675 \text{ V.}$$

Example 12. *Two circuits A and B connected to identical d.c. sources each of e.m.f. 12 V differ greatly in their self inductance. Circuit A has self inductance 12 H and B has 0·04 H. The total external resistance in each circuit which includes the resistance of the inductor itself is 25 Ω.*

(a) Are the steady current values in each circuit equal ? If so, what is the value ?

(b) Compare the time required for the currents in the two circuits to reach $\left(1 - \frac{1}{e^2}\right)$ *of their steady values.*

(c) Which circuit requires greater energy consumption of the source to build up its current to the steady value ?

(d) After the steady state is reached, do the circuits dissipate the same power in the form of heat ? Calculate their values.

Solution. (*a*) Yes, the steady current values in each circuit are equal.

$$I_0 = \frac{V}{R} = \frac{12}{25} = 0 \cdot 48 \text{ A}$$

(*b*) The current in L-C circuit grows according to the relation,

$$I = I_0 \left(1 - e^{-\frac{R}{L}t}\right)$$

For $$I = I_0 \left(1 - \frac{1}{e^2}\right)$$

$$t = \frac{2L}{R}$$

$$\therefore \quad \frac{t_A}{t_B} = \frac{L_A}{L_B} \quad \text{(since R is given to be same)}$$

$$= \frac{12}{\cdot 04}$$

$$= 300 : 1.$$

(*c*) The energy consumption in an inductor to build up steady current (I_0) is given by

$$E = \frac{1}{2} L I_0^2$$

Since I_0 is same for both the circuits A and B but inductance of the circuit A is more than that of B, so energy consumption in circuit A will be more than that in B.

(*d*) The power dissipated (in the form of heat) in a d.c. circuit is given by

$$P = I_0^2 R$$
$$= (0{\cdot}48)^2 \times 25$$
$$= 5{\cdot}76 \text{ watt}$$

is same for both the circuits A and B.

Example 13. *An L-C circuit contains 4 mH inductor and 10 μf capacitor with an initial charge of 20 mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be t=0*

(*a*) *What is the total energy stored initially ? Is it conserved during the L-C oscillations ?*

(*b*) *What is the natural frequeucy of the circuit ?*

(*c*) *At what times is the energy store (i) completely electrical (stored in the capacitor), (ii) completely magnetic (stored in the inductor)*

(*d*) *At what times is the total energy shared equally between the inductor and the capacitor ?*

(*e*) *If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat ?*

Solution. (*a*) Initial total energy,

$$E = \frac{Q_0^2}{2C}$$
$$= \frac{(20 \times 10^{-3})^2}{2 \times 10 \times 10^{-6}}$$
$$= 20 \text{ J}$$

Yes, it is conserved if R=0

(*b*)
$$\nu = \frac{1}{2\pi} \frac{1}{\sqrt{LC}}$$
$$= \frac{7}{2 \times 22} \frac{1}{\sqrt{(4 \times 10^{-3})(10 \times 10^{-6})}}$$
$$= \frac{10^4 \times 7}{2 \times 2 \times 22}$$
$$= 795 \text{ Hz.}$$

(*c*) $Q = Q_0 \cos \omega t$ and $T = \frac{1}{\nu} = \frac{1}{795} = 1{\cdot}26$ ms

The energy stored is (i) completely electrical at

$$t=0, \frac{T}{2}, \frac{2T}{2}, \frac{3T}{2}, \ldots\ldots$$

$$=0, 0{\cdot}63 \text{ ms}, 1{\cdot}26 \text{ ms}, 1{\cdot}89 \text{ ms}, \ldots\ldots$$

(ii) It is completely magnetic (*i.e.*, electrical energy is zero) at

$$t=\frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \ldots\ldots$$

$$=0{\cdot}315 \text{ ms}, 0{\cdot}945 \text{ ms}, 1{\cdot}575 \text{ ms}, \ldots\ldots$$

(d) The total energy E is equally shared between inductor and capacitor.

$$\therefore \quad \frac{1}{2}E=\frac{q^2}{2C}$$

$$\frac{1}{2}\frac{Q_0^2}{2C}=\frac{q^2}{2C}$$

$$q=\frac{Q_0}{\sqrt{2}}$$

But $$q=Q_0 \cos \omega t$$

$$\therefore \quad \cos \omega t=\frac{q}{Q_0}=\frac{1}{\sqrt{2}}$$

$$\therefore \quad \omega t=\frac{\pi}{4}$$

$$t=\frac{\pi}{4\omega}$$

$$=\frac{\pi T}{4\times 2\pi}$$

$$=\frac{T}{8}$$

$$\therefore \quad t=\frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \ldots\ldots$$

$$=0{\cdot}1575 \text{ ms}, 0{\cdot}4725 \text{ ms}, 0{\cdot}7875 \text{ ms}, \ldots\ldots$$

(e) Resistance damp out the oscillations eventually . So whole of the initial energy of 20 J is dissipated as heat.

Example 14. *A series LCR circuit with* $L=0{\cdot}2H$, $C=5\ \mu f$, $R=25\ \Omega$ *is connected to a 250 V variable frequency supply.*

(a) *What is the source frequency for which current amplitude is maximum ? Obtain this maximum value.*

(b) *What is the source frequency for which average power absorbed by the circuit is maximum ? Obtain the value of this maximum power.*

(*c*) *For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?*

(*d*) *What is the Q-factor of the circuit?*

Solution : (*a*) I_0 is maximum when

$$\omega_0=\frac{1}{\sqrt{LC}}$$

$$=\frac{1}{\sqrt{0{\cdot}2\times(5\times10^{-6})}}$$

$$=10^3\text{ Hz}$$

$$=1000\text{ Hz}$$

$$\therefore \qquad \nu_0=\frac{\omega_0}{2\pi}=\frac{1000}{2\times3{\cdot}14}=159\text{ Hz}$$

$$I_0=\frac{V_0}{R}=\frac{\sqrt{2}\,V_{eff}}{R}$$

$$=\frac{1{\cdot}414\times250}{25}$$

$$=14{\cdot}14\text{ A.}$$

(*b*) $P_{av}=\frac{1}{2}I_0^2R$ which is maximum at the same frequency of 159 Hz.

$$\therefore \qquad P_0=\frac{1}{2}(I_0^{max})^2R$$

$$=\frac{1}{2}(14{\cdot}14)^2\times25$$

$$=2500\text{ watt.}$$

(*c*) The power transferred to the circuit will be half the power at resonant frequency at

$$\nu=\left(\nu_0\pm\frac{1}{2\pi}\,\frac{R}{2L}\right)$$

$$=\left(159\pm\frac{1}{2\times3{\cdot}14}\,\frac{25}{2\times{\cdot}2}\right)$$

$$=(159\pm10)$$

$$\nu=169\text{ Hz and }149\text{ Hz}$$

At these frequencies, the current amplitude is $\frac{1}{\sqrt{2}}$ times I_0.

$$\therefore \qquad I'=\frac{I_0}{\sqrt{2}}$$

$$=\frac{14{\cdot}14}{1{\cdot}414}$$

$$=10\text{ A.}$$

(*d*)
$$Q=\frac{\omega_0 L}{R}$$

$$=\frac{1000\times{\cdot}2}{25}$$

$$=8.$$

Example 15. *A small d.c. motor operating at 250 V draws a current of 8 A at its full speed 2500 rev/min. The resistance of the armature is 10 Ω. Calculate (a) the back e.m.f. of the motor, (b) power input, power output (mechanical) and the efficiency of the motor.*

Solution. (*a*) The rotation of the motor changes the magnetic flux and hence an e.m.f. E′ (called back e.m.f.) is induced in a direction opposite to applied e.m.f. E. (Lenz's law).

$$\therefore \quad I=\frac{E-E'}{R} \qquad \text{(Ohm's law)}$$

$$8=\frac{250-E'}{10}$$

$$E'=250-10\times 8$$

$$E'=170\text{ V}$$

(*b*) ∴ Power input,

$$P=EI$$

$$=250\times 8$$

$$=2000\text{ Watt}$$

Power output,

$$P'=ER-I^2R$$

$$=E'I$$

$$=170\times 8$$

$$=1360\text{ watt}$$

$$\therefore \text{ Efficiency } = \frac{P'}{P}\times 100$$

$$=\frac{1360}{2000}\times 100$$

$$=68\%$$

Example 16. *At a hydroelectric power plant, the water pressure head is at a height of 400 m and water flow available is 60 $m^3 s^{-1}$. If the turbine generator efficiency is 70%. Calculate the electric power available from the plant.*

Solution. Power Input,

$$P = F \times v$$
$$= (P \times A) \times v$$
$$= (h\rho g) \times Av$$
$$= h\rho g\ V$$

$\therefore$ $$P = 400 \times 1000 \times 9{\cdot}8 \times 60$$
$$= 235{\cdot}2 \times 10^6\ W$$
$$= 235{\cdot}2\ MW$$

$\therefore$ output power,

$$P' = \text{Efficiency} \times \text{Input power}$$
$$= \frac{70}{100} \times 235{\cdot}2$$
$$= 164{\cdot}64\ MW$$

Example 17. *A small town with a demand of 500 KW of electric power at 220 V is situated 25 km away from the electric plant generating power at 550 V. The resistance of the two wire line carrying power is 0·4 Ω per km. The town gets power from the line through a 5000 V—220 V step down transformer at a sub-station in the town. (a) Calculate the power loss in the line in the form of heat (b) How much power must the plant supply, assuming there is negligible power loss due to leakage. (c) Characterize the transformer at the plant. (d) What advantage will we have if the step down transformer on line is 50,000 V—220 V.*

Solution. Resistance of the line

$$= 0{\cdot}4 \times 50$$
$$= 20\ \Omega$$

Current in the line,

$$I = \frac{P}{V} = \frac{(500 \times 1000)}{5000}$$
$$= 100\ A$$

(a) Power loss in line

$$= I^2 R = 100^2 \times 20 = 200\ KW.$$

(b) $\therefore$ Power supplied by the plant

$$= 500 + 200$$
$$= 700\ KW$$

$\therefore$ Percentage power loss

$$P = \frac{200}{700} \times 100$$
$$= 28{\cdot}6\%.$$

(*c*) **Voltage drop on the line,**

$$V = 100 \times 20$$
$$= 2000 \text{ V}$$

$\therefore$ output voltage of the transformer at power plant

$$= 5000 + 2000 = 7000 \text{ V}$$

Input voltage of transformer

$$= 550 \text{ V}$$

$\therefore$ The transformer is a step up one.

(*d*) $$I' = \frac{(500 \times 1000)}{50{,}000}$$
$$= 10 \text{ A}$$

$\therefore$ Power loss in line

$$10^2 \times 20$$
$$= 2 \text{ KW}$$

$\therefore$ *Percentage power loss,*

$$P' = \frac{2}{602} \times 100$$
$$= 0{\cdot}33\%.$$

By high voltage transmission, the (percentage) power loss is greatly reduced.

Example 18. *Consider the infinite network shown in Fig. 8·3(a).*

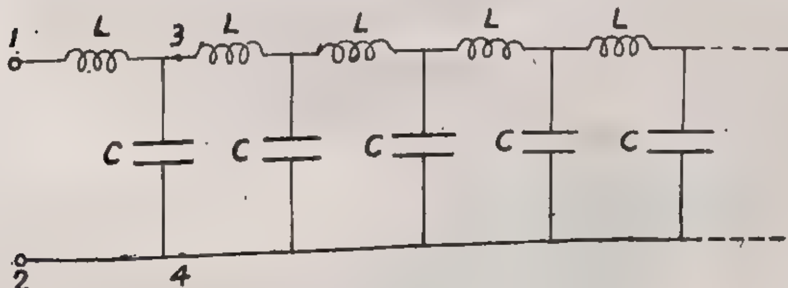


Fig. 8·3 (*a*)

Obtain its equivalent impedance. Hence show that for $\omega < \frac{2}{\sqrt{LC}}$, *the network absorbs energy from the source continuously even though there is no explicit resistor in the network.*

Solution. Let the equivalent impedance of the network between the terminal 1 and 2 be Z. If we take out first L and C, the

left out network between the terminal 3 and 4 will still have the impedance Z as the network is infinite. So the equavalent circuit will be the one as shown in Fig. 8·3 (*b*). This circuit is parallel combination of Z and $\frac{1}{j\omega C}$ which is connected in series with $j\omega L$ and will result in Z again.

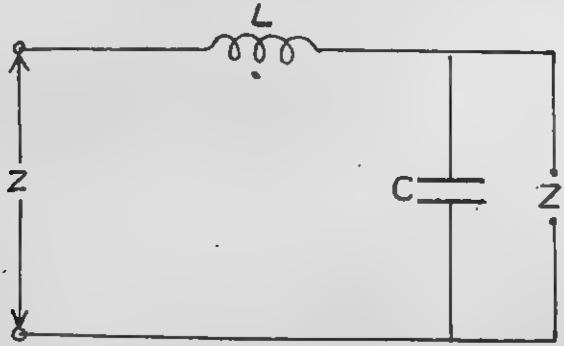


Fig. 8·3 (*b*)

$$\therefore \quad Z = j\omega L + \frac{Z \times \frac{1}{j\omega C}}{\left(Z + \frac{1}{j\omega C}\right)}$$

$$= j\omega L + \frac{Z}{(j\omega CZ + 1)}$$

$$Z = \frac{j^2\omega^2 LCZ + j\omega L + Z}{(j\omega CZ + 1)}$$

or $$(j\omega CZ^2 + Z) = (j^2\omega^2 LCZ + j\omega L + Z)$$

or $j\omega CZ^2 - j^2\omega^2 LCZ - j\omega L = 0$ $\quad [\because j^2 = -1]$

Dividing by $j\omega C$, we have

$$Z^2 - j\omega LZ - L/C = 0$$

It is a quadratic equation in Z whose solution will be

$$Z = \frac{-(-j\omega L) \pm \sqrt{(-j\omega L)^2 - 4 \times 1 \times \left(-\frac{L}{C}\right)}}{2}$$

or $$Z = j\omega \frac{L}{2} \pm \sqrt{\frac{L}{C} - \frac{\omega^2 L^2}{4}}$$

Now if $$\frac{\omega^2 L^2}{4} < \frac{L}{C}$$

or $$\omega < \frac{2}{\sqrt{LC}},$$

z will have a real part and we know real part of an impedance is a resistance.

$\therefore$ The network behaves like a resistor in series with an inductance $\frac{L}{2}$ even without an explicit resistor if $\omega < \frac{2}{\sqrt{LC}}$

$\therefore$ The network absorbs power like any resistor in the circuit. The energy absorbed from source is continuously propagated endlessly along the infinite network. The energy absorbed can not be dissipated as heat as the circuit does not have an actual resistor.

Example 19. *Two resistances one of 100 Ω and another of 200 Ω and an inductance of 10 H is connected to a 3 V battery through a key K as shown in Fig. 8·4 below. What is :*

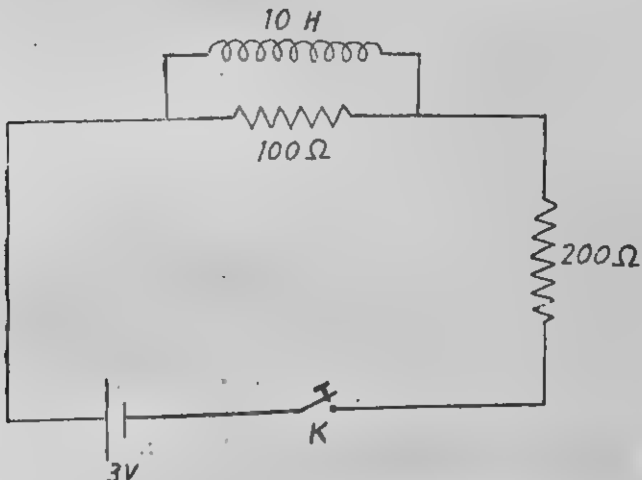


Fig. 8·4.

(*a*) *the initial value of the current drawn from the battery ?*
(*b*) *the initial potential drop across the inductor ?*
(*c*) *the final current drawn from the battery.*
(*d*) *the final current through 100 ohm resistance ?*

[*A.I.S.S.E. 1987*]

Solution. (*a*) Just at the close of the key, the inductor will behave as a large resistance.

$$\therefore \qquad I = \frac{V}{R} = \frac{3}{(100+200)} = 0{\cdot}01 \text{ A}$$

(*b*) So initial P.D. across the inductor,

$$V = IR = 0{\cdot}01 \times 100 = 1 \text{ volt}$$

(*c*) Finally the inductor will behave as a short of zero resistance.

$$\therefore \quad I = \frac{V}{R}$$

$$= \frac{3}{200} = 0{\cdot}015\text{A}.$$

(*d*) Final current through 100 Ω resistance will be zero.

EXERCISE 8

1. The electric supply in a house is marked 220 V, 50 Hz. Give the equation for instantaneous voltage.
2. If the effective current in a 50 c/s a.c. circuit is 5 A, what is (*a*) the peak value of current ? (*b*) the value of current 1/300 second after (*i*) it was zero (*ii*) it was maximum ?
3. What is the inductive reactance of a coil if the current through it is 0 A and the voltage across it is 250 V.
4. At what frequency will a 0·5 henry inductor have a reactance of 2000 ohms.
5. The current through 1·0 henry inductor varies sinusoidally with an amplitude of 0·5 A and a frequency of 50 c/s. Calculate the potential difference across the terminals of the inductor.
6. What is the capacitive reactance of a 5 μf capacitor when it is a part of circuit whose frequency 10 MHz.
7. When 200 volt a.c. is applied across a coil, a current at 2 A flows through it. When 200 V a.c. of 50 Hz is applied to the same coil only 1·0 A flows. calculate the inductance of the coil.
8. Calculate the inductive reactance and inductance of a coil if the a.c. current flowing throught it is 1·00 A when an a.c. voltage applied across it is 100 volt, 50 Hz.
9. The inductance of a coil is 15 mH. Calulate its inductive reactance at (*a*) 50 Hz (*b*) 4 MHz.
10. The reactance of a coil is 100 Ω at 50 Hz. Calculate the reactance of the coil at 10 KHz.
11. What is the reactance of a capacitor of 15 μf at (*a*) 50 Hz. (*b*) 4KHz ?
12. A coil has a resistance of 15 ohms and inductance of 8 mH. Calculate the inductive reactance and impedance of the coil at 50 Hz.
13. Calculate the current in an a.c. circuit containing a capacitance of 25 μf and a resistor of 25 Ω in series. The a.c. source applied to the circuit has voltage 220 volts, 50 Hz.

14. When a coil is connected to a 60 volt d.c. supply, the current is 0·5 A. But if the same coil is connected to 60 volt a.c. supply at 50 Hz, the current becomes 0·2 A. Calculate the resistance, impedance, reactance and inductance of the coil.

15. An inductance of 0·1 henry connected in series with a resistance of 15 ohms is joined with an a.c. supply of 200 volt, 50 Hz. Calculate (*a*) the current in the circuit, (*b*) the voltage across the inductance and (*c*) the voltage across the resistance.

16. In a LCR series circuit, the value of L, C and R are 5 mH, 5 μf and 10 ohm respectively. An a.c. emf of 200 volt, 50 Hz is applied across the ends of the circuit. Calculate the amplitude and phase of the current which flows in the circuit.

17. An emf $E=311{\cdot}13 \sin 314\ t$ volts is applied to a coil of resistance 125 ohms and self inductance 0·24 H. Calculate the amplitu e and the phase of the current flowing in the circuit. What is the value of voltage across inductor and resistor ?

18. An emf $E=282{\cdot}8 \sin 100\ \pi t$ volts is applied to a LCR series circuit with L=0·05 H, C=10 μf and R=25 ohms. Calculate the following :

(*a*) Frequency of emf (*b*) Net reactance (*c*) Impedance (*d*) Current in the circuit (*e*) Phase angle by which current leads or lags the emf (*f*) Expressson for instantaneous current (*g*) Effective voltages across the resistor, capacitor and inductor (*h*) The additional inductance to make the circuit resonant.

19. The tuning of a radio receiver contains a 15 mH coil in series with a variable capacitor. Calculate the value of the capacitance required to tune the radio at a frequency of 1000 KHz.

20. The current in a resistance is 2·5 A when it is connected across a 200 V, 50 Hz supply. How much capacitance is required to reduce the current to 2·0 A ?

21. An electric lamp in a railway compartment is marked 40 V, 40 W. Calculate the inductance of a choke coil required to use it at 220V, 50 Hz a.c. domestic supply. How will you use the same lamp safely at 220 volt d.c. supply ?

22. How can we use a 30 volt bulb on an alternating current supply of 210 volt.

23. A resistance of 100 Ω is connected in series with an inductance of 10 H and a capacitor of 0.1 μf. All these elements are connected to a 220 volt, 50 Hz a.c. supply. Calculate the total impedance of the circuit. [*A.I.S.S.E 1981*]

24. Calculate the resistance of a 200 V, 50 W bulb. [*A.I.S.S.E. 1981*]

25. When a wheel with metal spokes 1·2 m long is rotated in a magnetic field of flux density 5×10^{-5} tesla normal to the plane of the wheel, an emf of 10^{-2} V is induced between the rim and axle. Find the rate of rotation of the wheel. [*A.I.S.S.E. 1982*]

26. An electric lamp which runs at 80 volts d.c. and consumes 10 A is conneeted to 100 V, 50 cycles s^{-1} alternating current mains Calculate the inductance of the choke required. [*A.I.S.S.E. 1982*]

27. A 25 μf capacitor, a 10 H inductor and a 25 Ω resistor are connected in series with an A. C. source. E.M.F. of this source is given in volt by $E = 310 \sin (314t)$. Calculate (*a*) frequency of the emf, (*b*) reactance of the circuit, (*c*) impedance of the circuit, (*d*) current in the circuit, (*e*) the effective voltage across capacitor. [*D.S.S.E. 1981*]

28. A resistor of 100 ohm, an inductance of 0·7 henry and a capacitor of [illegible] microfarad are connected in series to a valtage source of 220 V, 50 Hz. Find the (*a*) impedance of the circuit, (*b*) the current in the circuit. [*D.S.S.E. 1982*]

29. A resistance of 30 ohms is placed in series with a capacitor of reactance 60 ohms and a coil of inductance 7/22 henry. The seriers combination is placed across a 200 V, 50 Hz A.C. supply. Calculate (*a*) the impedance, (*b*) the current, (*c*) phase angle between the current and voltage, which leads the current or voltage. [*D.S.S.* (*Comp.*) *E 1983*]

30. A closely wound rectangular coil of 200 turns and size 0·3 m × 0·1 m is rotating in a magnetic field of induction 0·005 Wb m^{-2} with a frequency of 1800 rpm about an axis normal to the field. Calculate the maximum value of induced emf. [*D.S.S.E. 1985*]

31. An RLC series circuit consists of a resistance of 10 ohm, a capacitor of reactance 60 ohms and an inductor coil. The circuit is found to resonate when put across a 300 volt 100 Hz supply. Calculate (*a*) the inductance of the coil, (*b*) the current in the the circuit. [Take $\pi = 3$] [*D.S.S.* (*Comp.*) *E 1984*]

32. Flux ϕ (in Weber) in a closed circuit of resistance 10 ohms varies with time t (in seconds) according to the equation

$$\phi = 6t^2 - 5t + 1$$

What is the magnitude of the induced current at 0·25 seconds. [*NCERT 1983*]

33. The average e.m.f. induced in a coil in which the current changes from 2 A to 4 A in 0·05 seconds is 8 V. What is the self inductance of the coil ? [*NCERT 1984*]

34. An alternating voltage E (in Volts)$=200\sqrt{2}\sin(100t)$ is connected to a 1 microfarad capacitor through an a.c. ammeter. What shall be the reading of the ammeter ? [*NCERT 1984*]

35. A resistor of 50 Ω, an inductor of $\left(\frac{20}{\pi}\right)$ H and a capacitor of $\left(\frac{5}{\pi}\right)$ μf are connected in series to a voltage supply of 230 V; 50 Hz. Find the impedance of the circuit. [*AISSE 1986*]

36. A coil having number of turns as 100 and area as 0·20 m^2 is placed normally in a magnetic field. The magnetic field changes from 0.2 Wb m^{-2} to 0·6 Wb m^{-2} at a uniform rate over a period of 0·01 s. Calculate the induced e.m.f. across the ends of the coil. [*DSSE 1986*]

37. A resistance of 25 Ω, inductance of 0·01 H and capacitance of 25 μf are connected in series with an oscillator. The emf varies sinusoidally with a maximum value of 310 V. At resonance determine the value of (*a*) frequency; (*b*) maximum current (*c*) voltage across inductance and (*d*) voltage across capacitance. [*DSSE 1987*]

38. An a.c. voltage $E=310\sin 314\,t$ is applied to a series combination of 25 μf capacitor, a 0·1 H inductor and a 24·0 Ω resistor. Calculate the (*a*) reactance; (*b*) impedance; (*c*) r.m.s. value of the current and (*d*) phase angle of the current with respect to applied voltage. [*D.S.S.E. 1988*]

39. An a.c. circuit consists of a 220 volt, 50 Hz supply connected across a 100 ohm resistance. What inductance should be introduced in the circuit, in series with the resistance, so that the current is reduced to half ? [*DSSE 1989*]

40. A 50 μf capacitor, a 0·05 H inductor and a 47·93 Ω resistor are connected in series with an a.c. source of e.m.f. by $E=310\sin 314\,t$. Calculate the reactance of the circuit and its character. What is the phase angle between the current and the applied e.m.f. [*AISSE 1989*]

41. A 100 μf capacitor, a 0·5 inductor and a 50 Ω resistor are connected in series with a 220 V, 50 Hz source. Calculate (*a*) the impedence of the circuit and (*b*) the current through the circuit. [*D.S.S.E.* (*Comp*) *1989*]

42. Calculate the following for the circuit in Fig. 8.5 :
    (*a*) Impedence of the circuit.
    (*b*) Current in the circuit and its equation.
    (*c*) Currents amplitudes in L, C and R.
    (*d*) Average power transferred to L, C and R.

(e) Total power input to the circuit.
(f) Power factor of the circuit.

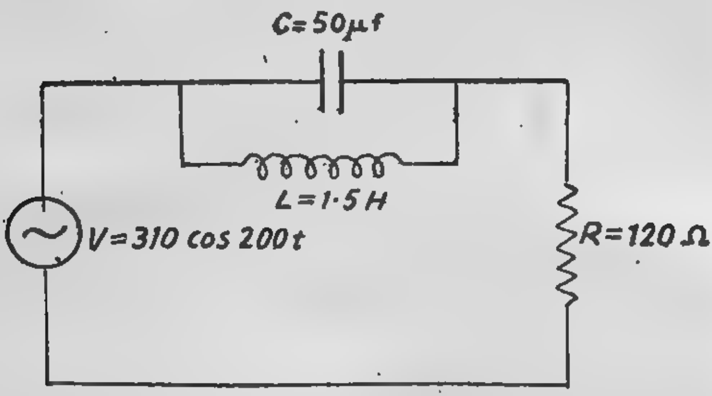


Fig. 8·5.

43. Calculate the values from (*a*) to (*f*) in Q. No. 42 above if the circuit in Fig. 8·5 has L=1 H, C=50μf, R=150 Ω and V=300 sin 100 *t*.

44. Two capacitors 2μf and 3μf in series are connected through a resistance 5 k Ω to a 12 V battery of negligible internal resistance. After a time of about 5 S, the battery is disconnected and the capacitors are allowed to discharge through the resistance. Determine the voltage across each capacitors after a time lapse of 12 ms.

45. Two circuits A and B connected to identical d.c. sources each of e.m.f. 8 V differ greatly in their self inductance. Circuit A has small self inductance=0·04 H, while the self inductance of B is much larger equal to 15 H. The total external resistance in each circuit which also includes the resistance of the inductor itself is 50 Ω. (*a*) Are the steady current values in each circuit equal ? If so, what is the value ? (*b*) Compare the times required for the currents in the two circuits to reach $(1-1/e)$ of their steady value (*c*) Which circuit requires greater energy consumption of the source to build up its current to the steady value (*d*) After the steady state is reached, do the circuits dissipate the same power in the form of heat ?

46. An L-C circuit contains a 2·5 mH inductor and 25μf capacitor with an initial charge of 40 mC. The resistance of the circuit in negligible. Let the instant the circuit is closed be $t=0$, (*a*) What is the total energy stored initially ? Is it conserved during the L-C oscillations ? (*b*) What is the natural frequency of the circuit ? (*c*) At what times is the energy stored

(*i*) completely electrical (*ii*) completely magnetic ? (*d*) At what times is the total energy shared equally between the inductor and the capacitor ? (*e*) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat ?

47. A series LCR circuit with L=0·7H, C=70 μf, R=20 Ω is connected to a 200 V variable frequency supply. (*a*) What is the source frequency for which current amplitude is maximum ? Obtain this maximum value (*b*) what is the source frequency for which average power absorbed by the circuit is maximum. Obtain the value of this maximum power. (*c*) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency ? What is the current amplitude at these frequencies ? (*d*) What is the Q-factor of the circuit ?

48. A d.c. motor operating at 220 V draws a current of 10A at its full speed of 3600 rev./min. The resistance of the armature of the motor is 8 Ω. (*a*) Determine the back e.m.f. of the motor (*b*) Obtain the power input, power output and the efficiency of the motor.

49. If V=300V, I=12 A, No. of revolution per minute=2400 and R=15 Ω in Q. No. 48 above, calculate the quantities (*a*) and (*b*).

50. At a hydroelectric power plant, the water pressure head is at a height of 350 m and water flow available is 80 $m^3 s^{-1}$. If the turbine generator efficiency is 75%. Calculate the electric power available from the plant.

51. A hydroelectric power plant generating 420 MW electric power has its water pressure head at a hight of 360 m and water flow available is 140 $m^3 s^{-1}$. What is the efficiency of the turbine generator ?

52. A small town with a demand of 660 KW of electric power at 220 V is situated 200 km away from an electric plant generating power at 1100 V. The resistance of the two wire line carrying power is 0 2 Ω per km. The town gets power from the line through 13200 V–220 V step down transformer at a substation in the town. (*a*) Calculate the power loss in the line in the form of heat. (*b*) How much power must the plant supply. (*c*) Characterise the transformer at the plant. (*d*) What advantage we will have if the step down transformer on the line is 132000 V—200 V.

53. A 25·0 μf capacitor, *a* 0·10 henry inductor and *a* 25·0 Ω resistor are connector in series with an a.c. source whose e.m.f. is given by E=310 sin 314 *t* volt. What is (*a*) the frequency of e.m.f. ? (*b*) the reactance of the circuit ?

(*c*) the impedance of the circuit ? (*d*) the current in the circuit ? (*e*) the phase angle of current by which it leads or lags the applied e.m.f. ? (*f*) the expression for instantaneous value of current in the circuit ? (*g*) the effective voltages across the capacitor the inductor and the resistor ? (*h*) Construct a vector diagram for these voltages (*i*) What additional value of inductance will make the circuit resonant ?

OBJECTIVE TYPE QUESTIONS

54. When an electric device X is connected to an a.c. voltage, the current through it is in same phase as the applied voltage. What is the device X ?

(*a*) resistor (*b*) inductor
(*c*) capacitor (*d*) none of them.

55. When an electric device is connected to an a.c. voltage, the current through it is leading the voltage in phase by $\pi/2$. The device is

(*a*) resistor (*b*) inductor
(*c*) capacitor (*d*) none of them.

56. When an electric device is connected to an a.c. source, the current through it is lagging behind the voltage in phase by $\pi/2$. The device is :

(*a*) resistor (*b*) inductor
(*c*) capacitor (*d*) none of them.

57. An electric main supply is at 220 V a.c. The peak value of supply voltage is

(*a*) 110 V (*b*) 127 V
(*c*) 220 V (*d*) 310 V

58. An alternating current is $I = I_0 \sin \omega t$. The average value of current over the half cycle is

(*a*) Zero (*b*) $\frac{I_0}{2}$
(*c*) $\frac{2I_0}{\pi}$ (*d*) $\frac{I_0}{\sqrt{2}}$

59. The effective value of a.c. current in a circuit is 10 A. The peak value of the current is

(*a*) 5 A (*b*) 0·707 A
(*c*) 10 A (*d*) 14·14 A.

60. An alternating current is $I = I_0 \cos \omega t$. The r.m.s. value of current over the complete cycle is

(*a*) $\frac{I_0}{\sqrt{2}}$ (*b*) $\frac{2I_0}{\pi}$

(c) $\frac{I_0}{2}$ (d) Zero.

61. The average power dissipated in choking coil in a complete cycle is
(a) zero (b) nearly zero
(c) $I_0^2\,\omega L$ (d) $I^2_{r.m.s.}\,\omega L$.

62. What is the virtual value of a.c. e.m.f. applied in a circuit if its instantaneous value is given by $E=622 \cos 314\ t$
(a) 220 V (b) 311 V
(c) 440 V (d) 254 V

63. An a.c. ammeter has
(a) a linear scale (b) an exponential scale
(c) a square scale (d) none of the above.

64. A series LCR circuit is at resonance. Then the frequency of the oscillation of the circuit is
(a) $\frac{1}{2\pi\sqrt{LC}}$ (b) $\frac{2\pi}{\sqrt{LC}}$
(c) $\frac{1}{\sqrt{LC}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{L}{C}}$

65. A d.c. ammeter has
(a) a linear scale (b) an exponential scale
(c) a square scale (d) none of the above.

66. The core of a transformer is laminated to reduce energy losses due to
(a) hysteresis (b) eddy currents
(c) resistance in windings (d) none of the above.

67. If the resistance of a LCR circuit in resonance is reduced, then
(a) the circuit will not resonate.
(b) the resonance will bocome more sharp.
(c) the resonance will become flat.
(d) the sharpness of resonance will not by affected.

68. An inductance L is connected in series with a resistance R and the combination is subjected to an alternating e.m.f. $E=E_0 \sin \omega t$. Then the impedance of the circuit is
(a) $\sqrt{R^2+\omega^2L^2}$ (b) $\frac{1}{\sqrt{R^2+\omega^2L^2}}$
(c) $\frac{\omega L}{R}$ (d) 0

69. The Q-factor of a LCR circuit in resonance is

(a) $\sqrt{R^2+\omega^2L^2}$ (b) $\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$

(c) $\sqrt{R^2+\frac{1}{\omega^2C^2}}$ (d) $\frac{\omega L}{R}$.

70. An inductor L and resistance R in series are connected to an a.c. e.m.f. source. The admittance of the circuit is

(a) $\sqrt{R^2+\omega^2L^2}$ (b) $\frac{1}{\sqrt{R^2+\omega^2L^2}}$

(c) $\frac{\omega L}{R}$ (d) ωLR.

☐ ☐

Electromagnetic Waves

IMPORTANT FORMULAE

1. The conduction current through a capacitor is given by

$$I=\frac{d\phi}{dt}=C\frac{dV}{dt}$$

2. (*a*) The displacement current through a capacitor is given by

$$I_d=E_0\frac{d\phi_E}{dt}$$

where ϕ_E=Flux of the electric field.

which is found equal to the conduction current through the capacitor.

3. **Ampere's law.** The line integral of magnetic field 'B' around any closed circuit is given by

$$\oint_l B \cdot dl=\mu_0 I$$

4. The 'converage range' d is related to the height 'h' of the transmission tower by the relation

$$d=\sqrt{h^2+2hR}$$

where R=Radius of the earth.

5. The velocity of electromagnetic waves in vacuum or in free space,

$$c=\frac{1}{\sqrt{\mu_0\epsilon_0}};$$

$$\mu_0=4\pi\times10^{-7}$$

$$\epsilon_0=\frac{1}{4\pi K}$$

where $K=9\times10^9$

6. Wien's displacement law

$$\lambda_m T=0{\cdot}29 \text{ cm °K}$$

SOLVED EXAMPLES

Example 1. *The T.V. transmission tower station has a height of 160 m. (a) What is the coverage range ? (b) How much population is*

covered if the average density around the tower is 1200 Km⁻². By how much should the height be increased to double its coverage range? (Radius of the earth=6400 Km).

Solution. (*a*) The coverage range,

$$d=\sqrt{h^2+2hR}\simeq\sqrt{2hR}$$
$$=\sqrt{2\times 160\ (6400\times 10^3)}$$
$$=160\sqrt{2\times 40\times 10^3}$$
$$\doteq 160\times 200\times\sqrt{2}$$
$$=32000\times 1{\cdot}414 \text{ m}$$
$$\simeq 45248 \text{ m}$$
$$\simeq 45 \text{ km.}$$

(*b*) Population covered

$$=\text{Population density}\times\text{Area covered}$$
$$=1200\left(\frac{22}{7}\times 45^2\right)$$
$$=76{\cdot}37 \text{ lacs.}$$

To make the coverage range double,

$$d=2\times 45 \text{ Km}$$

$$\therefore \quad 90=\sqrt{h'^2+2Rh'}$$
$$\simeq 2Rh' \qquad (\because h'^2 <<<< 2Rh')$$

$$\therefore \quad h'\simeq\frac{90\times 90}{2\times 6400}$$
$$\simeq 0{\cdot}633 \text{ km.}$$
$$\simeq 633 \text{ m.}$$

It is nearly four times the original height of the T.V. tower.

Example 2. *In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency $1{\cdot}5\times 10^{10}$ Hz and amplitude 36 Vm^{-1}.*

(*a*) *What is the wavelength of the waves?*

(*b*) *What is the amplitude of the oscillating magnetic field?*

(*c*) *Show that the average energy density of the E field equals to the average density of the B field.*

Solution.

(*a*)
$$\lambda=\frac{c}{v}$$
$$=\frac{3\times 10^8}{1{\cdot}5\times 10^{10}}$$
$$=2\times 10^{-2} \text{ m}$$

(b)
$$B_0=\frac{E_0}{c}$$
$$=\frac{36}{3\times10^8}=1{\cdot}2\times10^{-7}\ \text{T}$$

(c) Energy density in E field,
$$U_E=\frac{1}{2}\ \epsilon_0\ E^2$$

Energy density in B field,
$$U_B=\frac{1}{2}\ \frac{1}{\mu_0}\ B^2$$

Now consider
$$U_E=\frac{1}{2}\ c_0\ E^2$$
$$=\frac{1}{2}\ \epsilon_0\ (cB)^2\ \left[\because\ \frac{E}{B}=c\ \right]$$
$$=\frac{1}{2}\ \epsilon_0\ c^2\ B^2$$
$$=\frac{1}{2}\ \epsilon_0\ \frac{1}{\mu_0\ \epsilon_0}\ B^2\left[\because\ C=\frac{1}{\sqrt{\mu_0\epsilon_0}}\right]$$
$$=\frac{1}{L}\ \frac{1}{\mu_0}\ B^2$$
$$\therefore\qquad U_E=U_B$$

Example 3. *A parallel plate capacitor made of circular plates each of radius 4 cm has a capacitance C=200 pf The capacitor is connected to a 220 V a.c. supply with an angular frequency of 250 rad s^{-1}.*

(a) *What is the effective value (r.m.s, value) of the conduction current.*

(b) *Is the conduction current equal to displacement current ?*

(c) *Find the amplitude of B at a point 2 cm. from the axis between the plates.*

Solution.

(a)
$$I_{eff}=\frac{V_{eff}}{Xc}$$
$$=V_{eff}\ \omega C$$
$$=220\times250\times(200\times10^{-12})\ \text{A}$$
$$=1{\cdot}10\times10^{-6}\ \text{A}$$
$$=1{\cdot}1\ \mu\text{A}$$

(b) The displacement current will be equal to the conduction current (I_{eff}), as the two are always equal to each other in case of a capacitor.

(c) $$I_0 = I_0$$
$$= \sqrt{2}\, I_{eff}$$
$$= 1{\cdot}414 \times 1{\cdot}1$$
$$= 1{\cdot}5554\ \mu A$$

Now $$r = 2\text{ cm} = 0{\cdot}02\text{ m},$$
$$R = 4\text{ cm} = 0{\cdot}04\text{ m}$$

and $$I_D{}^{\circ} = 1{\cdot}55\ \mu A$$

$$B_0 = \frac{\mu_0 r}{2\pi R^2}\, I_D{}^{\circ}$$
$$= \frac{4\pi \times 10^{-7} \times (0{\cdot}02) \times (1{\cdot}55 \times 10^{-6})}{2\pi\,(0{\cdot}04)^2}$$
$$= 3{\cdot}875 \times 10^{-12}\text{ T}$$

Example 4. *A capacitor made of two circular plates each of radius 9 cm. and separated by 4·0 mm. The capacitor is being charged by an external source. The charging current is constant and equal to 0·12 A.*

(a) Calculate the capacitance and the rate of change of potential difference between the plates.

(b) Obtain the displacement current across the plates.

(c) Calculate the magnetic field between the plates at (i) on the axis (ii) 6·00 om from the axis (iii) 12·0 cm from the axis.

(d) Calculate the magnetic field due to the conduction current outside the plates at points (i) 6 cm (ii) 10 cm (iii) 12 cm.

Solution.

(a) $$C = \frac{K'A}{4\pi K d}$$
$$= \frac{1 \times (\pi \times 0{\cdot}09^2)}{\Delta\pi\,(9 \times 10^9) \times (4 \times 10^{-3})}$$
$$= 57{\cdot}25 \times 10^{-12}\, f$$
$$= 56{\cdot}25\ pf$$
$$Q = CV$$
$$\frac{dQ}{dt} = C\,\frac{dV}{dt}$$

$\therefore$ $$\frac{dV}{dt} = \frac{I}{C} \qquad \left[\because \frac{dQ}{dt} = I\right]$$
$$= \frac{0{\cdot}12}{56{\cdot}25 \times 10^{-12}}\ \text{VS}^{-1}$$
$$= 2{\cdot}13 \times 10^9\ \text{VS}^{-1}$$

(*b*) The conduction current

$$=\text{Charging current}$$
$$=0{\cdot}12\text{ A}$$

The displacement current is found to be equal to the conduction current.

$\therefore$ $$I_D=0{\cdot}12\text{ A}$$

(*c*) $$B=\frac{\mu_0 r}{2\pi R^2}I_D \quad \text{for } r<R$$

$\therefore$ (*i*) For $r=0$,

$$B=0$$

and (*ii*) $r=6$ cm,

$$B=\frac{(4\pi\times10^{-7})\,(0{\cdot}06)\times(0{\cdot}12)}{2\pi\;(0{\cdot}09)^2}$$
$$=1{\cdot}60\times10^{-7}\text{ T}$$

(*iii*) $$B=\frac{\mu_0 I_D}{2\pi r} \quad \text{for } r>R$$

$\therefore$ For $r=15$ cm

$\therefore$ $$B=\frac{4\pi\times10^{-7}\times0{\cdot}12}{2\pi\times0{\cdot}12}$$
$$=2\times10^{-7}\text{ T}$$

(*d*) Magnetic field due to conduction current outside the plates in each case,

$$B=\frac{\mu_0 I_D}{2\pi r}$$

(*i*) $$B=\frac{4\pi\times10^{-7}\times0{\cdot}12}{2\pi\times0{\cdot}6}$$
$$=4\times10^{-7}\text{ T}$$

(*ii*) $$B=\frac{4\pi\times10^{-7}\times0{\cdot}12}{2\pi\times0{\cdot}10}$$
$$=2{\cdot}4\times10^{-7}\text{ T.}$$

and (*iii*) $$B=\frac{4\pi\times10^{-7}\times0{\cdot}12}{2\pi\times0{\cdot}12}$$
$$=2\times10^{-7}\text{ T}$$

Exercise 5. *Find the temperature for red and violet light in vissible part of sun's spectrum. The maximum intensity of these two light are found at wavelengths 7500 Å and 4000 Å respectively.*

Solution.

(*i*) For red colour,

$$\lambda_m=7500\text{Å}=7500\times10^{-8}\text{ cm}$$

Now $\qquad \lambda_m T = 0{\cdot}29$ cm k°

$$7500\times10^{-8}T=0{\cdot}29$$

$$\therefore \quad T=3866{\cdot}6\text{ K}$$

(*ii*) For violet colour,

$$\lambda_m=4000\text{Å}=4000\times10^{-8}\text{ cm.}$$

$$\therefore \quad 4000\times10^{-8}\ T=0{\cdot}29$$

$$T=7250\text{ K.}$$

Example 6. *A parallel plate capacitor with circular plates of radius 50 cm has a capacitance of 100 μf. At $t=0$ it is connected for charging in series with a resistor of 20 KΩ across a 5V battery. Calculate the magnetic field at a point in between the plates and half way between the centre and the periphery of the plates, after 2s.*

Solution. The time constant of CR circuit,

$$CR=(100\times10^{-6})\ (20\times10^{3})$$

$$=2s$$

$\therefore$ The charge on the plate at any time 't',

$$q=q_0\left(1-e^{-\frac{t}{CR}}\right)$$

$$=CV\left(1-e^{-\frac{t}{2}}\right)$$

$$=(100\times10^{-6}\times5)\left(1-e^{-\frac{t}{2}}\right)$$

$$q=5\times10^{-4}\left(1-e^{-\frac{t}{2}}\right).$$

The electric field in between the plates at time t,

$$E=\frac{q}{\varepsilon_0 A}$$

$$E=\frac{5\times10^{-4}\left(1-e^{-\frac{t}{2}}\right)}{\varepsilon_0(\pi\times0{\cdot}50^2)}$$

$$=\frac{2\times10^{-3}\left(1-e^{-\frac{t}{2}}\right)}{\varepsilon_0\pi}$$

$$\therefore \quad \frac{dE}{dt}=\frac{2\times10^{-3}}{\varepsilon_0\pi}\frac{\left(-e^{-\frac{t}{2}}\right)}{-\frac{1}{2}}=\frac{4\times10^{-3}}{\varepsilon_0\pi\ e^{t/2}}$$

Consider now a circular loop of radius ½ of that of plates of the capacitor *i e.* 25 cm passing through the point in the question. The magnetic field B at all points on this loop is along the loop and is of same value.

∴ Applying Ampere's law,

$$\oint Bdl=\mu_0\left(i+\epsilon_0\frac{dE}{dt}\right)$$

We have,

$$B\, 2\pi\,(0{\cdot}25)=\mu_0\left(0+\varepsilon^0\frac{dE}{dt}\right)$$

$$\therefore \qquad B=\frac{\mu_0\varepsilon_0}{2\pi\times 0{\cdot}25}\frac{dE}{dt}.$$

$$\therefore \text{ At } t=2\text{s},\quad B=\frac{\mu_0\varepsilon_0}{2\pi\times 0{\cdot}25}\times\frac{4\times 10^{-3}}{\varepsilon_0\pi\ e^{2/2}}$$

$$=\frac{(4\pi\times 10^{-7})\times 4\times 10^{-3}}{0{\cdot}5\pi^2\times(2{\cdot}718)}$$

$$=3{\cdot}746\times 10^{-10}\text{ T}$$

Example 7. *A plane radio wave has $E_m=5\times 10^{-5}$ volt/m. Calculate the intensity of the wave.*

Solution. The intensity of the wave,

$$I=\frac{1}{2}\frac{C}{\mu_0}B^2{}_m=\frac{1}{2}\frac{C}{\mu_0}\left(\frac{E_m}{C}\right)^2$$

$$=\frac{1}{2}\frac{E_m{}^2}{c\mu_0}$$

$$=\frac{1}{2}\times\frac{(5\times 10^{-5})^2}{(3\times 10^8)(4\pi\times 10^{-7})}$$

$$=3{\cdot}3\times 10^{-12}\text{ W/m}^2.$$

EXERCISE 9

1. A parallel plate capacitor made of circular plates each of radius 10 cm has a capacitance of 500 pf. The capacitor is connected to a 200 V a.c. supply with an angular frequency of 300 rad s^{-1}.
   (*a*) What is the virtual value (r.m.s. value) of the conduction current.
   (*b*) Is the conduction current equal to the displacement current.
   (*c*) Find the amplitude of B at a point 4 cm from the axis between the plates.

2. If in Q. No. 1 above, $R=6$ cm, $C=350$ pf., $V=250$ volt, $\nu=50$ Hz and $r=3{\cdot}6$ cm, calculate all the values from (*a*) to (*c*).

3. The r.m.s. value of current flowing through a capacitor is 4·84 A when it is connected across a a.c. supply of 220 V 50 Hz. What is the value of the capacitance of the capacitor ?

4. What is the peak value of displacement current across the capacitor of 400 pf when it is connected with an a.c. supply of 250 V, 300 rad s^{-1}.

5. In the radio we have a 30 metre bond. What is the frequency and the amplitude of oscillating electric field if that of magnetic field is 2×10^{-6} T.

6. In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 250 MHz and amplitude 24 V m^{-1}.

   (*a*) What is the wavelength of the waves ?

   (*b*) What is the amplitude of the oscillating magnetic field ?

   (*c*) Show that the average energy density of the E field equals to the average density of the B field.

7. If in Q. No. 6 above $\nu=5\times10^{14}$ Hz and $E_0=21$ Vm^{-1}, calculate the values (*a*) and (*b*).

8. What is the coverage range of a T.V. transmission tower 125 m high ? (Radius of the earth=6400 Km)

9. The coverage range of a T.V. tower is 80 Km, what is the height of the tower ?

10. The T.V. transmission tower station has a height 250 m. How much population is covered if the average density around the population is 1400 Km^{-2} ? (Radius of the earth=6400 Km)

11. The T.V. transmission tower has a height of 180 m (*a*) what is the coverage range (*b*) How much population is covered if the average density around the tower is 2100 Km^{-2}. (*c*) By how much should the height be increased to double its coverage range ? (Radius of the earth=6400 Km)

12. A capacitor is made of two circular plates each of radius 10 cm and separated by 2·0 mm. The capacitor is being charged by an external source. The charging current is constant and equal to 0·08 A. Calculate

    (*a*) the capacitance and the rate of change of potential difference between the plates.

    (*b*) the displacement current across the plates.

    (*c*) the magnetic field between the plates (*i*) on the axis (*ii*) 4 cm from the axis (*iii*) 16 cm from the axis.

(*d*) the magnetic field due to the conduction current outside the plates at points (*i*) 8 cm (*ii*) 10 cm and (*iii*) 16 cm.

13. If in Q. No. 12 above R=15 cm, d=5·0 mm, I=0·2 A, calculate the values from (*a*) to (*d*).

14. The wavelength corresponding to maximum intensity of light coming from moon is 14 microns. Estimate the temperature of the moon.

15. Find the temperature for yellow and green light in visible region of sun's spectrum. The maximum intensity of these two lights are found at the wavelengths 5600 A° and 5200 A° respectively.

16. The surface temperature of the sattelite 'Venus' is estimated to be about 800 K. What is the wavelength of light coming from the Venus which produces maximum intensity ?

17. A parallel plate capacitor with circular plates of radius 40 cm has a capacitance of 200 μf. At t=0 it is connected for charging in series with resistor of 10 KΩ across a 4V battery. Calculate the magnetic field at a point in between the plates and halfway between the centre and the periphery of the plates, after 2*s*.

18. If in Q. No. 17 above R=60 cm, C=500 μf, R=2 KΩ and V=6·0 volt, calculate the magnetic field at a point in between the plates of the capacitor and one-fourth way from the centre towards the periphery of plates, after 2 s.

19. A plane radio wave has maximum intensity of its magnetic field vector as 2×10^{-18} T. Calculate the intensity of the wave.

20. A plane radio wave has $E_m=3\times10^{-4}$ volt m^{-1}. Calculate the amplitude of the magnetic field vector and the intensity of the wave.

OBJECTIVE TYPE QUESTIONS

21. The velocity of electromagnetic waves in vacuum is given by

(*a*) $\sqrt{\dfrac{\mu_0}{\varepsilon_0}}$ (*b*) $\sqrt{\dfrac{\varepsilon_0}{\mu_0}}$

(*c*) $\sqrt{\mu_0\varepsilon_0}$ (*d*) $\dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$

22. If the radius of the earth is R, then the covering range of a T.V. transmitting tower of height 'h' will be

(*a*) $\sqrt{h^2+2Rh}$ (*b*) $\sqrt{h^2-2Rh}$

(*c*) $\sqrt{2Rh}$ (*d*) $\sqrt{h^2/2Rh}$

23. For an electromagnetic wave the amplitude of the oscillating electric and magnetic field are related to each other as
(a) $E_0=cB_0$ (b) $B_0=cE_0$
(c) $E_0=B_0$ (d) $c=E_0B_0$

24. Out of the four basic equations of electromagnetism, the equation that asserts that electrostatic field lines cannot form closed curves is
(a) Amperes modified law (b) Gauss's law
(c) Faraday's law (d) Bio-Savart law

25. Which of the following are not the electromagnetic waves
(a) Radio waves (b) X-rays
(c) Sound waves (d) Light rays

26. All types of electromagnatic waves in free space travel with
(a) same velocity (b) different velocity
(c) cannot be decided, data are insufficient.

27. The small ozone layer that prevents ultraviolet light of the sun from reaching the earth is present in the
(a) stratosphere (b) ionosphere
(c) troposphere (d) exposphere

28. The electromagnetic waves have the following nature :
(a) longitudinal (b) transverse
(c) mechanical (d) elastic

29. The electromagnetic waves travel in free space with a velocity
(a) $332\ ms^{-1}$ (b) $3000\ Kms^{-1}$
(c) $3\times10^9\ cms^{-1}$ (d) $3\times10^8\ ms^{-1}$

30. For X-rays astronomy to be practically possible, we must make our observations
(a) at the poles (b) at the equator
(c) in artificial sattelite orbiting the earth well above the atmospheric layers
(d) at the peak of a mountain of high altitude.

31. In the absence of the atmosphere around the earth, the average surface temperature of the earth will
(a) go up (b) go down
(c) steadily rise up (d) uneffected.

□ □

Wave Optics

IMPORTANT FORMULAE

1. Snell's law,

$$\frac{\sin i}{\sin r}=\frac{c_1}{c_2}=\mu$$

2. Optical path p, in terms of actual path, d,

$$p=\mu d$$

3 In Young's double slit experiment or in Fresnel's biprism expt. or in Lloyd's mirror expt., fringe width (the separation between two successive maxima or minima),

$$\omega=\frac{D\lambda}{d}$$

d=distance between two slits

D=Perp. distance of observation plane from two slits

λ=wavelength of the source of light.

4. For diffraction of light through a slit of width d,

$$d \sin \theta=n\lambda \text{ for minima}$$

$$d \sin \theta=(2n+1)\frac{\lambda}{2} \text{ for maxima}$$

5. The smallest angular separation that can be resolved by an optical instrument,

$$\sin \theta=\frac{\lambda}{d}$$

6. In a thin transparent film for oblique incidence,

(a) for maxima (bright fringe)

$$2\mu t \cos r=(2n+1)\frac{\lambda}{2}$$

r=angle of refraction.
t=thickness of film

(b) for minima (dark fringe)

$$2\mu t \cos r=n\lambda$$

The above equations for normal incidence ($r=0$) become :

$$2\mu t=(2n+1)\frac{\lambda}{2} \qquad \text{(for maxima)}$$

$$2\mu t=n\lambda \qquad \text{(for minima)}$$

7. When a transparent sheet of refractive index μ and thickness t is introduced in the path of one of the interferring waves, the fringe shift is given by $x=(\mu-1)t\times\frac{\omega}{\lambda}$

SOLVED EXAMPLES

Example 1. *Mercury green light has a wavelength 5460 Å. Calculate (a) frequency in Hz (b) the period in s. Convert them to megahertz and microsecond respectively.*

Solution. $\lambda=5460$ Å

$=5460\times10^{-10}$ m

velocity of light $=3\times10^{8}$ m s^{-1}

1 MHz$=10^{6}$ Hz and 1 μs$=10^{-6}$ s

(*a*) Now $c=\nu\lambda$

$\therefore$ $\nu=\frac{c}{\lambda}$

$$=\frac{3\times10^{8}}{5460\times10^{-10}}=5{\cdot}494\times10^{14}\text{ Hz}$$

$=5{\cdot}494\times10^{7}$ MHz

(*b*) $T=\frac{1}{\nu}$

$$=\frac{1}{5{\cdot}494\times10^{14}}=1{\cdot}82\times10^{-15}\text{ s}$$

$=1{\cdot}82\times10^{-9}$ μs.

Example 2. *In ratio waves, we have a '25 metre bond'. Calculate the corresponding frequency.*

Solution. $\lambda=25$ m

$c=3\times10^{8}$ m s^{-1}

$\therefore$ $\nu=\frac{c}{\lambda}$

$$=\frac{3\times10^{8}}{25}=12\times10^{6}\text{ Hz.}$$

$=12$ MHz.

Example 3. *In micro waves, we have a source of frequency 30,000 MHz. Calculate the corresponding wavelength.*

Solution. $\nu = 30{,}000 \text{ MHz} = 30{,}000 \times 10^6 \text{ Hz}$

$$= 3 \times 10^{10} \text{ Hz}$$

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

$$\therefore \quad \lambda = \frac{c}{\nu}$$

$$= \frac{3 \times 10^8}{3 \times 10^{10}} = 10^{-2} \text{ m} = 1 \text{ cm.}$$

Example 4. *The red light of 750 nm (nanometers) enters a glass plate having refractive index 1·5. Calculate in glass (a) the frequency of light, (b) velocity of light, (c) wavelength of light in Angstrom unit.*

Solution. $\lambda = 750 \text{ nm} = 750 \times 10^{-9}\text{m}$; $c = 3 \times 10^8 \text{ ms}^{-1}$.

$$\therefore \quad \nu = \frac{c}{\lambda}$$

$$= \frac{3 \times 10^8}{750 \times 10^{-9}} = 4 \times 10^{14} \text{ Hz.}$$

(*a*) Since frequency is independent of medium, the frequency in glass will remain the same.

$$\therefore \quad \nu = 4 \times 10^{14} \text{ Hz}$$

(*b*) $$\mu_g = \frac{c}{c_g}$$

$$\therefore \quad c_g = \frac{c}{\mu_g}$$

$$= \frac{3 \times 10^8}{1{\cdot}5} = 2 \times 10^8 \text{ ms}^{-1}$$

(*b*) $$\lambda_g = \frac{c_g}{\nu}$$

$$= \frac{2 \times 10^8}{4 \times 10^{14}} = 0{\cdot}5 \times 10^{-6} \text{ m}$$

$$= 5000 \times 10^{-10} \text{ m} = 5000 \text{ Å}$$

Example 5. *The light of wavelength 6000 Å illuminates a slit which in turn illuminates two narrow slits 2mm apart. If the screen is at a distance of 2m away from the entre of two narrow slits, calculate the separation between two adjacent bright bands.*

Solution. $\lambda = 6000\text{Å} = 6000 \times 10^{-10} \text{ m} = 6 \times 10^{-7} \text{ m}$

$\text{D} = 2\text{m}$ and $d = 2\text{mm} = 2 \times 10^{-3} \text{ m}$

$$\therefore \quad \omega = \frac{\lambda \text{D}}{d}$$

$$= \frac{6 \times 10^{-7} \times 2}{2 \times 10^{-3}} = 6 \times 10^{-4} \text{ m} = 0{\cdot}6 \text{ mm}$$

Example 6. *In a Young's double slit experiment, the light of wavelength 5000 Å is used. The third bright band on the screen is observed at a distance of 1 cm from the central bright band. If the screen is at a distance of 1·5 m away from the centre of two narrow slits, calculate the separation between the slits.*

Solution. $\omega=\frac{1}{3}\text{ cm}=\frac{1}{3}\times10^{-2}\text{ m}$

$$D=1{\cdot}5\text{ m}$$

$$\lambda=5000\text{ Å}=5000\times10^{-10}=5\times10^{-7}\text{ m}$$

Now $\omega=\dfrac{\lambda D}{d}$

$\therefore$ $d=\dfrac{\lambda D}{\omega}$

$$=\frac{5\times10^{-7}\times1{\cdot}5\times3}{1\times10^{-2}}=22{\cdot}5\times10^{-5}\text{ m}$$

$$=0{\cdot}225\text{ mm.}$$

Example 7. *If the two slits in Young's experiment have widths in the ratio 16 : 1, calculate the ratio of intensity at the maxima and minima in interference pattern*

Solution. The intensities due to separate slits are in proportion to the slit widths.

$$\therefore \quad \frac{I_1}{I_2}=\frac{16}{1}$$

$\therefore$ Amplitude ratio, $r=\sqrt{\dfrac{I_1}{I_2}}$

$$=\sqrt{\frac{16}{1}}=4$$

$$\therefore \quad \frac{I_{max}}{I_{min}}=\frac{(r+1)^2}{(r-1)^2}$$

$$=\frac{(4+1)^2}{(4-1)^2}=25:9$$

Example 8. *Two coherent sources of intensity ratio 121 : 1 interfere. Calculate the intensity ratio between the maxima and minima in interference pattern.*

Solution.

$$r=\sqrt{\frac{I_1}{I_2}}=\sqrt{\frac{121}{1}}=11$$

$$\therefore \quad \frac{I_{max}}{I_{min}}=\frac{(11+1)^2}{(11-1)^2}=36:25$$

Example 9. *A Fresnel's biprism is placed at a distance of 30 cm. in front of a narrow slit illuminated by a monochromatic light. The virtual images formed by the prism are 0·30 cm. apart. When the screen is placed 120 cm apart in front of the biprism, the fringe width is found to be 0·235 mm. Calculate the wavelength of light used.*

Solution.

$$d=0{\cdot}30 \text{ cm}=0{\cdot}30\times10^{-2} \text{ m}$$
$$D=(120+30) \text{ cm}=1{\cdot}5 \text{ m}$$
$$\omega=0{\cdot}235 \text{ mm}=0{\cdot}235\times10^{-3} \text{ m}$$

$$\therefore \quad \lambda=\frac{\omega d}{D}$$
$$=\frac{0{\cdot}235\times10^{-3}\times0{\cdot}3\times10^{-2}}{1{\cdot}5}$$
$$=4700\times10^{-10} \text{ m}$$
$$=4700 \text{ Å}.$$

Example 10. *A soap film with refractive index 4/3 is just thick enough to produce constructive interference of wavelength 4000 Å of light for normal incidence. Calculate the thickness of the film in microns.*

Solution. For constructive interference in normal incidence,

$$2\mu t+\frac{\lambda}{2}=n\lambda$$

By the film is just thick to produce interference,

$$\therefore \quad n=1$$
$$\therefore \quad 2\mu t+\frac{\lambda}{2}=\lambda$$

or
$$2\mu t=\frac{\lambda}{2}$$
$$t=\frac{\lambda}{4\mu}$$
$$=\frac{4000\times3}{4\times4}=750 \text{ Å}=750\times10^{-10} \text{ m}$$
$$=0{\cdot}075\times10^{-6} \text{ m}$$
$$=0{\cdot}075 \text{ microns.}$$

Example 11. *Using light of $\lambda=5880$ Å, it is found that 8·5 fringes appear between two point in a thin air film. Calculate the difference in thickness of the film between the points in microns.*

Solution. For observing a bright fringe in a thin film,

$$2\mu t+\frac{\lambda}{2}=n\lambda$$

or $$2\mu t=(2n-1)\frac{\lambda}{2}$$

$\therefore$ $$2\mu t_1=(2n_1-1)\ \lambda\ 2$$

and $$2\mu t_2=(2n_2-1)\ \lambda/2$$

$\therefore$ $$2\mu t_2-2\mu t_1=2(n_2-n_1)\ \lambda/2$$

$\therefore$ $$(t_2-t_1)=\frac{(n_2-n_1)\ \lambda}{2\mu}$$

$$=\frac{8{\cdot}5\times5880\times10^{-10}}{2\times1}\text{ m}$$

$$=24990\times10^{-10}\text{ m}$$

$$=2{\cdot}499\times10^{-6}\text{ m}$$

$$=2{\cdot}499\text{ microns.}$$

Example 12. *If an oil film has thickness 10^{-4} cm, calculate the wavelengths in visible region for which the reflection along the normal incidence will be (a) strong (b) weak (μ of oil=1 42).*

Solution. White light may be considered to have $\lambda=4000$ Å to 7500 Å

$$2\mu t=2\times1{\cdot}42\times10^{-4}\text{ cm}$$

$$=28400\times10^{-8}\text{ cm}$$

$$=28400\text{ Å}$$

(*a*) For strong reflection *i.e.*, constructive interference.

$$2\mu t=(2n-1)\ \lambda/2$$

$\therefore$ $$\lambda=\frac{2\times2\ \mu t}{(2n-1)}$$

or $$\lambda=\frac{2\times28400}{(2n-1)}$$

$\therefore$ Possible values of λ for white light,

$$\lambda=\frac{56800}{9},\ \frac{56800}{11},\ \frac{56800}{13}$$

$$=6311\text{ A}^\circ,\ 5164\text{ A}^\circ,\ 4369\text{ A}^\circ$$

[$\because$ $(2n-1)$ is always a odd number and it is here only 9, 11 and 13 for which λ lies between 4000 A° and 7500 A°]

(*b*) For weak reflection, *i.e.*, destructive interference,

$$2\mu t=n\lambda$$

$\therefore$ $$\lambda=\frac{2\mu t}{n}$$

or $$\lambda=\frac{28400}{n},\text{ where } n \text{ is an integer.}$$

$\therefore$ Possible values of λ for white light

$$=\frac{28400}{4}, \frac{28400}{5}, \frac{28400}{6}, \frac{28400}{7}$$

$$=7100 \text{ A}^\circ, 5680 \text{ A}^\circ, 4733 \text{ A}^\circ, 4057 \text{ A}^\circ.$$

Example 13. *The wavelength of mercury green light is 5460 A° in vacuum. Calculate the frequency and wavelength in water. (μ for water$=4/3$ and velocity of light$=3\times 10^8$ ms^{-1})*

Solution. $\lambda=5460 \text{ A}^\circ=5460\times 10^{-10}$ m

$$c=3\times 10^8 \text{ ms}^{-1}$$

$$\nu=\frac{c}{\lambda}=\frac{3\times 10^8}{5460\times 10^{-10}}$$

$$=5{\cdot}4945\times 10^{14} \text{ Hz.}$$

$$\mu_w=\frac{c}{c_w}$$

$$c_w=\frac{c}{\mu_w}$$

$$=\frac{3\times 10^8\times 3}{4}=2{\cdot}25\times 10^8 \text{ ms}^{-1}$$

$\therefore$ $$\lambda_w=\frac{c_w}{\nu}$$

$$=\frac{2{\cdot}25\times 10^8}{5{\cdot}4945\times 10^{14}}=4095 \text{ A}^\circ$$

N.B. $\left(\text{Also } \lambda_w=\frac{\lambda}{\mu_w}\right)$

Example 14. *In a certain region in a thin film, 7 fringes are observed with light of $\lambda=5880$ A°. How many fringes will be observed with the light of $\lambda=4340$ A°.*

Solution. $n_2 \lambda_2=n_1 \lambda_1$

$$n_2=\frac{n_1 \lambda_1}{\lambda_2}$$

$$=\frac{7\times 5880}{4340}=9{\cdot}5 \text{ fringes.}$$

Example 15. *A slit 2·5 cm wide is irradiated with microwaves of $\lambda=1{\cdot}0$ cm. Calculate the angular spread of the central maxima if the incidence is*

(*a*) *normal to the plane of the slit*

(*b*) *at an angle 15° with the normal.*

Solution.

(*a*) $$\sin\theta=\frac{\lambda}{d}$$

$$=\frac{1\cdot0}{2\cdot5}$$

or $\quad \sin\theta=0\cdot4000$

$\therefore \quad \theta=\pm23°\ 36'$, since the angular spread will be on either side of normal direction. [Fig. 10·1 (*a*)]

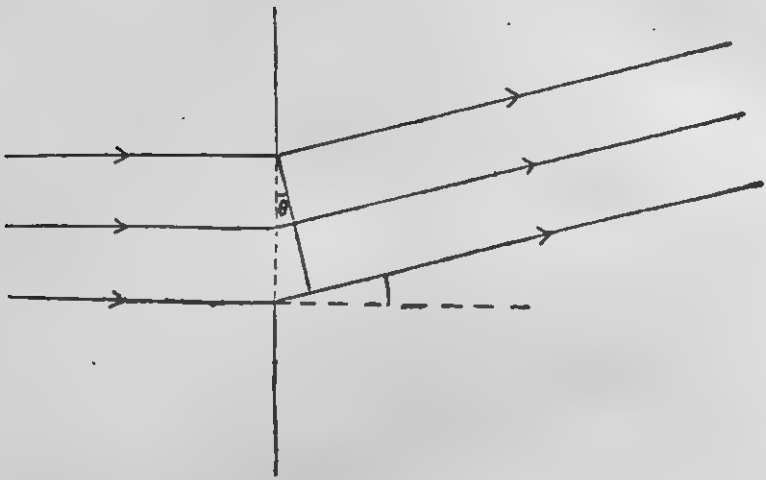


Fig. 10·1 (a)

(*b*) For incidence of waves at an angle $\alpha=15°$, angular spread will be given by

$(d\sin\theta\pm d\sin\alpha)=\lambda$

(*i*) $d(\sin\theta+\sin\alpha)=\lambda$ [See Fig. 10·1 (*b*)]

$$\therefore \quad \sin\theta=\frac{\lambda}{d}-\sin\alpha$$

$$=\frac{1\cdot0}{2\cdot5}-\sin 15$$

$$=\cdot4-0\cdot2588$$

or $\quad \sin\theta=0\cdot1412$

$\therefore \quad \theta=8°\ 7'$

(*ii*) $d(\sin\theta-\sin\alpha)=\lambda$ [See Fig 10·1 (*c*)]

$$\therefore \quad \sin\theta=\frac{\lambda}{d}+\sin\alpha$$

$$=0\cdot4+0\cdot2588$$

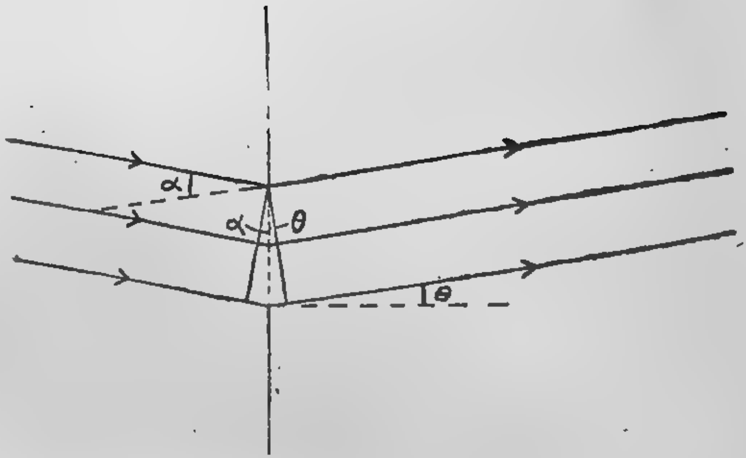


Fig. 10·1 (b)

or $\qquad \sin\theta = 0{\cdot}6588$

$\therefore \qquad \theta = 41°\ 12'$.

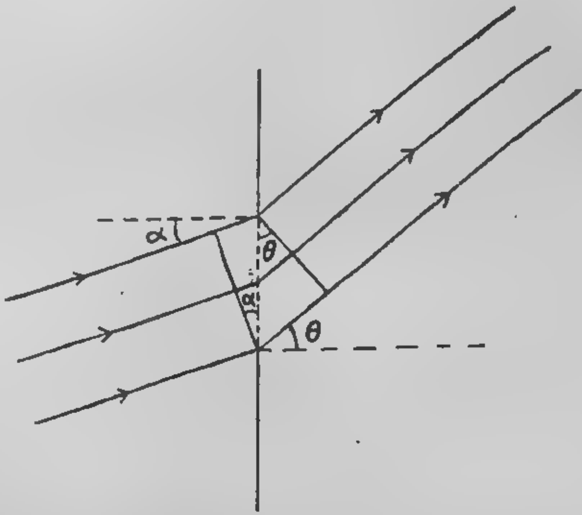


Fig. 10·1 (c)

Example 16. *A laser beam has a power of 10 mW. It has an aperture diameter of 4 mm and it emits light of wavelength λ=6800 Å. A lens of focal length 5 cm is used for focussing this beam. Calculate the intensity of the image.*

Solution. $\lambda = 6800$ Å$= 6800 \times 10^{-10}$ m; $d = 4$ mm$= 4 \times 10^{-3}$ m;
$D = 5$ cm.$= 5 \times 10^{-2}$ m and $P =$ mW$= 10^{-3}$ W.

The angular spread of the beam,

$$\theta = \frac{\lambda}{d}$$

$$= \frac{6800 \times 10^{-10}}{4 \times 10^{-3}} = 1{\cdot}7 \times 10^{-4} \text{ rad.}$$

$\therefore$ The areal spread,

$$A = (\theta . D)^2$$

$$= (1{\cdot}7 \times 10^{-4} \times 5 \times 10^{-2})^2.$$

$$= 7{\cdot}225 \times 10^{-11} \text{ m}^2$$

$\therefore$ The intensity,

$$I = \frac{P}{A}$$

$$= \frac{10^{-3}}{7{\cdot}225 \times 10^{-11}} = 1{\cdot}384 \times 10^{8} \text{ Wm}^{-2}$$

Example 17. *For sodium yellow light, the coherence length is 2·7 cm. Calculate (a) the coherence time, (b) the number of oscillations in this length. (λ for sodium yellow light$= 5880$ Å and $c = 3 \times 10^{8}$ ms^{-1}).*

Solution. The coherence length, $L = 2{\cdot}7$ cm$= 2{\cdot}7 \times 10^{-2}$ m

(*a*) The coherence time,

$$t = \frac{L}{c}$$

$$= \frac{2{\cdot}7 \times 10^{-2}}{3 \times 10^{8}} = 9 \times 10^{-11} \text{ S}$$

(*b*) The number of oscillations,

$$n = \frac{L}{\lambda}$$

$$= \frac{2{\cdot}7 \times 10^{-2}}{5880 \times 10^{-10}} = 4{\cdot}59 \times 10^{4}$$

Example 18. *A slit of width 'a' is illuminated by red light of wavelength 6500 A. For what value of 'a' will the first minimum fall at an angle of diffraction 30°.* [*A.I.S.S.E. 1983*]

Solution. Here, $\lambda = a \sin \theta$.

$$\therefore \quad a = \frac{\lambda}{\sin \theta} = \frac{6500}{\sin 30} = 6500 \times \frac{2}{1} = 13000 \text{ Å}$$

$$= 1{\cdot}3 \times 10^{-6} \text{ m.}$$

Example 19. *A beam of light consisting of two wavelengths 6500 Å and 5200 Å, is used to obtain interference fringes in a Young's double slit experiment. Find (a) The distance of the third bright fringe on the screen from the central maximum for the wavelength 6500 Å. (b) the least distance from the central maximum where the bright fringes due to both the wavelengths coincide. The distance between the slits is 2 mm and distance between the plane of the slits and the screen is 120 cm.* [*I.I.T. 1985*]

Solution.

(*a*) $d = 2$ mm $= {\cdot}2$ cm; $D = 120$ cm; $\lambda = 6500 \times 10^{-8}$ cm.

$\therefore$ Distance of the third bright fringe,

$$x = \frac{3D\lambda}{d}$$

$$= \frac{3 \times 120 \times 6500 \times 10^{-8}}{{\cdot}2}$$

$$= 0{\cdot}1170 \text{ cm.}$$

(*b*) If x is the least distance from the central maximum, it must contain n fringes of the longer wavelength and $(n+1)$ fringes of the shorter wavelength.

$$\therefore \quad x = \frac{n \times 6500 \times D}{d} = \frac{(n+1)\, 5200\, D}{d}$$

$$\therefore \quad 6500\, n = 5200\, (n+1)$$

or $$5n = 4\,(n+1)$$

or $$n = 4$$

$$\therefore \quad x = \frac{4 \times 6500 \times 10^{-8} \times 120}{{\cdot}2}$$

$$= 0{\cdot}1560 \text{ cm.}$$

Example 20. *A soap film held vertically and seen in the reflected white light shows red colour at the top, three intermediate red fringes and a bluish tinge at the bottom. The film then just breaks off at the top. Calculate the film thickness at the bottom, taking* $\lambda_{red} = 6500$ Å, $\lambda_{blue} = 5000$ Å *and* $\mu = 1{\cdot}33$.

Solution.

For the upper most red fringe R_0 (see Fig. 10·2),

$2\ \mu t=(0+\frac{1}{2})\ \lambda_{red}$

Then for the lowest red fring R_3

$2\mu t'=(3+\frac{1}{2})\lambda_{red}$

$=3{\cdot}5\times 6500\text{Å}$

$2\mu t'=22750\text{Å}$ ...(1)

For the bottom bluish fringe (B)

$2\mu t''=(n+\frac{1}{2})\lambda_{blue}$

$2\mu t''=(n+\frac{1}{2})\ 5000\text{Å}$ ...(2)

$\therefore$ Comparing eq^{ns} (1) and (2), smallest n is 5 since $t''>t'$.

$\therefore\quad 2\times 1{\cdot}33t''=(5+0{\cdot}5)\times 5000\ \text{Å}$

$\therefore\quad t''=10338\ \text{Å}$

$=1{\cdot}0338\times 10^{-4}$ cm.

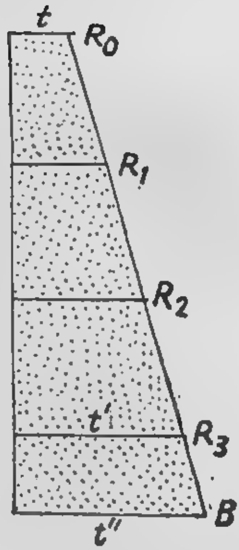


Fig. 10·2.

Example 21. *Calculate the largest distance from which the markings on a metre scule can be seen by using a light of $\lambda=5000$ Å by (a) the naked eye (aperture=2·0 mm) (b) a telscope whose objective has a diameter 2·5 cm.*

Solution. The markings on a metre scale have separation 1 mm. If the markings are seen from a distance of D metre, the angle subtended at the eye,

$$\theta=\frac{1\text{ mm}}{\text{D metre}}\qquad\left[\because\quad \theta=\frac{l}{r}\right]$$

$$=\frac{1\text{ mm}}{1000\text{ D mm}}$$

$$=\frac{1}{1000\text{ D}}\text{ rad}$$

Now the angular spread of markings $\left(\frac{\lambda}{d}\right)$ should be less than $\frac{1}{1000\text{ D}}$ to see them separately.

$$\therefore\quad \frac{\lambda}{d}<\frac{1}{1000\text{ D}}$$

where d=aperture of lens.

$$\therefore\quad \text{D}<\frac{d}{1000\ \lambda}$$

(*a*) For the eye,

$$D < \frac{0{\cdot}20 \text{ cm}}{1000\ (5000 \times 10^{-8} \text{ cm})}$$

$$< 4 \text{ m}$$

(*b*) For the telescope,

$$D < \frac{2{\cdot}5 \text{ cm}}{1000\ (5000 \times 10^{-8} \text{ cm})}$$

$$D < 50 \text{ m}.$$

Example 22. *Red light of wavelenath 6500 Å from a distant soucce falls on a slit 0 50 mm wide, (a) what is the distance between the two dark bands on each side of the central bright band of the diffraction pattern observed on a screen plaeed 1·8 m from the slit ? (b) what is the answer to (a) if the slit is replaced by a small circular hole of diameter 0·50 mm.*

Solution. (*a*) We know that angular diffraction

$$\theta = \frac{\lambda}{d}$$

∴ Angular separation between two dark bands on either side of the central band

$$= \frac{2\lambda}{d}$$

∴ Actual separation between the two dark bands

$$= \frac{2\lambda}{d}.D \text{ where } D = \text{Distance of the screen}$$

$$= \frac{2 \times 6500 \times 10^{-10} \times 1{\cdot}8}{0{\cdot}50 \times 10^{-3}}$$

$$= 4{\cdot}68 \times 10^{-3} \text{ m}.$$

$$= 4{\cdot}68 \text{ mm}$$

(*b*) A circular hole produces circular diffraction fringes. The angular separation between the central bright band and the first dark band according to Airy is found to be $1{\cdot}22 \frac{\lambda}{d}$.

∴ The separation between two dark circular fringes

$$= 1{\cdot}22 \frac{\lambda}{d} D$$

$$= 1{\cdot}22 \times \frac{6500 \times 10^{-10} \times 1{\cdot}8}{0{\cdot}50 \times 10^{-3}}$$

$$= 5{\cdot}71 \text{ mm}.$$

Example 23. *In a pin-hole camera, a box of length L has a hole of radius 'r' in one wall. When the hole is illuminated by a*

paralled beam, the size of the spot of light in large when r is large. Show that it is also very large when r is small, due to diffraction. Find the minimum size of the spot.

Solution. The diffraction angle,

$$\theta=\frac{\lambda}{r}$$

$\therefore$ The linear spreading of the size of the spot $dr=L\frac{\lambda}{r}$

$$\therefore \quad \boxed{dr \propto \frac{1}{r}}$$

$\therefore$ Total radius of the stot, $r'=\left(r+L\frac{\lambda}{r}\right)$ ...(1)

It means size of the spot is large when r is small.

$$\text{or} \quad r'=\sqrt{\left(r-L\frac{\lambda}{r}\right)^2+4.r\times L\frac{\lambda}{r}}$$

$$[\because \quad (a+b)^2=(a-b)^2+4\,ab]$$

For r' to be minimum,

$$\left(r-\frac{L\lambda}{r}\right)=0$$

$$\text{or} \quad r=\frac{L\lambda}{r}$$

$$\text{or} \quad r=\sqrt{L\lambda}$$

$$\therefore \quad r'_{min}=2r \; [\text{Eq}^n(1)]$$

$$=2\sqrt{L\lambda}$$

Example 24. *The wavelengths of the visible spectrum ranges from 4000 Å to 7500 Å. Find the angular breadth of the first order visible spectrum produced by a plane grating having 5000 lines per centimetre, when light is incident normally on the grating.*

Solution. The grating spacing,

$$d=\frac{1}{5000}\text{ cm}$$

$$=2\times10^{-4}\text{ cm}$$

The angular deviation of the red wavelengths,

$$\sin\theta_r=\frac{\lambda r}{d}$$

$$=\frac{7500\times10^{-8}}{2\times10^{-4}}$$

$$\text{or} \quad \sin\theta_r=0{\cdot}375$$

$$\therefore \quad \theta_r=22°\ 2'$$

Similarly for violet wavelengths,

$$\sin \theta_v = \frac{\lambda v}{d}$$

$$= \frac{4000 \times 10^{-8}}{2 \times 10^{-4}}$$

or $$\sin \theta_v = 0{\cdot}2$$

∴ $$\theta_v = 11° \ 32'$$

∴ Angular breadth of the first order spectrum

$$= \theta_r - \theta_v$$

$$= 22° \ 2' - 11° \ 32'$$

$$= 10° \ 30'$$

EXERCISE 10

1. The wavelength of X-rays is 10 A°. Calculate (*a*) frequency in Hz, (*b*) the period in s. Convert them into megahertz and microsecond respectively.
2. The frequency of cosmic rays is 4×10^{23} Hz. Calculate (*a*) wavelength in *m*, (*b*) the period in s. Convert them into Angstrom and microsecond respectively.
3. In radio waves, we have a '60 metre band'. Calculate the corresponding frequency in KHz.
4. In radio waves, we have a '15 MHz band'. Calculate the corresponding wavelength in metres.
5. Sodium orange light of wavelength 6000 A° enters a glass plate having refractive index 1·55. Calculate in glass (*a*) the frequency of light (*b*) velocity of light (*e*) wavelength of light.
6. The wavelength of violet rays of light is 4350 A° in vacuum. Calculate the frequency. Also calculate the wavelengh in glass and water (μ for water = 1·33 and for glass = 1·672).
7. Two coherent sources of intensity ratio 25 : 1 interfere. Calculate the ratio of intensity between the maxima and minima in the interference pattern.
8. The two slits in Young's double slit experiment have width ratio 1 : 16. Calculate the ratio of intensity at maxima and minima in interference pattern. [*AISSE 1981*]
9. In Young's double slit experiment the slit separation is 0·20 cm. The screen is 2m away from the centre of two slits. If the light of wavelength 5000 A° is used, calculate the separation between two adjacent bright bands.
10. With two slits spaced 0·30 mm apart and the screen at 1·5 m distance away in Young's double slit experiment, the fourth

bright fringe is found to be displaced 1·2 cm from the central bright fringe. Calculate the wavelength of the light used.

11. In Llyod's mirror experiment, the source slit and its virtual image are 3 mm apart. The perpendicular distance from the source to the screen is 1m. If $\lambda=7000$ A°, calculate the distance of the third bright fringe from the central bright fringe.

12. A source of light of $\lambda=5400$ A° is placed at one end of a table 2 m long and 3 mm above its flat well polished reflecting top. Find the fringe width of the interference bands located on the screen at the other end of the table.

13. In a Fresnel's biprism experiment, light of $\lambda=6000$ A° is used. The separation between two virtual sources is 5mm. If the observation screen is at a distance of 2m from the source, calculate the maximum slit width so that the fringes may not be obliterated.

14. A Fresnel's biprism is placed at a distance of 30 cm in front of a narrow slit illuminated by monochromatic sodium light of $\lambda=5894$ A°. The two virtual images of slit are found to be 0·55 cm apart. Calculate the width of the fringes formed on a screen placed 1·5 m apart from the biprism.

15. A soap film with $\mu=4/3$ is illuminated by light $\lambda=5820$ A° incident at an angle of 60° with a normal. Calculate the minimum thickness of film which will appear (*a*) dark by reflection (*b*) bright by reflection.

16. An oil film with $\mu=1{\cdot}30$ and thickness 2×10^{-4} cm is viewed in white light which may be considered to have λ from 4000A° to 7500 A°. Calculate the wavelength in visible region for which the reflection along the normal direction will be (*a*) weak, (*b*) strong.

17. By using light of $\lambda=6000$ A°, 8 fringes occur between two points in a thin film of air. Calculate the difference of the thickness of film between these two points.

18. In a certain region of a thin film, 10 firinges are observed by using light of $\lambda=4358$ A°. How many fringes would be observed in the same region with $\lambda=5893$ A°.

[*D.S S.E. (C) 1988*]

19. A slit 4 cm. wide is irradiated with micro waves of $\lambda=2{\cdot}0$ cm Calculate the angular spread of central maxima if the incidence is

(*a*) normal to the plane of the slit.

(*b*) at an angle of 10° with the normal.

20. A laser of 40 m W has aperture 3 mm and emits light of wavelength 5000A°. If it is focussed by a convex lens of focal length 6 cm, calculate the intensity of the image.

21. A laser beam with wavelength 6900 A° and aperture diameter 3 mm is used to study the details of the surface of the moon. Calculate angular spread. Also calculate the areal spread of beam as it reaches the moon (the distance of the moon from earth $=4\times10^8$ m).

22. The wavelength of yellow light is 6000 A°. The coherence length for this light is 2·4 cm. Calculate (*a*) the coherence time (*b*) the number of oscillations in this length.

23. The distance of the screen from the two slits is 1·0 m. When a light of wavelength 6000 A° is allowed to fall on the slits, the width of fringes obtained on the screen is 2·00 mm. Determine :
    (*a*) the distance between the two slits.
    (*b*) the width of the fringe if the wavelength of the incident light is 4800A°. [*A.I.S.S.E. 1982*]

24. A laser operates at a frequency of 3×10^{14} Hz and has an aperture of 10^{-2} m. What will be the angular spread ? [*D.S.S.E. 1982*]

25. In a Young's double slit experiment the distance between the slits and the screen is 1 metre. If the distance between the slits is 5 mm, the separation of succ ssive maxima is found to be 0·1092 mm. Calculate the wavelength of the light used. [*D.S.S.E. 1984*]

26. In Young's double slit experiment, the width of two slits are in the ratio 1 : 4. Calculate the ratio of intensities at the maxima and minima. [*A.I S S.* (*Comp*) *E 1985*]

27. The first diffra.tion minima due to a single slit diffraction is at $\theta=30°$ for a light of wavelength 5000 A°. What is the width of the slit ? [*C P.M.T. 1985*]

28. A light wave of frequency 5×10^{14} Hz enters a medium of refractive index 1 5. What is the velocity of light wave and its wavelength in the medium ? [*IIT 1983*]

29. In Young's double slit experiment using monochromatic light the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1·6 and thickness 1·964 microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance (D) between screen and the slits is doubled. It is found that the distance between the successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the monochromatic light used in the experiment. [*IIT 1983*]

30. If the two slits in Young's experiment have widths in the ratio 1 : 4, deduce the ratio of intensity at the maxima and minima in the interference pattern. [*A.I.S.S.E. 1985*]

31. In a Llyod's mirror interference experiment, the slit and its image have a separation 4·32 mm and the observations are made at a plane 2·00 m way show fringes of separation 0·260 mm. Calculate the wavelength of light.

32. In a biprism experiment with light of $\lambda = 5890$A°, We had $d = 4{\cdot}0$ mm, D = 1·50 m. Calculate the maximum slit width so that the fringes may not be obliterated.

33. Calculate the largest distance from which the markings on a metre scale can be seen by using a light of $\lambda = 6000$ A° by (*a*) the naked eye (aperture = 2·1 mm) (*b*) a telescope whose objective has a diameter 3·6 cm.

34. For sodium yellow light ($\lambda = 5900$A°) the coherence length is 2·4 cm. Calculate (*a*) the number of oscillations in this length. (*b*) the coherence time.

35. The wavelengths of the visible spectrum ranges from 400 A° to 7200 A°. Find the angular breadth of the first order visible spectrum produced by a plane grating having 7200 lines per centimetre, when light is incident normally on the grating.

36. Yellow light of wavelength 5890 A° from a distant source falls on a slit 0·40 mm wide (*a*) What is the distance between two dark bands on each side of the central bright band of the diffraction pattern observed on a screen 1·5 m from the slit ? (*b*) What is the answer to (*a*) if the slit is replaced by a small circular hole of diameter 0·40 mm.

37. The length of a pin hole camera is 40 cm, and it has a hole of radius 0.5 mm in one of its wall (*a*) What is the wavelength of the light with which spot is illuminated to produces the minimum size of the image of the spot on the opposite wall ? (*b*) What is that minimum size of the spot ?

OBJECTIVE TYPE QUESTIONS

38. In Young's double slits interference experiment if the slit is made 3 fold, the fringe width will become

(*a*) 3 times (*b*) $\frac{1}{3}$ times
(*c*) $\frac{1}{6}$ times (*d*) uneffected [*C.P.M.T. 1985*]

39. If the sodium lamp is replaced by a source of blue light in Young's double slit experiment, the fringe width will

(*a*) decrease (*b*) increase
(*c*) fringes will vanish (*d*) uneffected

40. If the distance between two slits in Young's experiment is doubled, the fringe width will become

(*a*) twice (*b*) one half
(*c*) one-fourth (*d*) uneffected

41. If the distance between the screen and the slits is doubled in Young's experiment, the fring width will become

(*a*) twice (*b*) one half
(*c*) one fourth (*d*) uneffected

42. When interference takes place, there is redistribution of

(*a*) energy (*b*) frequency
(*c*) amplitude (*d*) phase

43. If the amplitude ratio of two sources producing interference is 3 : 5, the ratio of intensity at maxima and minima is

(*a*) 16 : 25 (*b*) 5 : 3
(*c*) 16 : 1 (*d*) 1 : 25

44. In Young's double slit experiment, the intensity at central maxima is I_0. One of the two slits in now covered with black paper. The intensity at the same point is now

(*a*) $2I_0$ (*b*) I_0
(*c*) $I_0/2$ (*d*) $I_0/4$

45. Two coherent sources have same :

(*a*) amplitude (*b*) frequency
(*c*) phase difference (*d*) phase

46. In a Young's double slit experiment performed with ordinary filament lamp, the fringes obtained are

(*a*) black and white
(*b*) no fringe
(*c*) coloured with central fringe black
(*d*) coloured with central fringe white. [*I.I T. J.E E. 1987*]

47. The colours shown by a thin film viewed in sunlight is due to

(*a*) scattering (*b*) interference
(*c*) dispersion (*d*) diffraction

48. A bright spot at the centre of the geometrical shadow of a small circular disc placed in the path of light is due to the phenomenon of :

(*a*) diffraction (*b*) interference
(*c*) dispersion (*d*) polarisation.

49. For a grating of a grating element 'd' the nth order principal maxima is observed in a direction making an angle θ with the direction of incident light. Then :

(a) $d \sin \theta=(2n+1) \dfrac{\lambda}{2}$

(b) $d \sin \theta = n\lambda$

(c) $d \tan \theta=n\lambda$

(d) $d \cos \theta=n\lambda$.

50. Which one of the following waves cannot be polarised ?

(a) radio waves (b) X-rays
(c) micro waves (d) sound waves.

51. A polariod produces a strong beam of light which is :

(a) circularly polarised (b) elliptically polarised
(c) plane polarised (d) unpolarised.

52. Using a single slit, diffraction pattern is observed using the sodium lamp. If the sodium lamp is replaced by a blue light lamp, the diffraction pattern :

(a) becomes narrower (b) becomes broader
(c) disappears (d) does not change.

53. If the slit width is reduced in a single slit diffraction experiment, the diffraction pattern :

(a) does not change (b) becomes narrower
(c) becomes broder (d) disappears.

54. The several images of a distant lamp seen through a fine cloth rotate on rotating the cloth. It is due to the phenomenon of :

(a) interference (b) diffraction
(c) polarisation (d) scattering.

55. In Young's double slit experiment the two slit acts as coherent source of equal amplitude 'A' and of wavelength λ. In other experiment with the same set up the two slits are sources of equal amplitude 'A' and wavelength λ but are incoherent. The ratio of the intensity of light at the mid point of the screen in the first case to that in the second case is.........

[I.I.T J.E.E. 1986]

56. A monochromatic beam of light of wavelength 6000A° in vacuum enters a medium of refractive index 1·5. In the medium wavelength is......and its frequency is......

[I.I.T. J.E.E. 1985]

57. In a Young's double slit experiment, the interference pattern is found to have an intensity ratio between the bright and dark fringes as 9. This implies that :

(a) the intensities at the screen due to the two slits are 5 units and 4 units respectively

(b) the intensities at the screen due to the two slits are 4 units and 1 unit respectively.

(c) the amplitude ratio is 3.

(d) The amplitude ratio is 2. *[I.I.T.J.E.E. 1982]*

58. Two coherent monochromatic light beams of intensities I and 4 I are superposed. The maximum and minimum possible intensities in the resulting beam are :

(a) 5 I and I (b) 5 I and 3 I
(c) 9 I & I (d) 9 I and 4 I.

[I.I.T.J.E.E. 1984]

59. The idea of the quantum nature of light has emerged in an attempt to explain :

(a) the thermal radiations of a black body
(b) the interference of light
(c) radioactivity
(d) Photoelectric emission. *[C.P.M.T. 1980]*

60. The coherent sources of intensity ratio 4 : 1 interfere. The ratio of the intensity between the maxima and minima in the interference pattern will be :

(a) 9 : 1 (b) 16 : 7
(c) 2 : 1 (d) 1 : 9. *[A.I.S.S.E. 1989]*

61. Light is polarized to the maximum when a light is incident on a glass at an angle of :

(a) 37° (b) 57°
(c) 47° (d) 67°. *[D.P.M.T. 1985]*

62. A radar transmitter generates waves whose wavelength is 10 cm. The frequency of these waves is........ *[A.I.S.S.E. 1987]*

63. In a certain region of a thin film, we get 10 fringes with light of $\lambda = 4358$ A°. The number of fringes observed with the light of $\lambda = 5893$ A° in the same region will be........ *[D.S.S.E. (Comp.) 1988]*

64. Indicate the colour of light which travels through glass with the minimum speed.

(a) red (b) violet
(c) green (d) yellow.

[D.P.M.T. 1984]

65. Radio waves from an antenna travel with the velocity of
(a) light (b) ultrasonics
(c) sound (d) satellite.

[D.P.M.T. 1985]

66. The velocity of blue light in vacuum is :
(a) more than the velocity of green light
(b) less than the velocity of green light
(c) equal to velocity of green light
(d) half the velocity of green light.

67. Velocity of which of the following lights will be maximum while passing through a prism :
(a) violet (b) red
(c) green (d) yellow.

[D.P.M.T. 1987]

68. The fringes get displaced in :
(a) Fresnel's biprism (b) Llyod's mirror
(c) Young's double slit (d) thin film.

[D.P.M.T. 1987]

69. Water is transparent to visible light. Still it is not possible to see object at a distance in fog which consists of fine drops of water suspended in air. This is so because
(a) fine drops of water are opaque to visible light
(b) most of the light scattered away, hence apparant opacity.
(c) fog affects our vision adversely, the light rays suffer total interval.
(d) due to reflection the light cannot reach to the eye of the observer. [D.P.M.T. 1988]

70. The idea of the quantum nature of light has emerged in an attempt to explain
(a) the thermal radiation of a black body
(b) the interference of light
(c) radioctivity
(d) thermionic emission. [D.P.M.T. 1988]

71. Froussboffer's lines are evidence of :
(a) the complete absence of certain elements in the core of the sun.
(b) the absence of certain elements in the sun's surface layers.

(c) the presence of certein elements in the sun's surface layers.
(d) the presence of certain elements in the interior of the Sun.
[D.P.M.T. 1988]

72. Generally the approximate limits of visible spectrum are :
(a) 1000 to 4000 A° (b) 7000 to 10,000 A°
(c) 4000 to 7000 A (d) 10,000 to 13,000 A°.
[D.P.M.T. 1988]

73. A diffraction pattern is obtained using a beam of red light. What happens if the red light is replaced by blue light
(a) No change
(b) Bands becomes broader and farther apart
(c) bands disappear
(d) diffraction bands become narrower and crowded together.
[D.P.M.T. 1988 and 89]

74. What is the frequency of radiowaves corresponding to a wavelength of 10 m :
(a) $3{\cdot}4\times10^{-7}$ Hz (b) $3{\cdot}0\times10^{9}$ Hz
(c) 3×10^{7} Hz (d) $3{\cdot}3\times10^{-8}$ Hz
[D.P.M.T. 1988]

75. Newton postulated his corpuscular theory of light on the basis of :
(a) Newton's rings
(b) rectilinear propagation of light
(c) colour through thin film
(d) dispersion of white light into colours.
[D.P.M.T. 1989]

76. Rising and setting sun appears to be reddish because :
(a) The sun is colder at sunrise or at sunset
(b) diffraction sends red rays to the earth at these times.
(c) refraction is responsible for it.
(d) scattering due to dust particles and air molecules is resposible for it. [D.P.M.T. 1989]

□□

Ray Optics and Optical Instruments

IMPORTANT FORMULAE

1. Illuminance,

$$E=\frac{\phi}{A}=\frac{I}{r^2};$$

ϕ=Luminous flux

A=Area of surface

I=Luminous intensity

r=distance of the surface from the source.

The unit of E, ϕ and I are lux, lumen and candela respectively.

$$1 \text{ lux } (l_x)=\frac{1 \text{ lumen}}{1 \text{ m}^2}$$

and 1 candela (cd)$=\frac{1 \text{ lumen}}{1 \text{ steradian}}$

2. Luminous efficiency : The luminous efficiency of a lamp is defined as the ratio of luminous flux to electric power input *i.e.*

$$\eta=\frac{\phi}{P}$$

The unit of luminous efficiency is lumen/watt (lm w^{-1}).

3. Velocity of light

(*a*) Fizeau's method;

$C=4\,m\,n\,d$; m=no. of teeth in wheel.

n=no. of rotations of wheel per sec.

d=distance between the wheel and the concave mirror.

(*b*) Michelson's rotating mirror method :

$C=16\,nd$; n=no. of rotations of mirror per sec

d=distance between two concave mirrors

4. Spherical mirror :

(*a*) $f=\frac{r}{2}$; f=focal length

r=radius of curvature

(*b*) $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ u, v=the distances of the object and the image from the pole of the mirror respectively.

(*c*) $m=\frac{v}{u}=\frac{v-f}{f}=\frac{f}{u-f}$; m=linear magnification.

5. Refraction :

μ=refractive index

$\angle i$=angle of incidence

$\angle r$=angle of refraction

(*a*) $\mu=\frac{\sin i}{\sin r}$;

(*b*) ${}_w\mu_g={}_w\mu_a\times{}_a\mu_g$; w, g and a stands for media water glass and air respectively.

or ${}_w\mu_G=\frac{{}_a\mu_g}{{}_a\mu_w}$

(Note. When we write μ without prefix and suffix, it simply means the refractive index of the medium in which the light is entering from air.)

(*c*) $\mu=\frac{\text{Read depth}}{\text{Apparent depth}}$, for a medium surrounded by two parallel plane surfaces.

6. For a prism,

(*a*) $\angle i+\angle e=\angle A+\angle \delta$,

(*b*) If angle of deviation is mimimum

$\angle i=\frac{A+\delta m}{2}$ and $\angle r=\frac{A}{2}$

$\angle i$=angle of incidence

$\angle e$=angle of emergent

$\angle A$=refracting angle of prism

$\angle \delta$=angle of deviation

(*c*) The refractive index of the material of the prism,

$\mu=\frac{\sin \frac{1}{2}(A+\delta m)}{\sin \frac{1}{2} A}$; δ_m=Angle of minimum deviation

(*d*) For a thin prism, $\delta=(\mu-1)\,A$

(*e*) The dispersive power of a prism,

$\omega=\frac{\mu_v-\mu_r}{(\mu_y-1)}$

7. Spherical lenses :

(*a*) When light is going from one medium (refractive index $=\mu_1$) to another medium (refractive index$=\mu_2$) having spherical surface of radius of curvature 'r', then

$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{r}$;

u, v = distances of the object and the image from the optical centre of the spherical surface respectively.

(*b*) The focal length (f) of a lens surrounded by the two spherical surface of radius of curvature r_1 and r_2 is given by

$$\frac{1}{f}=\frac{(\mu_2-\mu_1)}{\mu_1}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

(*c*) If the lens is surrounded by air, $\mu_1=1$ and $\mu_2=\mu$, then

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

(*d*) $$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

(*e*) $$m=\frac{v}{u}=\frac{(f-v)}{f}=\frac{f}{(u+f)};$$

m = linear magnification.

(*f*) $$\text{P (in diopter)}=\frac{1}{f\text{(in metre)}};$$

P = Power of the lens.

(*g*) The focal length (f) of thin lenses of focal lengths $f_1, f_2, f_3, \ldots\ldots$ placed in contact of each other is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}+\ldots\ldots$$

(*h*) The focal length (f) of two thin lenses of focal lengths f_1 and f_2 separated by a distance x apart is given by

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{x}{f_1 f_2}.$$

8. Simple microscope :

$$M=\left(1+\frac{D}{f}\right);$$

M = Magnifying power

D = Least distance of distinct vision

f = focal length of the lens.

9. Compound microscope :

$$M=M_0\,M_e$$
$$=\frac{V}{U}\left(1+\frac{D}{f_e}\right);$$

o and e stands for objective lens and eye piece.

10. Astronomical telescope :

(a) For normal adjustment (the final image is formed at infinity),

$$M=\frac{f_0}{f_e}$$

and $L=(f_0+f_e)$; L=Length of the telescope.

(b) For the final image formed at the least distance of distinct vision,

$$M=\frac{f_o}{f_e}\left(1+\frac{f_e}{D}\right)$$

(c) The revolving power of a telescope,

$$\theta=\frac{1{\cdot}22\lambda}{d}$$

where λ=wavelength of light

d=diameter of the objective of the telescope

θ=angle subtended by the point object at the objective.

11. Sign convention for spherical mirrors and lenses :

(a) All distances are measured from the pole of the mirror or from the optical centre of the lens.

(b) The distances measured in the direction of the incident light are taken as positive.

(c) The distances measured in the direction opposite to the direction of the incident light are taken as negative.

SOLVED EXAMPLES

Example 1. *A lamp placed 90·0 cm from a screen on one side produces the same illumination as a standard 100 cd lamp placed 2 m away on the other side of the screen. What is the luminous intensity of the first lamp ?*

Solution. By principle of photometry,

$$\frac{I_1}{r_1^2}=\frac{I_2}{r_2^2}$$

$$\therefore \quad I_1=\frac{100}{2^2}\times(0{\cdot}90)^2$$

$$=20{\cdot}25 \text{ cd.}$$

Example 2. *A satisfactory photographic print was obtained when the exposure was for 20 S at a distance of 2 metre from a 20 cd lamp. At what distance must the same point exposed for 36 seconds from a 25 cd lamp in order to obtain a satisfactory print ?*

Solution. Luminance in first case

$$=\frac{20}{2^2}=5 \text{ cd m}^{-2}$$

$\therefore$ Amount of light received by the point in 20 s

$$=5\times20=100.$$

Let x be the distance of the lamp of 25 cd from the point for correct exposure in 36 s, then

$$\frac{25}{x^2}\times36=100$$

$$\therefore \quad x=\sqrt{\frac{25\times36}{100}}$$

$$=3 \text{ metre.}$$

Example 3. *In one of Fizeau's experiment the wheel has 620 teeth was rotating 12 times per second. The distance between the wheel and the concave mirror was 10 km. Calculate the velocity of light.*

Solution. Velocity of light,

$$C=4\,mnd$$

$$=4\times620\times12\times(10\times1000)\text{ m } s^{-1}$$

$$=2{\cdot}976\times10^8\text{ ms}^{-1}$$

Example 4. *In Michelson's method when the two mirrors were 37 km apart from each other, the velocity of light was determined as $2{\cdot}96\times10^8$ mS^{-1}. What is the minimum number of revolutions per minute of the octagonal mirror ?*

Solution. $C=16\,n\,d$

$$\therefore \quad 2{\cdot}96\times10^8=16\times n\times(37\times1000)$$

$$n=\frac{2{\cdot}96\times10^8}{16\times37\times1000}$$

$$=500\text{ rev. } s^{-1}$$

$$=500\times60\text{ rev. per minute}$$

$$=30{,}000\text{ r.p.m.}$$

Example 5. *Ray of light strike a horizontal plane mirror at an angle of 45°, show by a diagram how would you arrange a second mirror in order that the reflected ray may finally be reflected from the second mirror horizontally.*

Solution. In Fig. 11·1, PQ is the incident ray of light making an angle 45° with the horizontal plane mirror OM. Let the other mirror OM′ be inclined at an angle θ with OM. The final reflected ray RS is horizontal.

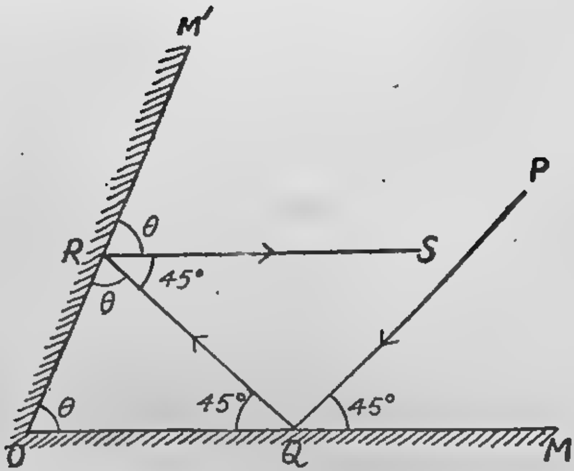


Fig. 11·1.

$$\angle RQO = \angle PQM = 45° \qquad \text{(laws of reflection)}$$

$\therefore$ $\quad \angle SRQ = \angle RQO = 45° \qquad$ (alternate angles)

Now $\quad \angle M'RS = \angle ROQ = \theta \qquad$ (corresponding angles)

and $\quad \angle ORQ = \angle M'RS = \theta \qquad$ (law of reflection)

$\therefore$ On straight line ORM',

$$\theta + 45 + \theta = 180$$

or $\quad 2\theta = (180 - 45)$

$\therefore$ $\quad \theta = \dfrac{135}{2}$

$$= 67 \cdot 5°.$$

Example 6. *An object 2 cm long is placed at a distance of 40 cm from a concave mirror of focal length 15 cm. What is (a) the position, size and nature of the image (b) the linear magnification. (c) By how much distance does the image move when the object is shifted towards the mirror through a distance of 20 cm?*

Solution. (*a*) $\quad u = -40$ cm, $f = -15$ cm

Now $\quad \dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$

$\therefore$ $\quad \dfrac{1}{v} = \dfrac{1}{-15} + \dfrac{1}{40}$

$$= \frac{-8+3}{120}$$

or $\quad \dfrac{1}{v} = \dfrac{-5}{120}$

$\therefore \qquad v = -24$ cm. The negative sign shows that the image is real.

$$\frac{I}{O} = \frac{v}{u}$$

$$I = \frac{-24}{-40} \times 2$$

$$I = 1{\cdot}2 \text{ cm}$$

(b) $$m = \frac{I}{O} = \frac{1{\cdot}2}{2} = 0{\cdot}6$$

(c) When the object is shifted 20 cm towards the mirror,

$$u_1 = -(40-20) = -20 \text{ cm}$$

$$\therefore \quad \frac{1}{v_1} + \frac{1}{u_1} = \frac{1}{f}$$

$$\frac{1}{v_1} = \frac{1}{f} - \frac{1}{u_1}$$

$$= \frac{1}{-15} + \frac{1}{20}$$

$$= \frac{-4+3}{60}$$

or $$\frac{1}{v_1} = -\frac{1}{60}$$

$$\therefore \quad v_1 = -60 \text{ cm.}$$

$\therefore$ The distance through which the image is shifted

$= 60 - 24 = 36$ cm. away from the mirror.

Example 7. *Find the position of an object placed in front of a concave mirror of focal length 24 cm so as to get an image magnified three times.*

Solution. $f = -24$ cm, $m = 3$

(i) When the image is real, $m = +3$

$$\therefore \quad \frac{v}{u} = 3$$

$$\therefore \quad v = 3u.$$

$$\therefore \quad \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{-24} = \frac{1}{u} + \frac{1}{3u}$$

$$u = -\frac{4}{3} \times 24$$

$$u = -32 \text{ cm}$$

(*ii*) When the image is virtual, $m = -3$

$$\therefore \quad \frac{v}{u} = -3$$

$$v = -3u$$

$$\therefore \quad \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{-24} = \frac{1}{v} - \frac{1}{3u}$$

$$\therefore \quad u = -16 \text{ cm.}$$

Example 8. *A convex mirror produces a magnification of $\frac{1}{3}$ when an object placed at a distance of 60 cm from it. What is the focal length of the lens ?*

Solution. $$m = -\frac{1}{3}$$

$$v = -\frac{1}{3}(u)$$

But $$u = -60 \text{ cm}$$

$$\therefore \quad v = -\frac{1}{3}(-60)$$

$$v = 20 \text{ cm}$$

$$\therefore \quad \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$= \frac{1}{-60} + \frac{1}{20}$$

$$= \frac{-1+3}{60}$$

$$\frac{1}{f} = \frac{2}{60}$$

$$f = 30 \text{ cm.}$$

Example 9. *An object is placed at a distance of 36 cm from a convex mirror. A plane mirror is placed such that the two virtual images coincide. If the plane mirror is at a distanceof 24 cm from the object, what is the radius of curvature of the convex mirror ?*

Solution. It is evident from Fig. 11·2.

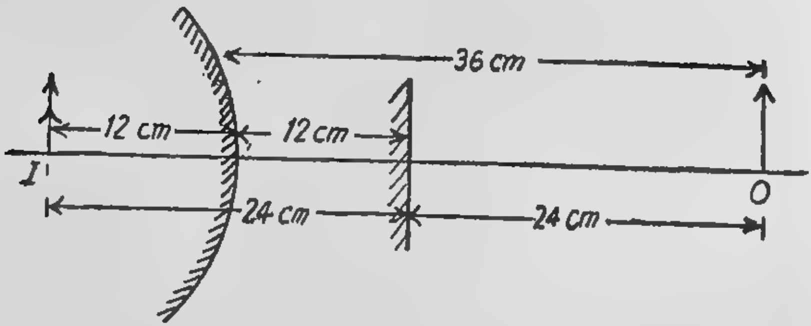


Fig. 11·2.

$$u = -36 \text{ cm}, \; v = 12 \text{ cm}.$$

$$\therefore \quad \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$= \frac{1}{-36} + \frac{1}{12}$$

$$= \frac{-1+3}{36}$$

$$\therefore \quad f = 18 \text{ cm}.$$

Example 10. *A convex lens has a focal length 25 cm. An object placed in front of the lens produces its image at 110 cm. On placing a convex mirror at 10 cm behind the lens, the image coincide with the object itself. What is the focal length of the convex mirror ?*

Solution. First the object at O produces its image at I by convex lens alone (see Fig. 11·3).

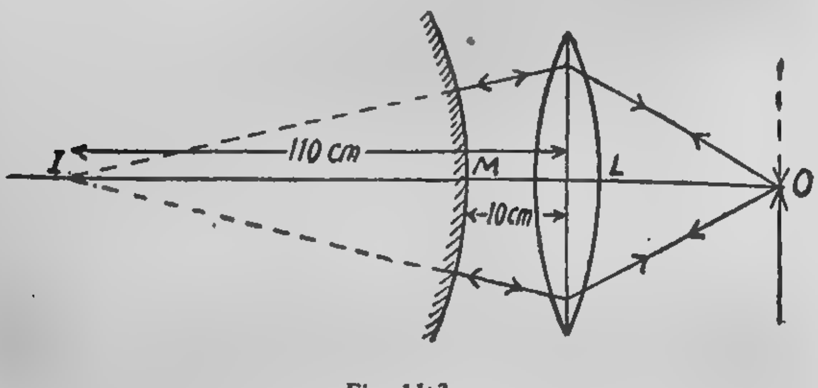


Fig. 11·3.

The rays emerging out from the lens incident on mirror **normally**, then they retrace their path back and produce the image at the object itself. It means I is the centre of curvature of the convex mirror.

$$\therefore \quad r = IM$$
$$= IL - ML$$
$$= 110 - 10$$
$$= 100 \text{ cm.}$$

Example 11. *A ray of light is incident on a glass slab at an angle of 60°. The refractive index of glass is 1·5. (a) What is the angle of refraction ? (b) If the glass slab is immersed in water ($\mu = \frac{4}{3}$). What is the angle of refraction ? (c) What is the cri ical angle of glass with respect to air ?*

Solution. (*a*) $\angle i = 60°$, ${}_a\mu_g = 1·5$

Now $${}_a\mu_g = \frac{\sin i}{\sin r}$$

$$\therefore \quad \sin r = \frac{\sin 60}{1·5}$$
$$= \frac{0·866}{1·5}$$
$$= 0·5733$$
$$\angle r = 34°59'$$

(*b*) $${}_w\mu_g = {}_w\mu_a \times {}_a\mu_g$$
$$= \frac{{}_a\mu_g}{{}_a\mu_w}$$
$$= \frac{1·5}{4/3}$$
$$= 1·125$$

$$\therefore \quad \sin r = \frac{\sin 60}{1·125}$$
$$= \frac{0\ 866}{1·125}$$
$$= 0·7698$$

$$\therefore \quad \angle r = 50°9'$$

(*c*) If C is the critical angle, $\angle i = C$ and $\angle r = 90°$ and the light is going from glass to air.

$$\therefore \quad {}_g\mu_a = \frac{\sin i}{\sin r}$$

$$\frac{1}{{}_a\mu_g} = \frac{\sin C}{\sin 90}$$

$$\therefore \qquad \sin C = \frac{1}{1{\cdot}5}$$

or $$\sin C = 0{\cdot}6667$$

$$\therefore \qquad \angle C = 41°43'$$

Example 12. *A microscope is focussed on a small object. On covering the object with a sheet of transparent glass the microscope must be raised through a distance of 2 1 mm to focus the small objects and a further distance of 4·5 mm. to focus on a scratch on the upper surface of the sheet. What is the refractive index of the glass ?*

Solution.

$$\mu = \frac{\text{Real depth of glass sheet}}{\text{Apparent depth of glass sheet}}$$

$$= \frac{(2{\cdot}1 + 4{\cdot}5)}{4{\cdot}5}$$

$$\mu = 1{\cdot}467.$$

Example 13. *A ray of light falls normally on the face of a prism of refractive index 1 5. Find the angle of the prism if the ray just fails to emerge from the second face.*

Solution. Let C be the angle of incidence on the second face. As the ray just fails to emerge the angle of refraction is 90°.

$$\therefore \quad \sin C = \frac{1}{\mu}$$

$$\left[\because \quad {}_a\mu_g = \frac{\sin 90°}{\sin C}\right]$$

$$= \frac{1}{1{\cdot}5}$$

or $$\sin C = 0{\cdot}6667$$

$$\therefore \qquad C = 41°\ 48'$$

Now $\angle A + \angle B = \angle B + \angle C = 90°$

$$\therefore \qquad \angle A = \angle C$$

$$\therefore \qquad \angle A = 41°\ 48'.$$

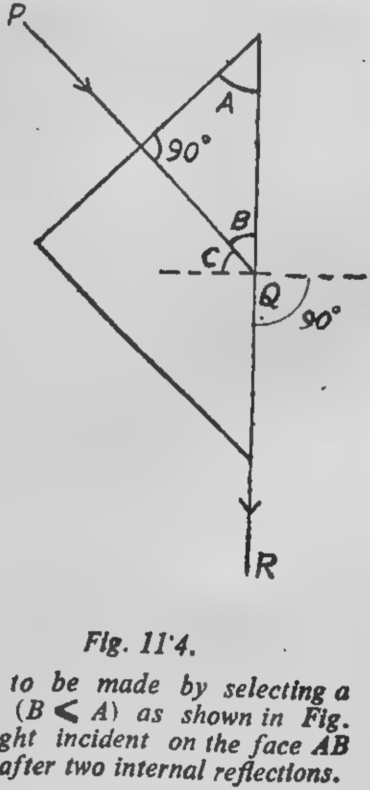


Fig. 11·4.

Example 14. *A right prism is to be made by selecting a proper materi l and th angles A and B (B < A) as shown in Fig. 11·5. It is desired that a ray of light incident on the face AB emerges parellel to the incident directi after two internal reflections.*

(*a*) *What should be the minimum refractive index* μ *for this to be possible ?*

(*b*) *For* $\mu=\frac{5}{3}$ *is it possible to achieve this with the angle B equal to 30 degrees ?* [*I.I.T. J.E.E., 1987*]

Solution. (*a*) Draw perpendiculars at the two points of incidence M and N on the line AC and CB respectively. Then angle of incidence at M and N will be A and B respectively.

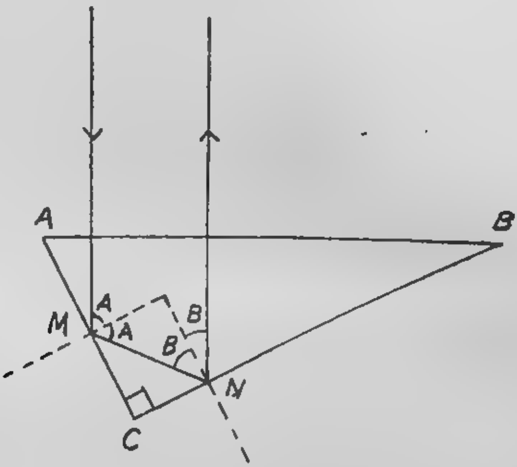


Fig. 11·5

For total internal reflection to take place at M and N,

$$\frac{1}{\sin A} < \mu \text{ and } \frac{1}{\sin B} < \mu$$

The maximum value of B is 45° since B < A so that minimum value of μ will be

$$\mu=\frac{1}{\sin B}=\frac{1}{\sin 45}=\sqrt{2}=1{\cdot}414$$

(*b*) When B=30°, for total internal reflection to take place at N,

$$\frac{1}{\sin B} < \mu$$

$$\frac{1}{\sin 30} < \mu$$

$\therefore$ $$\mu > 2$$

But given μ is $\frac{5}{3}$.

So it is not possible to achieve the reflection at N with $B=30°$ and $\mu=\frac{5}{3}$.

Example 15. *The minimum deviation produced by a glass prism of angle 60° is 56°. Calculate the refractive index of the glass.*

[D.S.S.E. 1982]

Solution.

$$\mu=\frac{\sin \frac{1}{2}(A+\delta m)}{\sin \frac{1}{2} A}$$

$$=\frac{\sin \frac{1}{2}(60+56)}{\sin \frac{1}{2}(60)}$$

$$=\frac{\sin 58}{\sin 30}$$

$$=\frac{0{\cdot}8480}{0{\cdot}5}=1{\cdot}6960$$

Example 16. *In a spectrometer, for the prism $A=60°$, calculate angle of minimum deviation if μ of prism for orange light is $\mu=1{\cdot}647$.*

Solution.

$$\mu=\frac{\sin \frac{1}{2}(A+\delta m)}{\sin \frac{1}{2} A}$$

$\therefore \quad \sin \frac{1}{2}(A+\delta m)=\mu \sin \frac{1}{2} A$

$\therefore \quad \sin\left(30+\frac{\delta m}{2}\right)=1{\cdot}647 \sin 30$

$$=0{\cdot}8235$$

$\therefore$ From table, $0{\cdot}8235=\sin 55° \, 28'$

$\therefore \quad \left(30+\frac{\delta m}{2}\right)=55° \, 28'$

$\therefore \quad \delta m=50° \, 56'$.

Example 17. *The angle of minimum deviation for yellow light in a prism of refractive index 1·6 as found to be 46°. Calculate the refracting angle of the prism.*

Solution.

$$\mu=\frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin \frac{A}{2}}$$

$$1{\cdot}6=\frac{\sin\left(\frac{A+46}{2}\right)}{\sin \frac{A}{2}}$$

or $$1{\cdot}6 \sin \frac{A}{2} = \sin\left(\frac{A}{2}+23\right)$$

or $$1{\cdot}6 \sin \frac{A}{2} = \sin \frac{A}{2} \cos 23 + \cos \frac{A}{2} . \sin 23$$

$$(1{\cdot}6 - \cos 23) \sin \frac{A}{2} = \sin 23 \cos \frac{A}{2}$$

$$\therefore \quad \tan \frac{A}{2} = \frac{\sin 23}{(1{\cdot}6 - \cos 23)}$$

$$= \frac{0{\cdot}3907}{(1{\cdot}6 - 0{\cdot}9205)}$$

$$= \frac{0{\cdot}3907}{0{\cdot}6795}$$

or $$\tan \frac{A}{2} = 0{\cdot}5750$$

$$\therefore \quad \frac{A}{2} = 29{\cdot}9°$$

$$A = 59{\cdot}8°.$$

Example 18. *A spectrometer measures angle correct upto 6′ of an arc. If an experiment gives $A=60°$ and $\delta_m=42°\ 24'$, calculate the percentage accuracy of the value of μ.*

Solution. Percentage accuracy of value of $\mu = \frac{d\mu}{\mu} \times 100$

Now $$\mu = \frac{\sin \frac{1}{2}(A+\delta_m)}{\sin \frac{1}{2} A}$$

$$\therefore \quad \frac{d\mu}{d\delta_m} = \frac{\cos \frac{1}{2}(A+\delta_m) . \frac{1}{2}}{\sin \frac{1}{2} A}$$

Now $A=60°$, $\delta_m=42°\ 24'$ and error in measuring $\delta_m = \pm 6'$

$\therefore$ Total range of error, $d\delta_m = 12'$

or $$d\delta_m = \frac{12}{60} \times \frac{\pi}{180} \text{ rad}$$

$$= \frac{11}{3150} \text{ rad}$$

$$\therefore \quad d\mu = \frac{\frac{1}{2} \cos \frac{1}{2}(60+42°\ 24')}{\sin \frac{1}{2}(60)} \times \frac{11}{3150}$$

$$= 0{\cdot}002188$$

$$\mu = \frac{\sin \frac{1}{2}(60°+42°\ 24')}{\sin \frac{1}{2}(60)} = 1{\cdot}5586$$

$\therefore$ % accuracy for value of $\mu = \frac{d\mu}{\mu} \times 100$

$$= \frac{0{\cdot}002188}{1{\cdot}5586} \times 100$$

$$= 0{\cdot}14\%.$$

Example 19. *A source of light and a screen are placed 1 m aport, where should a convex lens of focal length 25 cm be placed to form a real image of the source on the screen ?*

Solution. $f = 25$ cm

and $u + v = 100$ cm. $\Rightarrow$ $v = (100 - u)$

$$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{25} = \frac{1}{(100-u)} - \frac{1}{(-u)}$$

$$\frac{1}{25} = \frac{100}{(100-u)\,u}$$

$$100u - u^2 = -2500$$

$$u^2 - 100u + 2500 = 0$$

$$(u-50)^2 = 0$$

$$u = 50 \text{ cm}.$$

So lens should be placed 50 cm from the source.

Example 20. *An object when placed 15 cm in front of a lens forms a real image 2 times magnified. What is the focal length of the lens ? Is it a convex lens or concave lens ?*

Solution. $u = -15$ cm

$$m = \frac{v}{u} = 2$$

$$\therefore \quad v = 2u$$

$$= 2 \times 15$$

$$v = 30 \text{ cm}$$

$$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= \frac{1}{30} + \frac{1}{15}$$

$$= \frac{1+2}{30}$$

$$\frac{1}{f}=\frac{3}{30}$$
$$f=10 \text{ cm.}$$

Since f is +ive, it is a convex lens.

Example 21. *A lens of focal length 20 cm produces a virtual image which is $\frac{1}{4}$ times the size of the object. What kind of lens is it ? Determine the position of the object and the image.*

Solution. Since the image is virtual and smaller in size of that of the object, the lens in concave.

$$f=-20 \text{ cm}$$
$$\frac{v}{u}=\frac{I}{0}=\frac{1}{4}$$
$$\therefore \quad v=\frac{1}{4}u$$
$$\therefore \quad \frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$
$$-\frac{1}{20}=\frac{4}{(-u)}-\frac{1}{(-u)}$$
$$-\frac{1}{20}=\frac{3}{-4}$$
$$u=60 \text{ cm}$$
$$\therefore \quad v=\frac{1}{4}u=\frac{1}{4}\times 60=15 \text{ cm.}$$

Example 22. *A concave lens is placed on the axis of a concave mirror of radius of curvature 16 cm at a distance of 6 cm from the mirror. An object placed at 15 cm from the lens. The light from the object first passes through the lens, then gets reflected from the mirror, comes back through the lens to form an inverted image coincident with the object itself. What is the focal length of the lens ?*

Solution. Since the rays after refraction from the lens are reflected back by the mirror to the same incident path, so they are striking the mirror normally so on producing them backward they will meet on centre of curvature (c) of the mirror.

$$\therefore \quad MC=16 \text{ cm} \quad \text{(Given)}$$
$$ML=6 \text{ cm} \quad \text{(Given)}$$
$$\therefore \quad v=-LC=-(MC-ML)$$
$$=-(16-6)$$
$$\therefore \quad =-10 \text{ cm}$$

and also $u=-15$ cm

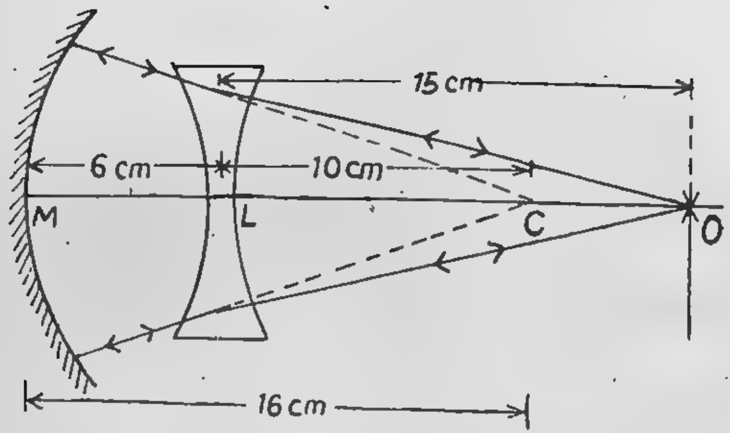


Fig. 11'6.

$$\therefore \quad \frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$=\frac{1}{-10}-\frac{1}{(-15)}$$

$$=\frac{-3+2}{30}$$

$$\frac{1}{f}=-\frac{1}{30}$$

$$f=-30 \text{ cm.}$$

Example 23. *An air bubble is left inside a solid sphere of glass at a distance of 1 cm from its centre. What will be the distance of air bubble if it is viewed from a surface nearer to it. The radius of the sphere is 7 cm and its glass has refractive index 1'4.*

Solution. $u=7-1=6$ cm.

$r=7$ cm.

$${}_g\mu_a=\frac{1}{{}_a\mu_g}$$

$$=\frac{1}{1\cdot4}$$

$$=\frac{10}{14}$$

$$=\frac{5}{7}$$

Now $\dfrac{\mu}{v}-\dfrac{1}{u}=\dfrac{\mu-1}{r}$

$$\frac{5/7}{v}-\frac{1}{6}=\frac{5/7-1}{7}$$

$$\frac{5}{7v}=\frac{1}{6}-\frac{2}{49}$$

$$\frac{5}{7v}=\frac{49-12}{49\times 6}$$

$$v=\frac{5}{7}\times\frac{49\times 6}{37}$$

$$=5{\cdot}67 \text{ cm.}$$

Example 24. *The image of an object at a distance of 100 cm from a convex lens is formed at 20 cm. from the lens on its other side. If radii of curvature of two surfaces of lens are 25 cm and 12·5 cm, calculate the refractive index of the glass of the lens.*

Solution. $u=-100$ cm, $v=20$ cm

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$=\frac{1}{20}+\frac{1}{100}$$

$$=\frac{5+1}{100}$$

$$f=\frac{100}{6} \text{ cm}$$

Now $r_1=25$ cm, $r_2=-12{\cdot}5$ cm

$\therefore$ $$\frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\frac{6}{100}=(\mu-1)\left(\frac{1}{25}+\frac{1}{12{\cdot}5}\right)$$

or $$\frac{3}{50}=(\mu-1)\left(\frac{1+2}{25}\right)$$

$\therefore$ $$(\mu-1)=\frac{3}{50}\times\frac{25}{3}$$

$$(\mu-1)=0{\cdot}5$$

$$\mu=1{\cdot}5.$$

Example 25. *A lens of focal length 20 cm is completely dipped in water. What will be its new focal length? The refractive indices of glass and water are 1·5 and 4/3 respectively.*

Solution. When lens is in air,

$$\frac{1}{20}=({}_a\mu_g-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \quad \text{...(1)}$$

when lens is in water,

$$\frac{1}{f}=\frac{({}_a\mu_g-{}_a\mu_w)}{{}_a\mu_w}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) \qquad \ldots(2)$$

Dividing eqn. (1) by eqn. (2), we have

$$\frac{f}{20}=\frac{({}_a\mu_g-1)}{({}_a\mu_g-{}_a\mu_w)}\times{}_a\mu_w$$

$$\therefore \qquad f=\frac{(1\cdot5-1)}{(1\cdot5-4/3)}\ 4/3\times20$$

$$f=80 \text{ cm.}$$

Example 26. *A convex lens placed over a plane mirror produces the image of a pin at a distance of 20 cm from the lens coincident with the pin. If the few drops of water are poured between the lens and the mirror the pin has to be displaced by 10 cm away from lens so that its image is again coincident with it. What is (a) the radius of curvature of lens (b) the refractive index of water? (The refractive index ef glass=1·5).*

Solution. (*a*) In first case

$$r_1=r, \qquad r_2=-r,$$

$$\mu=1\cdot5$$

$$f=20 \text{ cm}$$

(since rays after refraction from the lens becomes parallel to principal axis and falls on mirror normally so pin is at the focus of the lens)

$$\therefore \qquad \frac{1}{f}=(\mu-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\frac{1}{20}=(1\cdot5-1)\left(\frac{1}{r}+\frac{1}{r}\right)$$

$$r=20 \text{ cm.}$$

(*b*) In second case, the glass lens of focal length (f_1) and concavo-plane lens of water of focal length (f_2) are in contact with each other.

$$\therefore \qquad \frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$\frac{1}{(20+10)}=\frac{1}{20}+\frac{1}{f_2}$$

$$\therefore \qquad \frac{1}{f_2}=\frac{1}{30}-\frac{1}{20}$$

$$=\frac{2-3}{60}$$

$$\frac{1}{f_2}=-\frac{1}{60}$$

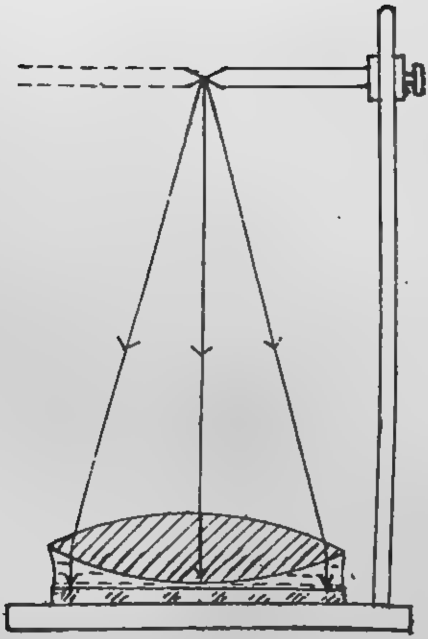


Fig. 11·7.

$\therefore$ For concavoplane lens of water,

$$\frac{1}{f_2}=({}_a\mu_w-1)\left(\frac{1}{r}-\frac{1}{\infty}\right)$$

$$-\frac{1}{60}=({}_a\mu_w-1)\left(-\frac{1}{20}\right)$$

$$\therefore \qquad ({}_a\mu_w-1)=\frac{1}{3}$$

$$\therefore \qquad {}_a\mu_w=\frac{4}{3}.$$

Example 27. *Two lenses of power +4 and −2 diopters are 10 cm. apart from each other. What is the focal length and power of the combination ?*

Solution.

$$f=\frac{100}{D}$$

$$\therefore \qquad f_1=\frac{100}{+4}=+25 \text{ cm}$$

$$f_2=\frac{100}{-2}=-50 \text{ cm}$$

and $$x=10 \text{ cm}$$

$\therefore$
$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{x}{f_1 f_2}$$
$$=\frac{1}{25}-\frac{1}{50}-\frac{10}{25\times 50}$$
$$=\frac{1}{25}-\frac{1}{50}-\frac{1}{125}$$
$$\frac{1}{f}=\frac{10-5-2}{250}$$
$$\frac{1}{f}=\frac{3}{250}$$

$\therefore$
$$f=\frac{250}{3}$$
$$=83{\cdot}3 \text{ cm.}$$

$\therefore$
$$P=\frac{100}{f}$$
$$=\frac{100\times 3}{250}$$
$$P=1{\cdot}2 \text{ diopter.}$$

Example 28. *Find the nature and focal length and power of a lens which must be placed in contact with a concave lens of focal length 20 cm in order that the lens combination may produce a real image twice the size of the object placed 30 cm from the combination.*

Solution. $$m=\frac{v}{u}=2$$

and $$u=-30 \text{ cm}$$

$\therefore$ $$v=2u=2\times 30=60 \text{ cm}$$

$\therefore$
$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$
$$=\frac{1}{60}+\frac{1}{30}$$

or $$\frac{1}{f}=\frac{1+2}{60}$$

$\therefore$ $$f=20 \text{ cm.}$$

Now $$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f}$$

and $$f_1=-20 \text{ cm (concave lens)}$$

$\therefore$ $$\frac{1}{-20}+\frac{1}{f_2}=\frac{1}{20}$$

$$\therefore \quad \frac{1}{f_2}=\frac{1}{20}+\frac{1}{20}$$

or $$f_2=10 \text{ cm.}$$

Since f_2 is +ive, lens is convex. The power of the lens,

$$P=\frac{100}{f}$$

$$=\frac{100}{10}$$

$$P=+10 \text{ diopter.}$$

Example 29. *A long sighted person cannot see objects nearer than 2 m. What kind of lens will he require in order that he can see objects clearly at all distances greater than 25 cm and what must be the focal length and power of that lens ?*

Solution. $$u=-25 \text{ cm}$$

$$v=-2 \text{ m}$$

$$=-200 \text{ cm}$$

$$\therefore \quad \frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$=-\frac{1}{200}+\frac{1}{25}$$

or $$\frac{1}{f}=\frac{-1-8}{200}$$

$$f=\frac{200}{7}$$

$$f=+28{\cdot}57 \text{ cm,}$$

convex lens (since $f=+$ive)

$$P=\frac{100}{f}$$

$$=\frac{100}{200}\times 7$$

$$P=+3{\cdot}5 \text{ diopter.}$$

Example 30. *A short sighted person can only see objects distinctly if they lie between 8 cm and 100 cm. from the eye. What kind of lens would be required to see a star clearly and what would be the focal length and power of the lens ?*

Solution. $$u=\infty$$

$$v=-100 \text{ cm,}$$

$$\therefore \quad \frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

or
$$\frac{1}{f}=-\frac{1}{100}-\frac{1}{\infty}$$

$\therefore$
$$f=-100 \text{ cm},$$

concave lens (since f is —ive)

$$P=\frac{100}{f}=\frac{100}{-100}=-1 \text{ diopter}.$$

Example 31. *A man whose least distance of distinct vision is 24 cm uses a convex lens of focal length 1·2 cm as the magnifying glass. Find the mognification he obtains.*

Solution.
$$D=24 \text{ cm}.$$
$$f=+1{\cdot}2 \text{ cm}.$$

$\therefore$
$$m=\left(1+\frac{D}{f}\right)$$
$$=\left(1+\frac{24}{1{\cdot}2}\right)$$
$$=21.$$

Example 32. *The focal length of the objective and the eye piece of a compound micro scope is 4 mm and 25 mm respectively. The length of the tube is 16 cm. If the final image is formed at the least distance of distinct vision 25 cm., find out the magnifying power of the microscope.*

Solution. For eye-piece,
$$f=24 \text{ mm}=2{\cdot}4 \text{ cm}.$$
$$v=-25 \text{ cm}.$$

$\therefore$
$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$
$$\frac{1}{2{\cdot}4}=\frac{1}{-25}-\frac{1}{u}$$

$\therefore$
$$\frac{1}{u}=-\frac{1}{25}-\frac{1}{2{\cdot}4}$$
$$=\frac{-2{\cdot}4-25}{25\times 2{\cdot}4}$$
$$\frac{1}{u}=\frac{-27{\cdot}4}{60{\cdot}0}$$
$$u=-2{\cdot}2 \text{ cm}.$$

$\therefore$ **For objective,**
$$V=(16-2{\cdot}2)$$
$$V=14{\cdot}8 \text{ cm},\quad f=0{\cdot}4 \text{ cm},$$
$$U=P$$

$\therefore$
$$\frac{1}{f}=\frac{1}{V}-\frac{1}{U}$$

$$\frac{1}{0\cdot4}=\frac{1}{14\cdot8}-\frac{1}{U}$$

$$\frac{1}{U}=\frac{1}{14\cdot8}-\frac{1}{0\cdot4}$$

$$\therefore \quad U=\frac{14\cdot8\times0\cdot4}{(0\cdot4-14\cdot8)}$$

$$=\frac{14\cdot8\times0\cdot4}{-14\cdot4}$$

$$U=-0\cdot41 \text{ cm.}$$

Magnifying power of the microscope,

$$m=\frac{V}{U}\left(1+\frac{D}{f_e}\right)$$

$$=\frac{14\cdot8}{0\cdot41}\left(1+\frac{25}{2\cdot5}\right)$$

$$=397.$$

Example 33. *The focal lengths of the objective and eye piece of an anstronomicat telescope are 50 cm. and 2·5 cm. respectively. Find the magnifying power and length of the telescope.*

Solution.

$$m=\frac{f_o}{f_e}$$

$$=\frac{50}{2\cdot5}$$

$$=20$$

$$L=(f_o+f_e)=(50+2\cdot5)=52\cdot5 \text{ cm.}$$

Example 34. *The focal lengths of the objective and eye piece of an astronomical telescope are 30 cm and 2 cm. The image of an object at a far distance is formed at 30 cm from eye. Find out the magnifying power of the telescope and the length of the telescope.*

Solution. For eye piece,

$$f=2 \text{ cm}, \ =v-30 \text{ cm}$$

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

$$\frac{1}{2}=\frac{1}{-30}-\frac{1}{u}$$

$$\therefore \quad u=\frac{30\times2}{(-2-30)}$$

$$=\frac{-60}{32}=-1\cdot875 \text{ cm.}$$

$\therefore$ magnifying power,

$$m=\frac{f_o}{u}$$

$$=\frac{30}{60}\times 32$$

$$=16$$

Length of the telescope,

$$L=(f_o+u)$$

$$=30+1{\cdot}875$$

$$L=31{\cdot}875 \text{ cm.}$$

EXERCISE 11

1. A book placed at a distance of 50 cm from a candle (Intensity of illumination=1 *cd*) can be read. What is the maximum distance at which a lamp of 100 C*d* can be placed for reading the book ?
2. Two electric lamps of 64 C*d*. and 16 C*d* respectively are placed 2 m apart. Where should a screen on the line joining in between the two be placed in order that it may be equally illuminated by each of them.
3. In a grease spot photometer light from a lamp with a dirty chimney is exactly balanced by that of a candle 10 cm from the spot. When the chimney is cleared the candle has to be shifted by 2 cm. to obtain a balance. Calculate the% light absorbed by the dirty chimney.
4. A photographic print is satisfactory when the exposure was for 15 *s* at a distance of 2 m from a 16 C*d* lamp. At what distance must it be held from a 32 C*d* lamp in order that an exposure of 20 S will give the same result ?
5. In Fizeau's experiment the distance between the source and the reflector is 8·333 km and these are 720 teeth in the toothed wheel. What should be the lowest speed of the wheel so that we may be able to measure the velocity of light.
6. In Michelson's null method, an octagonal mirror was used. The first reappearance of image occurred when the octagonal mirror was rotating at the speed of 600 revolutions per second. If the distance between the two mirrors was 31 km, find out the velocity of light.
7. What will be the angle between two mirrors when a ray incident on one and parallel to the other, after reflection at the second goes paralled to the first ?

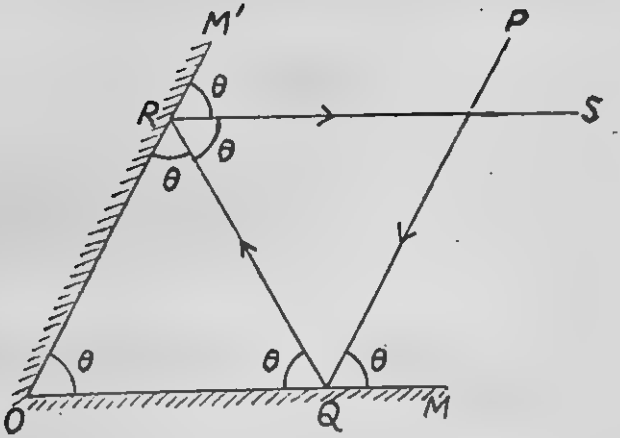


Fig. 118

8. A ray of light makes an angle of 30° with the horizontal and strikes the mirror of a microscope. What should be the inclination of the mirror with the horizontal so that the ray is reflected vertically.

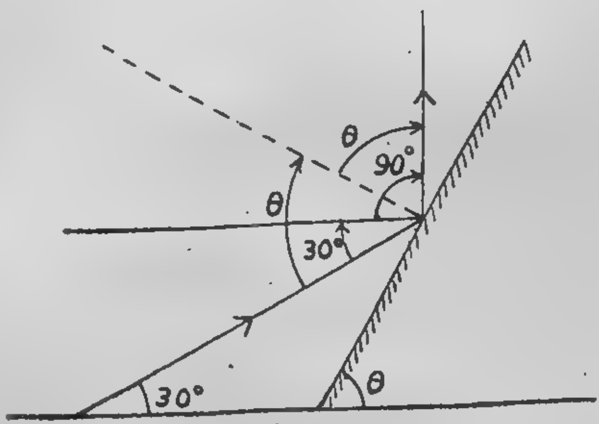


Fig. 11·9

9. An object 2 cm. long is placed 20 cm from a concave mirror of focal length 15 cm (*a*) What is the position, size and nature of the image and what is its magnification ?

(*b*) A concave mirror of focal length 10 cm is placed at a distance of 35 cm from a wall. How far from the wall should an object be placed to get its real image on the wall ?

10. An object placed at a distance of 40 cm from a convex mirror produces its image at 15 cm. What is the focal length of the mirror ?

11. An object is placed 18 cm from a concave mirror whose focal length is 10 cm. Find the position and size of the image if the object is 4 mm broad and 12 mm long.

12. A concave mirror is so placed that a candle flame situated on its principal axis at a distance of 18 cm from it produces an inverted image 3 times as long as the candle flame. (*a*) What is the radius of curvature of the mirror ? (*b*) What will be the focal length of the mirror if the image is erect in place of inverted ?

13. An object 3 cm high is placed at a distance of 120 cm from a convex mirror of radius of curvature 60 cm. What is the size of the image ?

14. An object is placed at a distance of 25 cm from a convex mirror. A plane mirror is placed such that the two virtual images coincide. If the plane mirror is at a distance of 20 cm from object, what is the focal length of the convex mirror ?

15. An object 3 cm long is placed at a distance of 45 cm from a concave mirror of focal length 20 cm. What is (*a*) the position, size and the nature of the image (*b*) the linear magnificaiion (*c*) By how much distance does the image move then object is shifted away the mirror through a distance of 15 [redacted] ?

16. A convex lens has a focal length 15 cm. An object placed in front of the lens produces its image at 70 cm. Now if a convex mirror placed at 20 cm. from lens, the image coincide with the object itself. What is the radius of curvature of the convex mirror.

17. The image formed by a convex mirror is only $\frac{1}{4}$ of the size of the object. If the focal length of the mirror is 12 cm, where is the object and the image ?

18. Show that if a ray is incident normally on a glass slab, it will pass undeviated.

19. A ray of light is incident on a glass slab at an angle of 45°. The refractive index of glass is 1·60. (*a*) What is the angle of refraction (*b*) If the glass slab is immersed in water ($\mu=4/3$). What is the angle of refraction (*c*) What is the critical angle of glass with respect to air ?

20. The critical angle of a liquid is 30°, find its refractive index.

21. A ray is travelling from diamond to glass. Calculate the value of critical angle for the ray if the refractive index of glass is 1·51 and that of diamond is 2·47.

22. A rectangular glass slab rests at the bottom of a trough of water. A ray of light incident on water surface at an angle of 50° passes through water into glass. Calculate the angle of refraction in glass given that μ for water is 1·333 and that for glass is 1·5.
23. A rectangular tank 1·6 m deep is full of water. By how much does the bottom appear to be raised (μ of water=4/3).
24. A ray of light falling normally on one of the sides of a right angles isoscles prism suffers total internal reflection. What is the refractive index of the prism ?

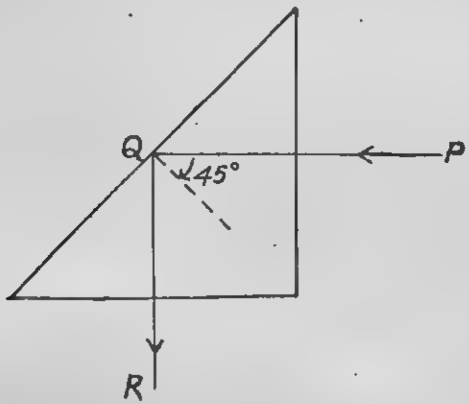


Fig. 11·10.

25. A prism of refractive index 1·414 has refracting angle 30°. One of its face is silver polished. At what angle a ray of light should be incident on one of the unsilvered face so that

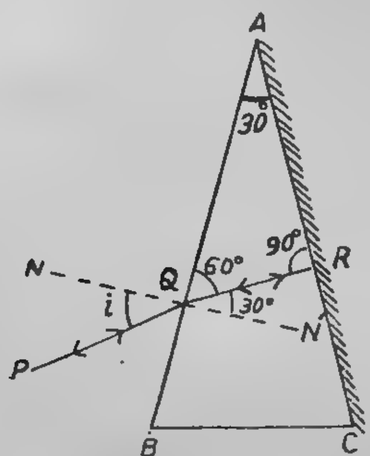


Fig. 11·11.

it retraces its path back after refraction and reflection from silvered face ?

26. A hollow prism filled with a transparent liquid has angle of minimum deviation as 30°. If the refracting angle of prism is 60°, what is the refractive index of liquid ?

27. When a narrow beam of sodium light is sent through a prism, refracting angle of prism and angle of minimum deviation are found to be 59° and 41° respectively. Calculate the refractive index of glass for sodium light.

28. The angle of minimum deviation is 42° and refractive index of the glass of prism is 1·55. Calculate the refracting angle of the prism.

29. Calculate the angle of minimum deviation for a sodium light when it passes through a dense flint prism with refracting angle of 60°. (μ of dense flint glass for sodium light $=1{\cdot}65$)

30. A spectrometer measures angles correct upto 6′ of an arc. If the experiment with this spectrometer gives $A=60°$ and $\delta_m=46°\ 42'$, calculate the percentage accuracy of the value of μ.

31. For a particular wavelength of light the angle of minimum deviation for a prism is found to be 50°. If the angle of the prism is 60°, what is the refractive index of the prism ? [*A.I.S.S.E. 1984*]

32. For a glass prism, the angle $A=60°$ and the refractive index $=1{\cdot}658$, calculate the angle of minimum deviation. [*A.I.S.S E. 1985*]

33. A monochromatic beam of light strikes a face of prism normally. If the refracting angle of prism is 30° and its refractive index is 1·50, what is the angle of deviation and angle of emergent ?

34. A spectrometer measures angle correct to 6′ of an arc. If an experiment gives $A=60°\ 0'$ and $\delta_m=48°\ 36'$, calculate the percentage accuracy of the value of μ.

35. A convex lens of 5 cm focal length is placed at a distance of 4 cm from an object. Find the position, nature and relative size of the image

36. A source of light and a screen are placed 90 cm apart. Where should a convex lens of focal length 20 cm be placed in order to form a real image of the source on the screen.

37. Find the distance at which an object should be placed in front of a convex lens of focal length 10 cm to obtain an image of double the size.

38. A lens of 12 cm focal length produces a virtual image whose linear dimensions are 1/3 that of the object. What kind of lens is it ? Find the position of the object and image.

39. The real image formed by a lens is twice the size of the object and 18 cm. from it. Find the focal length of the lens.

40. It is required to project a 3 cm square slide to give a 60 cm square picture on a screen at a distance 180 cm from the slide. What should be the focal length of the projecting lens ?

41. A concave lens whose focal length is 12 cm is placed on the axis of a concave mirror of 12 cm radius of curvature at a distance of 6 cm from the mirror. An object is so placed that the light coming from it first passes through the lens, then gets reflected from the mirror, comes back through the lens to form an inverted image coincident with the object itself. Determine the position of the object.

42. A hollow glass sphere of radius 9 cm is filled with water. If a point [illegible] cm from the centre is viewed from the nearer surface, what will be its distance from the surface ? (μ for water=4/3).

43. The refractive index of glass of a convex lens is 1·5. If the radii of curvature of its two surfaces are 20 cm and 30 cm, what is the focal length of the lens ?

44. An object 50 cm. from a lens forms its image of size 1/4 of that of the object. If radii of curvature of the two surfaces of lens are 30 cm and 25 cm respectively, what is the refractive index of glass of the lens ?

45. A lens has its focal length 80 cm and refractive index of its glass 1·5. If the lens is immersed in a liquid of refractive index 1·40, what is the new focal length of the lens ?

46. If the plane surface of a plano convex lens is silver polished, it behaves like a concave mirror of focal length 25 cm. If the convex surface is polished, it behaves as a concave mirror of focal length 9 cm. What is the refractive index of the material of the lens ?

47. Calculate the refractive index of the material of a equibi convex lens of focal length 10 cm. The radius of curvature of the convex surface is also 10 cm.

48. A convex lens placed over a plane mirror produces the image of a pin at distance of 15 cm. from the lens coincident with the pin. If the few drops of water are poured between the lens and the mirror, the pin has to be displaced by 7·5 cm away from the lens so that its image is again coincident with it. What is (*a*) the radius of curvature of the lens ? (*b*) the refractive index of water ? (The refractive index of glass=1·5).

49. Two lenses of power +6 and −2 diopter are placed in contact. Find the power and focal length of the combination.

50. Two lenses of power -2.5 and $+4$ diopter are 5 cm apart. Find the power and focal length of the combination.

51. Find the nature, focal length and power of a lens which must be placed in contact with a concave lens of focal length 25 cm in order that the combination may produce a real image 3 times the size of the object placed 20 cm from the combination.

52. What is the power of a lens which when combined with a convex lens of focal length 25 cm gives a system whose effective power is 3 diopter ?

53. A student with defective eye sight can see clearly nothing that is farther than 50 cm. from his eyes. What is the number of his correcting lens that will enable to see distant objects distinctly.

54. The far point of a short sighted person is 1·0 m. Find the power of the lens which enable him to see the distant objects distinctly.

55. A person's near point is 50 cm and far point is 1·5 m. What spectacles will be required (*a*) for reading purposes aud (*b*) for seeing distant objects ? Least distance of distinct vision is 25 cm.

56. The near point of a long sighted person is 100 cm. Find the power of the lens which enable him to see the objects at a distance 20 cm or more than 20 cm. distinctly.

57. The convex lens working as a simple microscope has the focal length 3 cm. Calculate the magnifying power of the microscope if the least distance of distinct vision is 27 cm.

58. In a compound microscope the focal length of the objective and the eye lens are 2 mm and 20 mm respectively and length of the tube is 10 cm. The least distance of distinct vision is 25 cm, Calculate the magnifying power of the microscope if the final image is formed (*a*) at infinity. (*b*) at the least distance of the distinct vision. (*c*) at 8 cm from the eye lens on the objective side.

59. Calculate the magnifying power and length of the telescope having objective of focal length 40 cm and eye piece of focal length 2 cm.

60. An astronomical telescope has magnifying power equal to 8 and length equal to 18 cm. Calculate the focal length of the objective and the eye piece.

61. In a astronomical telescope the focal length of the objective and eye lens are 75 cm and 5 cm respectively. The final image of a distant object is formed at least distance of distinct vision 30 cm. Calculate the magnifying power of the telescope.

62. The focal length of the objective and eye piece of a telescope are 60 cm and 4 cm respectively. Find the magnifying power and the length of the telescope for normal adjustment.

63. The focal lengths of the objective and eye piece of the astronomical telescope are 40 cm and 8 cm respectively. The final image of an object at a distance of 8 m from the objective is formed at 25 cm from the eye piece. Calculate the magnifying power and length of the telescope.

64. The refractive indices of crown glass for blue and red are 1·523 and 1·513 respectively and the corresponding values for flint glass are 1·665 and 1·645. Calculate the dispersive power of the two materials.

65. Calculate resolving power of a giant telescope whose objective has a diameter 5 m. ($\lambda = 5750 A°$)

OBJECTIVE TYPE QUESTIONS

66. A thin lens of refractive index 1·5 has a focal length of 15 cm. in air. When the lens is placed in a medium of refractive index $\frac{4}{3}$, its focal length will become......cm.

[*I.I.T., J.E.E. 1987*]

67. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror. The size of the image is approximately equal to

(*a*) $b\left(\frac{u-f}{f}\right)^{1/2}$ (*b*) $b\left(\frac{f}{u-f}\right)^{1/2}$

(*c*) $b\left(\frac{u-f}{f}\right)$ (*d*) $b\left(\frac{f}{u-f}\right)$

[*I.I.T., J.E.E. 1988*]

68. A convex lens A of focal length 20 cm. and a concave lens B of focal length 5 cm. are kept along the same axis with a distance d between them. If a parallel beam of light falling on A leaves B as a parallel beam of light, then d is equal to.... cm. [*I.I.T., J.E.E. 1985*]

69. A concave lens of glass is put in a medium whose refractive index is same at that of lens. It will behave as

(*a*) converging (*b*) diverging

(*c*) plane slab (*d*) none.

[*C.P.M.T. 1981*]

70. The focal length of a double convex lens of glass (${}_a\mu_g = 1·5$) is 20 cm in air. Its focal length in water (${}_a\mu_w = 4/3$) is :

(*a*) 20 cm (*b*) 80/3 cm

(*c*) 75 cm (*d*) 80 cm. [*C.P.M.T. 1978*]

71. The radius of curvature of the convex surface of an equibiconvex lens is 15 cm. If the refractive index of the material of the lens is 1·5, the focal length of the lens :

(a) 5 cm (b) 10 cm
(c) 15 cm (d) 20 cm.

72. The relation between the focal length (f) of an equibiconcave (or equibiconvex) lens ($\mu=1·5$) and the radius of curvature (r) of its curved surface is :

(a) $f=r$ (b) $f=r/2$
(c) $f\neq r$ (d) undecided.

73. Light energy has the nature of :

(a) electromagnetic wave
(b) particle
(c) transverse wave
(d) wave and particle both.

74. Speed of light in water ($\mu=4/3$) is :

(a) 3×10^8 ms^{-1} (b) 4×10^8 ms^{-1}
(c) $2·25\times10^8$ ms^{-1} (d) 340 ms^{-1}.

75. A student 1·5 m tall stands in front of a plane mirror. The minimum size of the mirror to produce full size image of student is :

(a) 75 cm (b) 3·0 m
(c) 2·0 m (d) 1·5 m.

76. A plane mirror is rotated by an angle 20°. If the direction of the incident ray remains the same, the reflected ray will rotate by an angle :

(a) 20° (b) 40°
(c) 10° (d) 30°.

77. A girl runs towards a plane mirror with a speed of 10 Kmh^{-1}. The speed with which the image moves towards the girl is :

(a) 20 Kmh^{-1} (b) 15 Kmh^{-1}
(c) 10 Kmh^{-1} (d) 5 Kmh^{-1}.

78. The reflector of the torch is a :

(a) plane mirror (b) concave mirror
(c) convex mirror (d) parabolic mirror.

79. The radius of curvature of a plane mirror is :

(a) zero (b) infinite
(c) finite (d) undecided.

80. The image formed by a mirror is erect and diminished, the mirror is :

(a) plane (b) concave
(c) convex (d) parabolic.

81. When light passes from one medium to another, the quantity that remains constant is :

(a) velocity (b) frequency
(c) wavelength (d) amplitude.

[D.P.M.T 1986]

82. The tarcoal road in summer appears to be covered with water. It is due to the phenomenon of :

(a) refraction (b) reflection
(c) total internal reflection (d) scattering.

83. Two thin lenses of +6D and −4D power are placed in contact. The focal length of the combination is :

(a) 50 cm (b) 2 m
(c) 1·5 m (d) 100 cm.

84. For a planoconcave lens ($\mu=1{\cdot}5$), the relation between its focal length (f) and radius of curvature (r) is :

(a) $f=\frac{r}{2}$ (b) $f=r$

(c) $f=\frac{3r}{2}$ (d) $f=2r$.

85. The minimum distance between the object and its real image formed by a convex lens is :

(a) f (b) $2f$
(c) $3f$ (d) $4f$.

86. When a ray is incident at 48° on a prism of refracting angle 60°, it suffers minimum deviation. The angle of minimum deviation is :

(a) 108° (b) 12°
(c) 36° (d) 54°.

87. A ray of light incident at an angle 50° on a prism of refracting angle 60° gets deviated by 30°. The angle of emergent is :

(a) 80° (b) 55°
(c) 45° (d) 40°.

88. The dispersion in a prism is greatest for :

(a) violet (b) blue
(c) orange (d) red.

89. When yellow rose is seen in blue light it appears :

(*a*) red (*b*) green
(*c*) white (*d*) black.

90. Among the following which part of spectrum of white light produces maximum visibility :

(*a*) red (*b*) yellow
(*c*) green (*d*) violet.

91. The formation of rainbow is due to the phenomenon :

(*a*) dispersion
(*b*) total internal reflection
(*c*) interference
(*d*) dispersion and total internal reflection.

92. Sky appears blue because blue light is scattered :

(*a*) least (*b*) maximum
(*c*) regularly (*d*) irregularly.

93. According to Rayleigh's law of scattering, the intensity of the scattered light is proportional to :

(*a*) λ^{-4} (*b*) λ^{-2}
(*c*) λ^{2} (*d*) λ

94. The amount of light entering the eye is controlled by :

(*a*) eye lid (*b*) eye lens
(*c*) the iris (*d*) the pupil.

95. A myopia eye (short sighted) can't see distinctly the objects placed at :

(*a*) short distances
(*b*) long distances
(*c*) least distance of distinct vision
(*d*) short distances and long distances both.

96. To correct the hypermetropia defect of vision one uses :

(*a*) convex lens
(*b*) concave lens
(*c*) combination of convex lens and concave lens
(*d*) cylindrical lens.

97. The far point of a short sighted person is 4 metre. The lens used in spectacles has the power :

(*a*) +4D (*b*) −4D
+0·25D (*d*) −0·25D.

98. A telescope in normal adjustment whose objective and eyepiece has focal lengths 50 cm and 5 cm respectively, has magnifying power :
(a) 250 (b) 55
(c) 20 (d) 10.

99. A simple microscope consisting of a lens of focal length 'f' forms image at the least distance of distinct vision 'D'. The magnifying power is
(a) $\left(1+\frac{D}{f}\right)$ (b) $\left(1-\frac{D}{f}\right)$
(c) $\frac{f}{D}$ (d) $\frac{D}{f}$.

100. If in Q. No. 99 the image is formed at infinity, the magnifying power is :
(a) $\left(1+\frac{D}{f}\right)$ (b) $\left(1-\frac{D}{f}\right)$
(c) $\frac{f}{D}$ (d) $\frac{D}{f}$

101. The distance of the object (d) from the objective (focal length $=f$) of a compound microscope is such that :
(a) $d=f$ (b) $d<f$
(c) $d>f$ (d) $d \ll f$.

102. Binoculars are based on the function of :
(a) $m=Df$ (b) $m=\left(1+\frac{D}{f}\right)$
(c) astronomical telescope (d) simple microscope.
[*D.P.M.T. 1984*]

103. A concave lens is immersed in water. It will behave as a convergent lens only if :
(a) ${}_a\mu_g > {}_a\mu_w$ (b) ${}_a\mu_g \neq {}_a\mu_w$
(c) ${}_a\mu_g < {}_a\mu_w$ (d) ${}_a\mu_g = 2\,{}_a\mu_w$ [*D.P.M.T. 1984*]

104. Two lenses of focal length 10 cm and 15 cm when put in contact forms an achromatic convex lens if the ratio of dispersive powers of the material of these lenses is :
(a) 3/2 (b) 2/3
(c) $-3/2$ (d) $-2/3$. [*D.P.M.T. 1984*]

105. Telephoto lens for a movie camera differs from the usual lens by :
(a) a shorter aperture (b) a longer aperture
(c) a longer focal length (d) a shorter focal length.
[*D.P.M.T. 1985*]

106. The refractive index of glass with respect to water is :

(a) equal to 1

(b) more than 1·5

(c) equal to 1·5

(d) more than 1 but less than 1·5. [D.P.M.T. 1986]

107. A man suffering from short sight is unable to see objects distinctly at a distance greater than 2 metres. The power of the lens required to correct this defect should be :

(a) −0·50 D (b) +0·50 D

(c) +2 D (d) −2 D. [D.P.M.T. 1986]

108. A person can see objects upto 25 cm. He wants to see distant objects distinctly. He will have to use lens of power :

(a) +4 D (b) −4 D

(c) −0·25 D (d) +0 25 D. [D.P.M.T. 1986]

109. When a light my passes through a prism, which of the following remains unchanged :

(a) frequency (b) amplitude

(c) wavelength (d) velocity. [C.P.M.T. 1987]

110. Magnifying power of the telescope is M. If the focal length of its eye piece is doubled, then the magnifying power is :

(a) 2 M (b) $\frac{M}{2}$

(c) $\sqrt{2}$ M (d) 3 M. [D.P.M.T. 1987]

111. A prism whose angular dispersion is 30° deviates light by 60°. The dispersive power of the prism is :

(a) 0·5 (b) 0·02

(c) 0·05 (d) 0·2. [C.P.M.T. 1987]

112. A prism of angle 60° deviates light by 15°. The refractive index will be :

(a) 1·25 (b) 12·5

(c) 5 (d) 0·5. [C.P.M.T. 1987]

113. Astigmatism is corrected with the help of :

(a) bifocal glasses (b) cylindrical glasses

(c) concave lens (d) convex lens. [D.P.M.T. 1988]

114. In a pinhole camera, the effect of doubling the diameter of the hole from 0·5 mm to 1·00 mm is to :

(a) double the magnification of the image

(b) worsen the chromatic aberation of the image

(c) increase burring of the image caused by diffraction.

(*d*) cut the necessary exposure time to one-fourth its previous value. [*D.P.M.T. 1988*]

115. At what angle does a driver see the setting sun ?

(*a*) at 0° to horizontal (*b*) at 41° to the horizontal
(*c*) at 90° to the horizontal
(*d*) at 60° to the horizontal. [*D.P.M.T. 1988*]

116. The refractive index of glass is least for

(*a*) red light (*b*) yellow light
(*c*) violet light (*d*) green light.
[*D.P.M.T. 1988*]

117. A post driven into a river bed stands 1 metre above the water surface. If the sun is 30° over the horizon and the river is 2 metre deep, then the length of the shadow thrown by the post on the bottom of river is :

(*a*) 1·73 metre (*b*) 3·46 metre
(*c*) 3·00 metre (*d*) 4·50 metre
[*D.P.M.T. 1988*]

118. 'An opera glass (Galelio telescope) measures 9 cm. from the objective to the eye piece. The focal length of the objective is 15 cm. Its magnifying power is :

(*a*) 2·5 (*b*) 5/3
(*c*) 2/5 (*d*) 0·4.
[*D.M.P.T. 1988*]

119. In a movie hall the distance between the projector and the screen is increased by 1%. The illumination on the screen is :

(*a*) increased by 1% (*b*) increased by 2%
(*c*) decreased by 1% (*d*) decreased by 2%.
[*D.P.M.T. 1988*]

120. The magnifying power of an astronomical telescope can be increased if we :

(*a*) increase the focal length of the objective
(*b*) increase the focal length of the eye piece
(*c*) decrease the focal length of the objective
(*d*) decrease the focal length of the objective and at the same time increase the focal length of the eye piece.
[*D.P.M.T. 1988*]

121. The unit of luminous efficiency of electric bulb is :

(a) watt (b) lumen
(c) lumen/watt (d) lux.

[D.P.M.T. 1989]

122. The time taken by the ray of light to travel through glass slab of thickness 2 cm. and refractive index 1·5 will be :

(a) 1×10^{-13} s (b) 10^{-10} s
(c) 2×10^{-6} s (d) 10^{-8} s.

[D.P.M.T. 1989]

123. If a red rose is observed in a background with red light, then it will appear :

(a) red (b) greenish-yellow
(c) blue (d) invisibe.

[D.P.M.T. 1989]

□ □

Physics of the Atom

IMPORTANT FORMULAE

1. Energy gained by the cathode rays,

$$E = eV = \tfrac{1}{2}mv^2$$

2. Velocity of the undeflected cathode rays in electric and magnetic fields,

$$v = \frac{E}{B}$$

3. In Bohr's model of the atom,

The angular momentum of an electron in an orbit is an integral multiple of $\frac{h}{2\pi}$.

$$mv_n a_n = \frac{nh}{2\pi}.$$

4. The radius of nth orbit of an atom,

$$= \frac{n^2h^2}{4\pi^2mkze^2} = n^2a_1$$

For hydrogen atom $a_1 = 0{\cdot}53$ A°.

5. The velocity of electron in nth orbit of an atom,

$$v_n = \frac{2\pi ke^2}{nh} = \frac{v_1}{n}$$

For hydrogen, $v_1 = 2{\cdot}18 \times 10^6\ ms^{-1}$.

6. The energy of an electron in the nth orbit of an atom,

$$E_n = -\frac{2\pi^2mk^2z^2e^4}{n^2h^2} = -\frac{E_1}{n^2}$$

For hydrogen atom, $E_1 = 13{\cdot}6$ eV.

7. (a) Frequency of radiation,

$$\nu = R'Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where $$R' = \frac{2\pi^2 mk^2e^4}{h^3} \simeq 3 \cdot 3 \times 10^{15} \text{ Hz}$$

(*b*) Rydberg's Constant,

$$R = \frac{R'}{C} = \frac{2\pi^2 mk^2e^4}{Ch^3} \simeq 1 \cdot 1 \times 10^7 \, m^{-1}$$

8. Bragg's law :

$$2d \sin \theta = n\lambda$$

9. The intensity of electromagnetic radiations after passing through a material of thickness x,

$$I = I_0 \, e^{-\mu x}$$

I_0=Intensity of incident radiations

μ=absorption coefficient

10. In J.J. Thomson's experiment,

(*a*) $$\frac{e}{m} = \frac{E^2}{2B^2V}$$

(*b*) $$\frac{e}{m} = \frac{yE}{B^2L\left(D + \frac{L}{2}\right)}$$

E=electric field

B=magnetic field

V=P.D. between cathode and anode

y=deflection on screen

L=length of the plates

D=distance of the screen from the centre of the plates.

11. Millikon's oil drop experiment :

(*a*) Radius of the drop,

$$r = 3\sqrt{\frac{\eta v_0}{2(\rho - \sigma)g}}$$

(*b*) Charge on the drop,

$$q = \frac{6\pi\eta r \, (v + v_0) \, d}{V};$$

η=viscosity of air

v_0=downward terminal velocity

ρ=density of oil

σ_0=density of air

v=upward terminal velocity
V=P.D. between the plates
d=distance between the plates

12. The pressure of a gas by kinetic theory of gases,

$$p=\frac{1}{3}\frac{mN}{V}\overline{C^2}$$

and $$pV=RT$$

m=mass of one molecule
N=No. of molecules
$\overline{C^2}$ =mean square velocity of gas molecules.

13. The root mean square speed of gas molecule

$$C=\sqrt{\frac{3KT}{m}}$$

where $$K=\frac{R}{N}$$

14. Energy of photon,

$$E=h\nu$$

15. For a X-ray tube,

$$eV=h\,\nu_{max}=h\frac{C}{\lambda_{min}}$$

16. Einstein's equation for photo-electric effect,

$$\tfrac{1}{2}mv^2=h\nu-W$$

17. Threshold frequency (ν_0) is related with work function (W) as:

$$W=h\nu_0$$

18. de Broglie wavelength,

$$\lambda=\frac{h}{p}=\frac{h}{mv}$$

SOLVED EXAMPLES

Example 1. *An electron beam is moving with a velocity of 6×10^7 ms^{-1}. It passes between two parallel plates having electric field as 30 volt/cm. Calculate the magnetic field required to keep the electric beam undeflected.*

Solution. E=30 V cm^{-1}=3000 V m^{-1} ; $v=6\times10^7$ ms^{-1}.

For an undeflected electron beam,

$$v=\frac{E}{B}$$

$\therefore \quad B=\frac{E}{v}$

$$v=\frac{3000}{6\times10^{7}}=0{\cdot}5\times10^{-4}\text{ Tesla}$$

Example 2. *Calculate the radius of first Bohr orbit of hydrogen atom and velocity of electron in this arbit.*

Solution. $n=1$; $h=6{\cdot}63\times10^{-34}$ Js ; $m=9{\cdot}11\times10^{-31}$ Kg ;
$k=9\times10^{9}$ Nm^2 coulomb^{-2} ; $z=1$
and $e=1{\cdot}6\times10^{-19}$ coulomb,

Now
$$a_n=\frac{n^2h^2}{4\pi^2mkze^2}$$

$$\therefore \quad a_1=\frac{1^2\times(6{\cdot}63\times10^{-34})^2}{4\times(3{\cdot}14)^2\times9{\cdot}11\times10^{-31})\times(9\times10^{9})\times1\times(1{\cdot}6\times10^{-19})^2}$$
$$=5{\cdot}295\times10^{-11}\text{ m}$$
$$=0{\cdot}5295\text{ A}^\circ$$

$$V_n=\frac{2\pi\, kze^2}{nh}$$

$$\therefore \quad V_1=\frac{3\times3{\cdot}14\times(9\times10^{9})\times1\times(1{\cdot}6\times10^{-19})^2}{1\times6{\cdot}63\times10^{-34}}$$
$$=2{\cdot}18\times10^{6}\text{ ms}^{-1}.$$

Example 3. *The radius of first Bohr orbit of hydrogen atom is $5{\cdot}3\times10^{-11}$ m. What is the radius of fourth orbit and the velocity of the electron in this orbit if that in first orbit is $2{\cdot}18\times10^{6}$ ms^{-1}.*

Solution.
$$a_n=n^2.a_1$$
$$=4^2\times5{\cdot}3\times10^{-11}$$
$$=8{\cdot}48\times10^{-10}\text{ m}$$
$$=8{\cdot}48\text{ A}^\circ$$

$$V_n=\frac{V_1}{n}$$
$$=\frac{2{\cdot}18\times10^{6}}{4}$$
$$=5{\cdot}45\times10^{5}\text{ ms}^{-1}.$$

Example 4. *Show that the ionisation potential for a hydrogen atom is 13·6 eV.*

Solution. The energy of an electron in the nth orbit of hydrogen atom,

$$E_n=-\frac{1}{n^2}\times\frac{2\pi^2mk^2z^2e^4}{h^2}$$

$$=-\frac{1}{n^2}\frac{2(22)^2}{(7)^2}\frac{(9\cdot11\times10^{-31})(9\times10^9)^2\times1^2\times(1\cdot6\times10^{-19})^4}{(6\cdot63\times10^{-34})^2}$$

$$=-\frac{13\cdot6}{n^2}\times1\cdot6\times10^{-19}\text{ J}$$

$$=-\frac{13\cdot6}{n^2}\ eV$$

For the hydrogen atom, the energy required to remove an electron from $n=1$ to $n=\infty$ orbit is called ionisation potential.

∴ Ionisation potential,

$$=\left[\frac{-13\cdot6}{\infty}-\left(\frac{-13\cdot6}{1^2}\right)\right]$$

$$=13\cdot6\ eV$$

Example 5. *Calculate the value of Rydberg constant 'R'.*

Solution. $R=\dfrac{2\pi^2mk^2e^4}{C\,h^3}$

Now $m=9\cdot11\times10^{-31}$ Kg,

$k=9\times10^9$ Nm^2C^{-2},

$e=1\cdot6\times10^{-19}$ C,

$h=6\cdot63\times10^{-34}$ Js,

and $C=3\times10^8$ ms^{-1}

$$\therefore\quad R=\frac{2(3\cdot14)^2\times9\cdot11\times10^{-31}\times(9\times10^9)^2(1\cdot6\times10^{19})^4}{(3\times10^8)\times(6\cdot63\times10^{-34})^3}$$

$$=1\cdot093\times10^7\ m^{-1}$$

Example 6. *Calculate the ionisation potential for lithium atom ($z=3$) if the ionisation potential for hydrogen atom is 13·6 eV.*

Solution. The ionisation potential,

$$E=\frac{2\pi^2mk^2e^4}{h^2}\ \frac{z^2}{n^2}$$

Now $z=3$; $n=2$

and $\dfrac{2\pi^2mk^2e^4}{h^2}=13\cdot6\ eV$

$$\therefore\quad E=13\cdot6\times\ \frac{3^2}{2^2}$$

$$=30\cdot6\ eV.$$

Example 7. *In a head on collision between α-particle and a gold nucleus, the minimum distance of approach is $4\cdot5\times10^{-14}$ m. Calculate the energy of the α-particle (z for gold=79).*

Solution. $Z=79$, $e=1{\cdot}6\times10^{-19}$ C, $k=9\times10^{9}$ Nm^{2}C^{-2} and $r_0=4{\cdot}5\times10^{-14}$ m.

For a head-on collision between α-particle and the gold nucleus, the energy of α-particles;

$$E=\frac{2kze^2}{r_0}$$

$$=\frac{2\times(9\times10^{9})\times79\times(1{\cdot}6\times10^{-19})^2}{4{\cdot}5\times10^{-14}}\text{ J}$$

$$=\frac{2\times9\times79\times1{\cdot}60\times10^{-2}}{4{\cdot}5}\times(1{\cdot}6\times10^{-13})\text{ J}$$

$$=5{\cdot}056(1{\cdot}6\times10^{-13})\text{ J}$$

$$=5{\cdot}056\text{ Mev.}$$

Example 8. *Calculate the maximum frequency and minimum wavelength of the continuous X-rays tube whose operating voltage is 40,000 volts.*

Solution. $h\nu_{max}=eV$

$\therefore$ $\nu_{max}=\frac{eV}{h}$

Now $e=1{\cdot}6\times10^{-19}$ c;

$V=40{,}000$ Volts

and $h=6{\cdot}63\times10^{-34}$ Js.

$\therefore$ $$\nu_{max}=\frac{1{\cdot}6\times10^{-19}\times40{,}000}{6{\cdot}63\times10^{-34}}$$

$$=9{\cdot}65\times10^{18}\text{ Hz}$$

Now $\lambda_{min}=\frac{c}{\nu_{max}}$

$$=\frac{3\times10^{8}}{9{\cdot}65\times10^{18}}=0{\cdot}31\times10^{-10}\text{ m}$$

$$=0{\cdot}31\text{ A}^{\circ}.$$

Example 9. *The angle of reflection for monochromatic X-rays from a crystal whose atomic spacing is 2·5 A° is 14°. Calculate the wavelength of the X-rays.*

Solution. $d=2{\cdot}5$ A°, $\theta=14°$

Now $2d\sin\theta=n\lambda$

Take $n=1$

$\therefore$ $\lambda=2d\sin\theta$

$$=2\times2{\cdot}5\times\sin14°=1{\cdot}2095\text{ A}^{\circ}.$$

Example 10. (*a*) *A monoenergetic electron beam with electron speed of* $5{\cdot}20\times10^6$ ms^{-1} *is subject to a magnetic field of* $1{\cdot}30\times10^{-4}$ T *normal to the beam velocity. What is the radius of the circle traced by the beam, given e/m for electron equals* $1{\cdot}76\times10^{11}$ $c\ kg^{-1}$.

(*b*) *Is the formula you employ in* (*a*) *valid for calculating radius of the path of a 0·20 MeV electron beam? If not, in what way is it modified and calculate the new radius?*

Solution. (*a*) The force due to magnetic field provides the electron necessary centripetal force to go around in circular path.

$$\therefore \qquad \frac{mv^2}{r}=\text{B}\,e\,v$$

$$\therefore \qquad r=\frac{v}{e/m\ \text{B}}$$

$$=\frac{5{\cdot}20\times10^6}{1{\cdot}76\times10^{11}\times1{\cdot}30\times10^{-4}}$$

$$=0{\cdot}227\ \text{m}$$

$$=22{\cdot}7\ \text{cm}$$

(*b*) If a particle moves with a very large velocity ($\sim10^8$ ms^{-1}), its relativistic mass is given by

$$m=\frac{m_0}{\sqrt{1-v^2/C^2}}$$

$\therefore$ The formula for calculating radius modifies to

$$r=\frac{m_0v}{e\text{B}\sqrt{1-v^2/C^2}}$$

$$v=\sqrt{\frac{2\text{E}}{m_0}} \qquad \left[\because\ \text{E}=\frac{1}{2}m_0v^2\right]$$

$$=\sqrt{\frac{2\times(0{\cdot}20\times1{\cdot}6\times10^{-13})}{9\times10^{-31}}}$$

$$v=\frac{8}{3}\times10^8\ \text{ms}^{-1}$$

$$r=\frac{v}{e/m_0\text{B}\sqrt{1-\left(\frac{v}{\text{C}}\right)^2}}$$

$$=\frac{8\times10^8}{3\times(1{\cdot}76\times10^{11})\sqrt{1-\left(\frac{1\times10^8}{3\times3\times10^8}\right)^2}}$$

$$=\frac{8\times10^{-3}}{3\times1{\cdot}76\times4{\cdot}123}$$

$$=0{\cdot}367\times10^{-3}\ \text{m}$$

$$=0{\cdot}367\ \text{mm.}$$

Example 11. *In a Thomson's set-up for determination of e/m, a uniform electric field E=24·KV m⁻¹ set up between two parallel plates of length 6·0 cm produces a deflection of 10·9 cm on the floures-cent screen. A magnetic field is then switched on and adjusted to the value B=8·0×10⁻⁴ T to restore the beam to its undeflected position. The distance of the screen from the centre of the plates is 40·0 cm. Calculate the value of e/m.*

Solution.
$$\frac{e}{m}=\frac{y\,E}{B^2\,L\left(D+\frac{l}{2}\right)}$$
$$=\frac{(10{\cdot}9\times10^{-2})\times(24\times1000)}{(8\times10^{-4})^2(0{\cdot}06)(0{\cdot}40+0{\cdot}03)}$$
$$=1{\cdot}58\times10^{11}\ \text{C kg}^{-1}.$$

Example 12. *In a Millikan's oil drop experiment, a charged oil drop of mass density 880 kg m⁻³ is held stationary between two parallel plates 6·0 mm apart held at a potential difference of 157 V. When the electric field is switched off, the drop is observed to fall a distance of 2·00 mm in 35·7 s. (a) What is the radius of the drop? (b) Calculate the charge on the drop. (Viscosity of air=1·8×10⁻⁵ Pas, density of air=1·3 kg m⁻³)*

Solution. (*a*)
$$r=3\sqrt{\frac{\eta\,v}{2(\rho-\sigma)g}}$$
$$=3\sqrt{\frac{(1{\cdot}8\times10^{-5})}{2(880-1{\cdot}3)9{\cdot}8}\left(\frac{2\times10^{-3}}{35{\cdot}7}\right)}$$
$$=3\times0{\cdot}0024\times10^{-4}\ \text{m}$$
$$=7{\cdot}2\times10^{-7}\ \text{m}$$

(*b*) $qE=\frac{4}{3}\pi r^3\,(\rho-\sigma)g$

$$q=\frac{4\pi r^3\,(\rho-\sigma)gd}{3\ V}\qquad\left[\because\ E=\frac{V}{d}\right]$$
$$=\frac{4\times3{\cdot}14\times(7{\cdot}2\times10^{-7})^3(880-1{\cdot}3)(9{\cdot}8)\times(6\times10^{-3})}{3\times157}$$
$$=5{\cdot}14\times10^{-17}\ \text{C}.$$

Example 13. *In an experiment on photoelectric emission by γ-rays on platinum, the energy distribution of photoelectrons exhibits peaks at a number of discrete energies 270 keV, 339 keV and 354 keV. The binding energies of K, L and M shells in platinum are known to be 77 keV, 13 keV and 3·5 keV respectively. What is the wavelength of the γ-rays with which the data are consistent?*

Solution. $(h\nu-B)=E\ \Rightarrow\ \nu=\frac{(B+E)}{h}$

where B is the binding energy of the level from which electron is emitted after absorption of photon and E is the energy of the photo electron emitted.

$$(B+E) \text{ for K shell}=(77+270)=347\ keV$$
$$(B+E) \text{ for L shell}=(13+339)=352\ keV$$
and $$(B+E) \text{ for M shell}=(3{\cdot}5+354)=357{\cdot}5\ keV$$

Thus we find that (B+E) is almost same for all the shells.

$$\therefore \qquad (B+E)=350\ keV \text{ approximately}$$
$$=(350\times 1{\cdot}6\times 10^{-16})\ J$$
$$=56\times 10^{-15}\ J$$

$\therefore$ The wavelength of γ-rays,

$$\lambda=\frac{C}{\nu}$$
$$=\frac{hC}{(B+E)}$$
$$=\frac{(6{\cdot}6\times 10^{-34})\times(3\times 10^{8})}{56\times 10^{-15}}$$
$$=3{\cdot}5\times 10^{-12}\ m.$$

Example 14. (*a*) *Find the typical de Broglie wavelength associated with a helium atom in helium gas at room temperature 27°C and 1 atmospheric pressure.* (*b*) *Compare it with the mean separation between two atoms under these conditions.*

Solution. (*a*) de Broglie wavelength,

$$\lambda=\frac{h}{mC}$$

But $$C=\sqrt{\frac{3kT}{m}}$$

$$\therefore \qquad \lambda=\frac{h}{\sqrt{3mkT}}$$

Now mass of one helium atom

$$=\frac{4\times 10^{-3}}{6\times 10^{23}}\ kg$$
$$=\frac{2}{3}\times 10^{-26}\ kg$$
$$\therefore \quad \lambda=\frac{6{\cdot}6\times 10^{-34}}{\sqrt{3\times\left(\frac{2}{3}\times 10^{-26}\right)\times(1{\cdot}38\times 10^{-23})300}}$$
$$\lambda=0{\cdot}73\times 10^{-10}\ m=0{\cdot}73\ Å$$

(*b*) If mean separation is r, each molecule will occupy a space equal to volume of a cube of r metre size.

$$\therefore \qquad \frac{V}{N}=r^3$$

But $\qquad PV=RT$ for 1 mole of the gas

$$\therefore \qquad \frac{RT}{NP}=r^3 \quad \Rightarrow \quad r=\left(\frac{kT}{P}\right)^{\frac{1}{3}}$$

$$=\left(\frac{1\cdot38\times10^{-23}\times300}{1\cdot01\times10^5}\right)^{\frac{1}{3}}$$

$$=3\cdot4\times10^{-9} \text{ (by log method)}$$

$$\frac{r}{\lambda}=\frac{3\cdot4\times10^{-9}}{0\cdot73\times10^{-10}}$$

$$=46\cdot6$$

$\therefore$ It follows $r>>\lambda$

Example 15. *Monochromatic X-rays of $\lambda=1\cdot2$ A° are reduced to $\frac{1}{3}$ of their original intensity in passing through a gold foil of 2 mm thickness. Calculate absorption coefficient for the X-rays.*

Solution. $\qquad x=2$ mm$=0\cdot2$ cm.

$$\frac{I}{I_0}=e^{-\mu x}$$

$$\frac{1}{3}=e^{-0\cdot2\,\mu}$$

or $\qquad 3=e^{0\cdot2\,\mu}$

$$\therefore \qquad \log_{10}3=0\cdot2\,\mu\log_{10}e$$

$$0\cdot4771=0\cdot2\,\mu\times0\cdot4343$$

$$\therefore \qquad \mu=\frac{0\cdot4771}{0\cdot2\times0\cdot4343}=5\cdot5\text{ cm}^{-1}.$$

Example 16. *The wavelength of K_α line is 1·36 A° for copper. Calculate the ionisation potential of a K shell electron in copper. ($h=6\cdot63\times10^{-34}$ Js ; $c=3\times10^8$ ms^{-1})*

Solution. For K_α line, $n_1=1$ and $n_2=2$

$$\therefore \qquad \nu=Rz^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) \text{ becomes}$$

$$\nu=Rz^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right)$$

or $\qquad Rz^2=\frac{4}{3}\nu$

For ionisation potential of a K shell electron,

$$n_1=1 \text{ and } n_2=\infty$$

$$\therefore \quad E=Rz^2h\left(\frac{1}{1^2}-\frac{1}{\infty^2}\right)$$

$$\therefore \quad E=\frac{4}{3}\,\nu h\,(1)$$

$$=\frac{4}{3}\,\frac{c}{\lambda}\,h$$

$$=\frac{4\times3\times10^8}{3\times1{\cdot}36\times10^{-10}}\times6{\cdot}63\times10^{-34}\text{ J}$$

$$=19{\cdot}5\times10^{-16}\text{ J.}$$

Example 17. *What is the frequency of a photon whose energy is 66·3 eV. ($h=6{\cdot}63\times10^{-34}$ Js)*

Solution. $E=66{\cdot}3\ eV=66{\cdot}3\times1{\cdot}6\times10^{-19}\text{ J}$

$$\therefore \quad \nu=\frac{E}{h}$$

$$=\frac{66{\cdot}3\times1{\cdot}6\times10^{-19}}{6{\cdot}63\times10^{-34}}=16\times10^{15}\text{ Hz.}$$

Example 18. *Calculate the de Broglie wavelength of an electron moving under a potential difference of 500 volt. ($h=6{\cdot}63\times10^{-34}$ Js $e=1{\cdot}6\times10^{-19}$ C and $m=9\times10^{-31}$ kg.)*

Solution. From $eV=\frac{1}{2}\,mv^2$ and $\lambda=\frac{h}{mv}$, we have

$$\lambda=\frac{h}{\sqrt{2emV}}$$

$$=\frac{6{\cdot}63\times10^{-34}}{\sqrt{2\times1{\cdot}6\times10^{-19}\times9\times10^{-31}\times500}}$$

$$=0{\cdot}55\times10^{-10}\text{ m}=0{\cdot}55\text{ A}^\circ.$$

Example 19. *The wavelength of a photon is 1·5 A°. It collides with an electron. Its wavelength after collision becomes 2·0 A°. Calculate the energy of scattered electron in eV ($h=6{\cdot}63\times10^{-34}$ Js).*

Solution. $E_1=h\nu_1=h\frac{c}{\lambda_1}$

and $E_2=h\nu_2=h\frac{c}{\lambda_2}$

$\therefore$ Energy of the scattered electron,

$$\therefore \quad E_1-E_2=hc\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)$$

$$=(6{\cdot}63\times10^{-34})\,(3\times10^{8})\times\left(\frac{1}{1{\cdot}5\times10^{-10}}-\frac{1}{2{\cdot}0\times10^{-10}}\right)$$

$$=3{\cdot}315\times10^{-16}\text{ J}$$

$$=\frac{3{\cdot}315\times10^{-16}}{1{\cdot}6\times10^{-19}}\text{ eV}$$

$$=2072\text{ eV}=2{\cdot}072\text{ KeV.}$$

Example 20. *Calculate the energy in eV of a photon whose (a) wavelength is 5000 A°, (b) wavelength is 1·0 A°, (c) frequency is 1200 KHz. ($h=6{\cdot}63\times10^{-34}$ Js).*

Solution. $E=h\nu$

$$=h\,\frac{c}{\lambda}$$

$$=6{\cdot}63\times10^{-34}\times\frac{3\times10^{8}}{5000\times10^{-10}}$$

$$=3{\cdot}978\times10^{-19}\text{ J}$$

$$=\frac{3{\cdot}978\times10^{-19}}{1{\cdot}6\times10^{-19}}\text{ eV}$$

$$=2{\cdot}486\text{ eV.}$$

(b) $$E=\frac{6{\cdot}63\times10^{-34}\times3\times10^{8}}{1\times10^{-10}\times1{\cdot}6\times10^{-19}}\text{ eV}=12{\cdot}43\text{ KeV}$$

(c) $$E=h\nu$$

$$=(6{\cdot}63\times10^{-34})\times(1200\times10^{3})$$

$$=7{\cdot}956\times10^{-28}\text{ J}$$

$$=\frac{7{\cdot}956\times10^{-28}}{1{\cdot}6\times10^{-19}}\text{ eV}$$

$$=4{\cdot}97\times10^{-9}\text{ eV.}$$

Example 21. *Calculate the threshold frequency of photons which can remove photo-electrons from sodium. (work function for sodium=2·5 eV, $h=6{\cdot}63\times10^{-34}$ Js).*

Solution. $W=2{\cdot}5\text{ eV}=2{\cdot}5\times1{\cdot}6\times10^{-19}\text{ J}$

For threshold frequency,

$$h\nu_0=W$$

$$\nu_0=\frac{W}{h}$$

$$=\frac{2{\cdot}5\times1{\cdot}6\times10^{-19}}{6{\cdot}63\times10^{-34}}=0{\cdot}603\times10^{15}\text{ Hz.}$$

Example 22. *Calculate the frequency of the incident radiation on o nickel metal if the speed of a photo-electron is 10^5 ms^{-1}. (K for nickel=5·9 eV ; $h=6{\cdot}63\times10^{-34}$ Js and mass of electron=9×10^{-31} Kg.)*

Solution. $W=5{\cdot}9\ eV=5{\cdot}9\times1{\cdot}6\times10^{-19}$ J

$$h\nu=\frac{1}{2}mv^2+W$$

$$\therefore \quad \nu=\frac{\frac{1}{2}\times(9\times10^{-31})\times(10^5)^2+5{\cdot}9\times1{\cdot}6\times10^{-19}}{6{\cdot}63\times10^{-34}}$$

$$=1{\cdot}43\times10^{15}\text{ Hz,}$$

Example 23. *Calculate the de Broglie wavelength for a proton moving with a speed of 10^4 ms^{-1} (mass of proton=$1{\cdot}6726\times10^{-27}$ Kg and $h=6{\cdot}63\times10^{-34}$ Js).*

Solution.

$$\lambda=\frac{h}{mv}$$

$$=\frac{6{\cdot}63\times10^{-34}}{1{\cdot}6726\times10^{-27}\times10^4}=3{\cdot}96\times10^{-11}\text{ m}$$

$$=0{\cdot}396\text{ A°}$$

Example 24. *Calculate the momentum of electrons if their wavelength is 0·2 A°. ($h=6{\cdot}63\times10^{-34}$ Js).*

Solution. $\lambda=0{\cdot}2\text{ A°}=0{\cdot}2\times10^{-10}$ m

$$\lambda=\frac{h}{p}$$

$$\therefore \quad p=\frac{h}{\lambda}$$

$$=\frac{6{\cdot}63\times10^{-34}}{0{\cdot}2\times10^{-10}}=3{\cdot}315\times10^{-23}\text{ Kg ms}^{-1}.$$

Example 25. *The threshold frequency of cesium metal is 4×10^{14} Hz. Calculate the energy in eV of electrons ejected by visible light of wavelength 6000 A°. ($h=6{\cdot}63\times10^{-34}$ J s).*

Solution. $E=h\nu-W$

But $W=h\nu_0$

$\therefore$ $E=h\nu-h\nu_0$

$$=h(\nu-\nu_0)$$

$$=h\left(\frac{c}{\lambda}-\nu_0\right)$$

$$=6{\cdot}63\times10^{-34}\left(\frac{3\times10^8}{6000\times10^{-10}}-4\times10^{14}\right)$$

$$=6{\cdot}63\times10^{-20}\text{ J}$$

$$=\frac{6{\cdot}63\times10^{-20}}{1{\cdot}6\times10^{-19}}\text{ eV}=0{\cdot}414\text{ eV.}$$

Example 26. *Calculate the number of photons in 6·62 Joules of radiation energy of frequency 10^{12} Hz ($h=6{\cdot}62\times10^{-34}$ J s).*

[*A.I.S.S.* (*Comp.*) *E. 1984*]

Solution. $$E=nh\nu$$

$\therefore$ $$n=\frac{E}{h\nu}$$

$$=\frac{6{\cdot}62}{6{\cdot}62\times10^{-34}\times10^{12}}$$

$$=10^{22}.$$

Example 27. *What is the energy of emitted photoelectrons if light of frequency 1×10^{15} Hz is incident on a sodium target? (Work function of sodium$=2{\cdot}5$ eV, $e=1{\cdot}6\times10^{-19}$ c, $h=6{\cdot}63\times10^{-34}$ J-s).*

[*A.I.S.S.E. 1983*]

Solution. Energy of incident photon,

$$h\nu=6{\cdot}63\times10^{-34}\times1\times10^{15}=6{\cdot}63\times10^{-19}\ \text{J}$$
$$W=2{\cdot}5\ eV=2{\cdot}5\times1{\cdot}6\times10^{-19}\ \text{J}$$
$$=4{\cdot}00\times10^{-19}\ \text{J}$$

$\therefore$ Energy of photo electrons emitted,

$$E=h\nu-W$$
$$=6{\cdot}63\times10^{-19}-4{\cdot}00\times10^{-19}$$
$$=2{\cdot}63\times10^{-19}\ \text{J}.$$

Example 28. *Hydrogen atom in its ground state is excited by means of mono chromatic radiation of wavelength 975 A°. How many lines are possible in the resulting spectrum? Calculate the longest wavelength amounst them. You may assume the ionization energy of hydrogen atom is 13 6 eV.*

[*I.I.T. 1983*]

Solution. $$\lambda=975\ \text{A}=975\times10^{-10}\ \text{m}$$

$\therefore$ $$E=\frac{hc}{\lambda}=\frac{(6{\cdot}63\times10^{-34})\times(3\times10^{8})}{975\times10^{-10}}\ \text{Joule}$$

$$=20{\cdot}4\times10^{-19}\ \text{J}$$

$$=\frac{20{\cdot}4\times10^{-19}}{1{\cdot}6\times10^{-19}}\ eV$$

$$=12{\cdot}75\ eV.$$

If this energy excite the H-atom to nth quantum state,

$$K\left(\frac{1}{1^2}-\frac{1}{n^2}\right)=12{\cdot}75$$

Now $$K=13{\cdot}6\ eV$$

$\therefore$ $$13{\cdot}6-\frac{13{\cdot}6}{n^2}=12{\cdot}75$$

or $$n^2=\frac{13{\cdot}6}{{\cdot}85}=16$$

$\therefore$ $$n=4$$

So following transitions of electron are possible from $n=4$ orbit to $n=3$, $n=2$ and $n=1$, from $n=3$ orbit to $n=2$ and $n=1$, from $n=2$ orbit to $n=1$ orbit. So in all six lines will be emitted in the spectrum.

The longest wavelength will be given by

$$\frac{1}{\lambda}=R\left(\frac{1}{3^2}-\frac{1}{4^2}\right)=\frac{7R}{144}$$

But $$\frac{1}{975}=R\left(\frac{1}{1^2}-\frac{1}{4^2}\right)$$

$\therefore$ $$R=\frac{16}{15}\times\frac{1}{975}$$

$\therefore$ $$\lambda=\frac{144}{7R}$$

$$=\frac{144\times15\times975}{7\times16}$$

$$=18803{\cdot}6\ \text{Å}.$$

EXERCISE 12

1. An electron beam in a cathode ray gun remains undeflected when it passes through the mutually perpendicular electric and magnetic fields of 50 V cm^{-1} and $2{\cdot}0\times10^{-4}$ Tesla respectively. Calculate the speed of electron beam.

2. A doubly ionised lithium atom is hydrogen like with atomic number 3. Find :

   (*a*) the wavelength of radiation required to excite the electron in Lithium from the first to the third Bohr orbit. (Ionisation energy of hydrogen atom equals 13·6 eV).

   (*b*) how many spectral lines are observed in the emission spectrum of the above excited system ? [*I.I.T. 1985*]

3. What is the wavelength of an electron moving with a velocity of 500 Km/s ? [*N.C.E.R T. 1984*]

4. Calculate the energy gained by an electron when it is accelerated in a Cathode Ray Tube through 50 Volts. [*D.S.S.E. 1986*]

5. An X-ray tube works at 20 KV. Calculate the maximum speed of electron striking the anode.

6. An α-particle possessing the energy of 5·5 MeV strikes a gold foil and gets scattered through an angle of 180°. Calculate the radius of gold nucleus (Z for gold=79).

7. For the electron in a hydrogen atom in the ground state, determine the following :

   (*a*) its speed, (*b*) its total energy, (*c*) its ionisation potential.

8. Calculate the radius of first Bohr orbit of hydrogen atom.

9. The radius of first Bohr orbit is $5 \cdot 3 \times 10^{-11}$ m. Calculate the radius of the third orbit.

10. Calculate the ionisation potential of sodium atom if the ionisation potential of hyderogen atom is 13·6 eV.

11. Calculate the energy required to raise hydrogen atom from the ground state to the second excited state if the ionisation potential of hydrogen atom is 13·6 eV.

12. An X-ray tube operates at 25 KV. Calculate :

    (*a*) the maximum speed of the striking electrons.
    (*b*) the maximum frequency of X-rays produced.
    (*c*) the shortest wavelength of X-rays produced.

13. The glancing angle for the first order spectrum is found to be equal to 5°. If the crystal spacing is 2·60 A°, calculate the wavelength of X-rays.

14. X-ray of wavelength of 1·4 A° are reduced to $\frac{4}{5}$ of their original intensity in passing through a gold foil of 4 mm. thickness. Calculate the absorption coefficient for the X-rays.

15. An X-ray tube operating at 22 KV emits continuous spectrum with a sharp wavelength limit of 0·565 Å. Calculate Planck's constant.

16. The wavelength of K_β line is 0·27 Å for a metal. Calculate the ionisation potential of a K-shell electron in the metal.

17. What is the frequency of a photon whose energy is (*a*) 50 eV, (*b*) 75 eV, (*c*) 250 eV.

18. Calculate the energy of a photon whose frequency is (*a*) $0 \cdot 5 \times 10^{15}$ Hz, (*b*) $1 \cdot 2 \times 10^{15}$ Hz, (*c*) $1 \cdot 5 \times 10^{15}$ Hz, (*d*) 10 MHz, (*e*) 20 KHz.

19. Calculate the energy of a photon in eV whose wavelength is (*a*) 5500 Å, (*b*) 7500 Å, (*c*) 1·6 Å.

20. Calculate the threshold frequency of photon which can remove photoelectrons from (*a*) cesium, (*b*) potassium, (*c*) zinc. (W for Ce, K and Zn=1·8, 2·3, 3·4 eV respectively).

21. Calculate the work function in eV of iron and sodium if the threshold frequency for them are $1{\cdot}16\times10^{15}$ Hz and $0{\cdot}6\times10^{15}$ Hz respectively.

22. An electron and a proton each has a wavelength 1 A°. Calculate their (*a*) momenta, (*b*) velocities and (*c*) energies in eV.

23. Calculate the frequency of the incident radiation on a potassium metal if the speed of photoelectron is 4×10^{1} m s^{-1} (W for potassium$=2{\cdot}3$ eV).

24. Calculate de Broglie wavelength for an electron and a proton each moving with a speed of 6×10^{4} m s^{-1}.

25. A charged oil drop is suspended in a uniform electric field of intensity 4×10^{4} Vm^{-1} so that it neither falls nor rises. Find the charge on the drop if its mass is $9{\cdot}75\times10^{-15}$ Kg.

26. What will be the de Broglie wavelength of an electron having kinetic enargy of 500 eV ? ($h=6{\cdot}63\times10^{-34}$ Js, $e=1{\cdot}6\times10^{-19}$ C and $m_e=9{\cdot}11\times10^{-31}$ Kg) [*D.S.S.E. 1982*]

27. The work function of a metal is $3{\cdot}45$ eV. Calculate what should be the maximum wavelength of a photon that can eject photoelectrons from the metal. [*A.I.S.S.E. 1982*]

28. Calculate the velocity of the electron in the first Bohr's orbit, given

$h=6{\cdot}6\times10^{-34}$ Js ; $m_e=9\times10^{-31}$ Kg ;

r (First orbit)$=5{\cdot}5\times10^{-11}$ m ; $\pi=3$.

[*D.S.S. (Compt.) E 1983*]

29. A metal surface is illuminated with light of wavelength 3×10^{-7} m. The work function of the metal is $3{\cdot}3$ eV. Calculate (*a*) threshold frequency of the photons (*b*) Maximum energy of photo electrons.

($h=6{\cdot}6\times10^{-34}$ Js ; $C=3\times10^{8}$ m s^{-1}) [*D.S.S.E. 1985*]

30. A uniform electric field of intensity 400 KV/m in between the two plates of length $1{\cdot}6$ cm is perpendicular to a uniform magnetic field of flux density $2{\cdot}17\times10^{-2}$ T. An electron moving perpendicularly to both fields experiences no net force. When the magnetic field is switched off and the potential difference between the cathode and the anode of the Thomson's tube is 1 KV (*a*) What is the deflection of electron beam on the screen ? (*b*) Calculate the value of $\frac{e}{m}$. (The distance of the screen from the centre of the plates is 15 cm.)

31. The following data were obtained in a Millikan's oil drop experiment :

Plate separation	$=0{\cdot}016$ m
Voltage across the plates	$=5085$ V
Distance of fall	$=1{\cdot}021$ cm

Viscosity of air $=1{\cdot}824\times10^{-5}$ Pa-s
Density of oil $=920$ kg m^{-3}
Density of air $=1{\cdot}2$ kg m^{-3}
Average time of fall (no field) $=11{\cdot}88$ s
Average time of rise (with field) $=19{\cdot}7$ s

(*a*) Calculate tne radius of the drop.

(*b*) Find the charge on the drop.

32. A charged oil drop falls 4·0 mm in 16·0 s at constant speed in air in the absence of an electric field. The relative density of oil is 0·80, that of air is $1{\cdot}3\times10^{-3}$ and the viscosity of air is $1{\cdot}81\times10^{-5}$ NS m^{-2}. Find (*a*) the radius of the drop and (*b*) the mass of the drop (*c*) if the drop carries one electronic unit of charge and is in an electric field of 2000 V/cm, what is the ratio of the force of the electric field on the drop to its weight.

33. (*a*) Find the typical de Broglie wavelength associated with a hydrogen atom in hydrogen gas at S. T. P. (*b*) Compare it with the mean separation between two atoms under these conditions.

34. In an experiment on photoelectric emission by γ-rays on tungsten, one of the peak in the energy distribution is at 350 KeV for M shall electron. The binding energy in M shell is 50 KeV. What is the wavelength of the γ-rays ?

35. Calculate the wavelength for the waves that are associated with electrons accelerated through a potential difference of 1000 V ($e=1{\cdot}6\times10^{-19}$ C, $m_e=9{\cdot}0\times10^{-31}$ kg; $h=6{\cdot}63\times10^{-34}$JS) [*A.I.S.S.E. 1986*]

36. The threshold frequency of a photosensitive surface is 5×10^{14} Hz. A photon of frequency 10^{15} Hz is incident on the surface. Calculate (*a*) work function (*b*) energy of the incident photon and (*c*) K.E. of emitted electron in eV.

37. A sheet of silver is illuminated by monochromatic unltraviolet radiation of wavelength 1810 A°. What is the maximum energy of the emitted electron ? Threshold wavelength of silver is 2640 A°. [*D.S S.E. 1987*]

38. Calculate the number of photons in 6·63 J of radiation energy of frequency 10^{12} Hz. [*D.S.S.E. (Compt.) 1988*]

39. Calculate the de Broglie wavelength of elements of kinetic energy 125eV ($e=1{\cdot}6\times10^{-19}$ C, $m_e=9{\cdot}0\times10^{-31}$ kg, $h=6{\cdot}6\times10^{-34}$ JS) [*A.I.S.S.E. 1989*]

40. Light of wavelength 4000 A° falls on a metal surface. The work function of the metal is 1·9 eV. What is the maximum velocity of the emitted photoelectrons ? [*D.S.S.E. 1989*]

OBJECTIVE TYPE QUESTIONS

41. In Bohr model of the hydrogen atom,
    (*a*) The radius of the *n*th orbit is proportional to n^2.
    (*b*) The total energy of the electron in the *n*th orbit is inversely proportional to *n*.
    (*c*) The angular momentum of the electron in an orbit is an integral multiple of $\frac{h}{2\pi}$.
    (*d*) The magnitude of the potential of an electron in any orbit is greater than its kinetic energy. [*I.I.T., J.E.E. 1984*]

42. An electron and a proton are moving with the same kinetic energy along the same direction When they pass through a uniform magnetic field perpendicular to the direction of their motion, they describe circular paths of the same radius. (True or false). [*I.I.T., J.E.E. 1985*]

43. In a photoelectric emission process the maximum energy of the photo-electrons increases with increasing intensity of the incident light (True or False). [*I.I.T., J.E.E. 1986*]

44. Four physical quantities are listed in column I. Their values are listed in Column II in a random order :

Column I	*Column II*
(*a*) Thermal energy of air molecules at room temperature.	(*e*) 0·02 eV
(*b*) Binding energy of heavy nuclei per nucleon	(*f*) 2 eV
(*c*) X-ray photon energy	(*g*) 1 keV
(*d*) Photon energy of visible light	(*h*) 7 MeV.

The correct matching of Column I and II is given by
(A) *a—e, b—h, c—g, d—f*
(B) *a—e, b—g, c—f, d—h*
(C) *a—f, b—e, c—g, d—h*
(D) *a—f, b—h, c—e, d—h.* [*I.I.T., J.E.E, 1987*]

45. Photoelectric effect supports quantum nature of light because :
    (*a*) There is a minimum frequency of light below which no photoelectrons are emitted.
    (*b*) The maximum kinetic energy of photoelectrons depends only on the frequency of light and not on its intensity.

(*c*) Even when the metal surface is faintly illuminated, the photoelectrons leave the surface immediately.

(*d*) The electric charge on photoelectrons is quantized.
[*I.I.T., J.E.E. 1987*]

46. Two particles X and Y having equal charges after being accelerated through the same potential difference, enter a region of uniform magnetic field and describe circular paths of radii R_1 and R_2 respectively. The ratio of the mass of X to that of Y is

(*a*) $\left(\frac{R_1}{R_2}\right)^{1/2}$ (*b*) $\frac{R_2}{R_1}$

(*c*) $\left(\frac{R_1}{R_2}\right)^2$ (*d*) $\frac{R_1}{R_2}$.

[*I.I.T., J.E.E. 1988*]

47. The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation,

(*a*) the intensity increases
(*b*) the minimum wavelength increased
(*c*) the intensity remains unchanged
(*d*) the minimum wavelength decreases. [*I.I.T., J.E.E. 1988*]

48. The frequency of a photon having energy 41·25 eV is.........
[*D.S.S.E. 1986*]

49. The X-ray beam coming from an X-ray tube will be—(A) monochromatic (B) having all wavelengths smaller than a certain maximum wavelength (C) having all wavelengths larger than a certain minimum wavelength (D) having all wavelengths lying between a minimum and maximum wavelength. Which of the above four statement is correct ? [*I.I.T. 1985*]

50. What are the dimensions of Planck's constant. [*I.I.T. 1985*]

51. By Millikan's oil drop experiment we measure :

(*a*) charge of electron and verifies quantum nature of charge
(*b*) the charge on positive ion
(*c*) $\frac{e}{m}$ of electron
(*d*) $\frac{e}{m}$ of ion.

[*D.P.M.T. 1984*]

52. Which of the following has least specific charge :

(*a*) electron (*b*) proton
(*c*) α-particle (*d*) β-particle. [*D.P.M.T. 1984*]

53. For the study of internal structure of crystals we use
(a) X-rays (b) Ultra-violet rays
(c) Infrared radiations (d) Yellow light. [*D.P.M.T. 1984*]

54. In photoelectric effect
(a) electric energy changes into mechanical energy
(b) light energy changes into electrical energy
(c) photon produces electrons
(d) photon changes into electrons. [*D.P.M.T. 1984*]

55. The size of the atom is approximately equal to
(a) 10^{-8} m (b) 10^{-6} m
(c) 10^{-14} m (d) 10^{-10} m. [*D.P.M.T. 1984*]

56. Rutherford investigated nuclear structure by bombarding foil with
(a) α-particles (b) γ-rays
(c) β-particles (d) none of these.
[*D.P.M.T. 1985*]

57. An electron emits energy
(a) when it escapes from the atom
(b) because it is in orbit
(c) when it jumps from one energy level to another
(d) when it falls into nucleus. [*D.P.M.T. 1985*]

58. The value of atomic mass unit cannot be expressed in
(a) electron volt (b) newton
(c) kilogram (d) joules. [*D.P.M.T. 1985*]

59. A black paper can be passed through by
(a) X-rays only (b) visible rays only
(c) ultraviolet rays only (d) all three kinds
[*D.P.M.T. 1985*]

60. The nature of X-rays and of light rays are alike, was discovered by
(a) Mosley (b) Bragg
(c) Max Von Lane (d) Roentgen. [*D.P.M.T. 1985*]

61. Highest energy electrons will be produced by
(a) X-rays (b) visible light
(c) gamma rays (d) ultraviolet rays.
[*D.P.M.T. 1984*]

62. Lymen Series in emission spectrum of hydrogen lie in the

(a) red region (b) visible spectrum
(c) ultraviolet region (d) intrared region.

[D.P.M.T. 1986]

63. Balmer series in the emission spectrum of hydrogen atom lie in the

(a) visible region (b) infrared region
(c) violet light (d) ultraviolet region.

[D.P M.T. 1986, 87]

64. The energy gap between successive energy levels in a hydrogen atom (in Bohr's theory)

(a) decreases as n decreases
(b) increases as n increases
(c) decreases as n increases
(d) remains constant. [D.P.M.T. 1986]

65. A moving electron has energy 728 eV. Its velocity is

(a) 728 ms^{-1} (b) 1.6×10^{7} ms^{-1}
(c) 1.6×10^{10} ms^{-1} (d) none of the above.

[D.P.M.T. 1986]

66. The atomic number and mass number of a isotope are 92 and 235 respectively. The number of electrons in the neutral atom would be

(a) 92 (b) 143
(c) 235 (d) 327. [D.P.M.T. 1986]

67. The phenomenon of photoelectric effect was explained by

(a) Hertz (b) Lenard
(c) Einstein (d) Hallwachs. [D.P.M.T. 1986]

68. The number of photons emitted from a metal foil are directly proportional to.........of the light falling on it.

(a) frequency (b) wavelength
(c) intensity (d) none of the above.

[D.P.M.T. 1987]

69. The energy required to knock out the electron in the 3rd orbit of an atom is equal to

(a) -13.6 eV. (b) $\frac{-13.6}{3}$
(c) $\frac{-13.6}{9}$ eV (d) $\frac{-3}{13.6}$ eV

[D.P.M.T. 1987]

70. The number of band lines in the outermost orbit of the Na atom when it forms a metallic bond is

(a) 10^{8} (b) 10^{18}

(c) 10^{14} (d) 10^{23} [D.P.M.T. 1987]

71. Wavelength of the electron moving with velocity 3×10^{6} m/s. is nearly equal to

(a) $0{\cdot}6\times10^{-11}$ m (b) $0{\cdot}06\times10^{-11}$ m

(c) 8×10^{-9} m (d) $0{\cdot}08\times10^{-10}$ m.

[D.P.M.T. 1987]

72. Which of the following has minimum wavelength :

(a) X-rays (b) ultraviolet rays

(c) γ-rays (d) infrared waves.

[D.P.M.T. 1987]

73. Penetration of X-rays increases if we

(a) increases both wavelength and voltage

(b) decrease both wavelength and voltage

(c) increase wavelength, decrease voltage

(d) decrease wavelength, increase voltage. [D P.M.T. 1987]

74. An electron jumps from orbit $n=2$ to $n=1$. The wavelength emitted correspond to which of the following series.

(a) Lyman (b) Balmer

(c) Paschen (d) Bracket. [D P M.T. 1987]

75. Planck's constant has the dimension of :

(a) energy. (b) frequency

(c) power (d) angular momentum.

[D.P.M.T. 1988]

76. Rutherford and Geiger and Marsden's experiments on scattering α-particles from metal foils suggested

(a) the electrons formed a hard, impenetrable shell around the nucleus.

(b) that an extremely small, positively charged nucleus existed.

(c) that positive charges in an atom are eventually distributed throughout the atom.

(d) neutron exist in the nucleus. [D.P M T. 1988]

77. According to classical theory the path of an electron in Rutherford atom model will be

(a) circula (b) parabolic

(c) straight line (d) spiral. [D.P.M.T. 1989]

Physics of the Nucleus

IMPORTANT FORMULAE

1. Mass number,

$$A=Z \text{ (No. of protons)}+N \text{ (No. of neutrons)}$$

2. Einstein's mass-energy equivalence principle,

$$E=mc^2$$

3. Atomic mass unit,

$$1 \ amu=931 \text{ MeV}$$

4. Nuclear radius,

$$r=r_0(A)^{1/3} \ ; \ (r_0=1{\cdot}2\times10^{-15} \text{ m})$$

5. Binding energy,

$$B.E.=931 \ [Zm_p+(A-Z)m_n+m_A]$$

6. For a cyclotron, the period of revolution,

$$T=2\pi \frac{m}{qB}$$

7. Radioactive decay,

m_f=final mass after disintegration upto time 't'

(a) $$\left(\frac{m_f}{m_i}\right)=\left(\frac{1}{2}\right)^{\frac{t}{T}}$$

m_i=Initial mass

T=Half life time

(b) $$\lambda=\frac{0{\cdot}693}{T};$$

λ=disintegration constant

(c) $$N=N_0 \ e^{-\lambda t}$$

N=final no. of atoms disintegrate

N_0=initial no. of atoms disintegrate

SOLVED EXAMPLES

(Where necessary the following data may be used :
Mass of ${}_0n^1(m_n)=1{\cdot}008665$ *amu* ; mass of ${}_1H^1$ $(m_H)=1{\cdot}007825$ amu).

Example 1. *What is the number of protons, the number of neutrons and number of electrons in each of the following atoms :*

(*a*) ${}_3Li^6$ (*b*) ${}_5B^{10}$ (*c*) ${}_{15}P^{31}$ (*d*) ${}_{56}Ba^{137}$

Solution.

(*a*) For ${}_3\text{Li}^6$

$$A=6\ ;\quad Z=3$$

$\therefore$ No. of neutrons,

$$N=A-Z$$
$$=6-3=3$$

No. of electrons are always the same as number of protons (Z)

$\therefore$ No. of electrons $=3$.

(*b*) For ${}_5\text{B}^{10}$,

$$A=10\ ;\ Z=5$$

$\therefore$ $$N=10-5=5$$

It means ${}_5\text{B}^{10}$ atom has 5 protons, 5 electrons and 5 neutrons.

(*c*) For ${}_{15}\text{P}^{31}$,

$$A=31\ ;\ Z=15$$

$\therefore$ $$N=31-15=16$$

It means ${}_{15}\text{P}^{31}$ atom has 15 protons, 15 electrons and 16 nuetrons.

(*d*) For ${}_{56}\text{Ba}^{137}$,

$$A=137 \text{ and } Z=56$$

$\therefore$ $$N=137-56=81$$

It means ${}_{56}\text{Ba}^{137}$ atom has 56 protons, 56 electrons and 81 neutrons.

Example 2. *What is the number of protons and number of neutrons in each of the following nuclei :*

(*a*) ${}_6\text{C}^{12}$ (*b*) ${}_{10}\text{Ne}^{22}$ (*c*) ${}_{38}Sr^{88}$ (*d*) ${}_{92}\text{U}^{238}$

Solution.

(*a*) For ${}_6\text{C}^{12}$,

$$A=12\ ;\ Z=6$$

$\therefore$ $$N=12-6=6$$

It means ${}_6\text{C}^{12}$ nucleus contains 6 protons and 6 neutrons.

(*b*) For ${}_{10}\text{Ne}^{22}$,

$$A=22\ ;\ Z=10$$

$\therefore$ $$N=22-10=12$$

It means ${}_{10}\text{Ne}^{22}$ nucleus contains 10 protons and 12 neutrons

(*c*) For ${}_{38}\text{Sr}^{88}$,

$$A=88\ ;\ Z=38$$

$\therefore$ $$N=88-38=50$$

It means ${}_{38}\text{Sr}^{88}$ nucleus contains 38 protons and 50 neutrons.

(*d*) For $_{92}U^{238}$

$$A=238\ ;\ Z=92$$

$$\therefore \quad N=238-92=146$$

It means $_{92}U^{238}$ nucleus contains 92 protons and 146 neutrons.

Example 3. *Calculate the nuclear radius of* $_8O^{16}$.

Solution. We know $r_0=1{\cdot}2\times10^{-15}$ m and for $_8O^{16}$, A=16

$$r=r_0\ A^{1/3}$$

$$r=1{\cdot}2\times10^{-15}\times16^{1/3}$$

$$=1{\cdot}2\times10^{-15}\times2{\cdot}52=3{\cdot}024\times10^{-15}\text{ m.}$$

Example 4. *The nuclear radius of* $_{82}Pb^{205}$ *is* 7×10^{-15} *m. Calculate the nuclear radius of* $_{11}Na^{24}$.

Solution.

$$r=r_0A^{1/3}$$

$\therefore$ For $_{82}Pb^{205}$,

$$7\times10^{-15}=r_0\ (205)^{1/3}$$

$$r_0=\frac{7\times10^{-15}}{205^{1/3}} \qquad \ldots(i)$$

$\therefore$ For $_{11}Na^{24}$,

$$r=r_0A^{1/3}$$

$$=\frac{7\times10^{-15}}{205^{1/3}}\ 24^{1/3}.$$

$$=7\times10^{-15}\left(\frac{24}{205}\right)^{1/3}=3{\cdot}355\times10^{-15}\text{m}$$

Example 5. *The mass of an atom of chlorine* $_{17}Cl^{35}$ *is 34 9800 amu. Calculate its binding energy. What is its binding energy per nucleon.*

[*A.I.S.S.E. 1980*]
[*D.S S.E. 1981*]
[*A.I.S.S.E.* (*Compt.*) *1985*]

Solution. For $_{17}Cl^{35}$,

$$A=35\ ;\ Z=17$$

$$\therefore \quad N=A-Z=35-17=18$$

It means $_{17}Cl^{35}$ atom has 17 protons plus 17 electrons which is equivalent to 17 $_1H^1$ and 18 neutrons.

Mass of 18 neutrons

$$=18\times1{\cdot}008665=18{\cdot}155970\ amu$$

Mass of 17 hydrogen atoms ($_1H^1$)

$$=17\times1\ 007825=17{\cdot}133025\ amu$$

Total mass $=35{\cdot}288995$ *amu*

But atomic mass of $_{17}Cl^{35}$ $=34{\cdot}980000$ *amu*

$\therefore$ Loss in mass $=0{\cdot}308995$ *amu*

$\therefore$ Binding energy $= 0{\cdot}308995 \times 931$ MeV

$= 287{\cdot}674$ MeV

B.E. per nucleon $= \frac{287{\cdot}674}{35} = 8{\cdot}219$ MeV/nucleon.

Example 6. *Calculate the binding energy per nucleon in the nuclei $_{26}Fe^{56}$ and $_{15}P^{31}$. (Given mass of $_{26}Fe^{56} = 55{\cdot}934932$ amu and that of $_{15}P^{31} = 30{\cdot}973763$ amu).* [D.S.S.E. 1979] [D.S.S.E. 1988]

Solution. For $_{26}Fe^{56}$,

$A = 56$; $Z = 26$

$\therefore$ $N = A - Z = 56 - 26 = 30.$

mass of 30 neutrons

$= 30 \times 1{\cdot}008665 = 30{\cdot}259950$ amu

mass of 26 hydrogen atoms

$= 26 \times 1{\cdot}007825 = 26{\cdot}203450$ amu

Total Mass $= 56{\cdot}463400$ amu

But atomic mass of $_{26}Fe^{56}$ $= 55{\cdot}934932$ amu

Loss in mass $= 0{\cdot}528468$ amu

$\therefore$ Binding energy per nucleon

$= \frac{931 \times 0{\cdot}528468}{56} = 8{\cdot}786$ MeV/nucleon

For $_{15}P^{31}$,

$A = 31$; $Z = 15$

$\therefore$ $N = 31 - 15 = 16$

Mass of 16 neutrons

$= 16 \times 1{\cdot}008665 = 16{\cdot}138640$ amu

Mass of 15 hydrogen atoms

$= 15 \times 1{\cdot}007825 = 15{\cdot}117375$ amu

Total mass $= 31{\cdot}256015$ amu

But mass of $_{15}P^{31}$ atom $= 30{\cdot}973763$ amu

Loss in mass $= 0{\cdot}282252$ amu

$\therefore$ Binding energy per nucleon

$= \frac{931 \times 0{\cdot}282252}{31} = 8{\cdot}477$ MeV/nucleon

Example 7. *Calculate the energy released in the following reaction.*

$$_0n^1 + {}_3Li^6 = {}_2He^4 + {}_1H^3$$

(Given that the mass of $_3Li^6 = 6{\cdot}015126$ amu, mass of $_2He^4 = 4{\cdot}002604$ amu and that of $_1H^3 = 3{\cdot}016049$ amu).

Solution.

mass of $_0n^1$ $= 1{\cdot}008665$ amu

mass of $_3Li^6$	$=6{\cdot}015126$ amu
Total initial mass	$=7{\cdot}023791$ amu
Mass of $_2He^4$	$=4{\cdot}002604$ amu
mass of $_1H^3$	$=3{\cdot}016049$ amu
Total final mass	$=7{\cdot}018653$
∴ Loss in mass	$=7{\cdot}023791-7{\cdot}018653$
	$=0{\cdot}005138$ amu
∴ Energy released	$=931\times 0{\cdot}005138$
	$=4{\cdot}7835$ MeV.

Example 8. *A neutron strikes a $_{12}Mg^{24}$ nucleus with the emission of a proton. Calculate the atomic number, mass number and chemical name of the remaining nucleus.*

Solution. We can write nuclear reaction as follows,

$$_0n^1+{}_{12}Mg^{24}\rightarrow {}_1H^1+{}_ZX^A$$

Applying laws of conservation of mass and charge respectively we have

$$1+24=1+A \text{ or } A=24$$

and
$$0+12=1+Z \text{ or } Z=11$$

$$\therefore \quad {}_ZX^A={}_{11}X^{24}$$

It means the atomic number is 11 and the mass number is 24. Then chemical name of the nucleus should be Sodium.

Example 9. *Insert the missing symbols in the following nuclear reactions :*

(*a*) $_6C^{12}+{}_1H^2\rightarrow {}_6C^{13}+\ldots\ldots$

(*b*) $_{15}P^{31}+\ldots\ldots\rightarrow {}_{16}S^{34}+{}_1H^1$

(*c*) $_6C^{14}\rightarrow {}_{-1}\beta^0+\ldots\ldots$

(*d*) *α-particle* $+{}_4Be^9\rightarrow {}_0n^1+\ldots\ldots$

Solution. (*a*) Let $_6C^{12}+{}_1H^2\rightarrow {}_6C^{13}+{}_ZX^A$

On applying the principles of conservation of mass and charge, we have

$$12+2=13+A \text{ or } A=1$$

and
$$6+1=6+Z \quad \text{or } Z=1$$

$$\therefore \quad {}_ZX^A={}_1X^1={}_1H^1$$

(*b*) Let $_{15}P^{31}+{}_ZX^A={}_{16}S^{34}+{}_1H^1$

On applying the principles of conservation of mass and charge, we have

$$31+A=34+1 \text{ or } A=4$$

$$15+Z=16+1 \text{ or } Z=2$$

$$\therefore \quad {}_ZX^A={}_ZX^A={}_2He^4$$

(*c*) Let $_6C^{14}\rightarrow {}_{-}\beta^0+{}_ZX^A$

On applying the principles of conservation of mass and charge, we have

$$14=0+A \text{ or } A=14$$

and $$6=-1+Z \text{ or } Z=7$$

$\therefore$ $$_ZX^A={}_7X^{14}={}_7N^{14}$$

(*d*) Let $_2He^4+{}_4Be^9\rightarrow{}_0n^1+{}_ZX^A$

On applying the principles of conservation of mass and charge, we have

$$4+9=1+A \text{ or } A=12$$

and $$2+4=0+Z \text{ or } Z=6$$

$\therefore$ $$_ZX^A={}_6X^{12}={}_6C^{12}$$

Example 10. *When $_{14}Si^{28}$ nuclei are bombarded by neutrons, protons eject from the target. Write down the reaction equation.*

Solution. Let $_0n^1+{}_{14}Si^{28}\rightarrow{}_1H^1+{}_ZX^A$

On applying the principles of conservation of mass and charge, we have

$$1+28=1+A \text{ or } A=28$$

and $$0+14=1+Z \text{ or } Z=13$$

$\therefore$ $$_ZX^A={}_{13}X^{28}={}_{13}Al^{28}$$

Example 11. *200 MeV energy is released in the fission of a single nucleus of $_{92}U^{235}$. How many fissions must occur per minute to produce a power of 2 KW.*

Solution. $t=1$ mt.$=60$ s ; P$=2$ KW$=2000$ W

and energy due to fission of single nucleus (E_1)

$$=200 \text{ MeV}$$
$$=200\times1{\cdot}6\times10^{-13} \text{ J}$$
$$=32\times10^{-12} \text{ J}$$

Now Total energy (E)

$$=P\times t$$
$$=2000\times60=12\times10^4 \text{ J}$$

$\therefore$ No. of fissions$=\dfrac{E}{E_1}$

$$=\frac{12\times10^4}{32\times10^{-12}}$$
$$=375\times10^{13}$$

Example 12. *The Uranium $_{92}U^{238}$ decays successively into $_{90}Th^{234}$, $_{91}Pa^{234}$, and $_{92}U^{234}$ $_{90}Th^{230}$. Find the radiation in each decay.*

Solution. (*i*) Let $_{92}U^{238}\rightarrow{}_{90}Th^{234}+{}_ZX^A$

For principles of conservation of mass and charge to hold good,

$$238 = 234 + A \text{ or } A = 4$$

and
$$92 = 90 + Z \quad \text{or } Z = 2$$

$\therefore$ $${}_ZX^A = {}_2X^4 = {}_2He^4 \text{ or } \alpha\text{-particle} \quad ...(i)$$

(*ii*) Let ${}_{90}Th^{234} \rightarrow {}_{91}Pa^{234} + {}_ZX^A$

For the reason stated in part (*i*),

$$A = 234 - 234 = 0$$
$$Z = 90 - 91 = -1$$

$\therefore$ $${}_ZX^A = {}_{-1}X^0 = {}_{-1}e^0 \text{ or } \beta\text{-particle} \quad ...(ii)$$

(*iii*) Let ${}_{91}Pa^{234} \rightarrow {}_{92}U^{234} + {}_ZX^A$

For the reason stated in part (*i*),

$$A = 234 - 234 = 0$$
$$Z = 91 - 92 = -1$$

$\therefore$ $${}_ZX^A = {}_{-1}X^0 = {}_{-1}e^0 \text{ or } \beta\text{-particle} \quad ...(iii)$$

(*iv*) Let ${}_{92}U^{234} \rightarrow {}_{90}Th^{230} + {}_ZX^A$

For the reason stated in part (*i*)

$$A = 234 - 230 = 4$$
$$Z = 92 - 90 = 2$$

$\therefore$ $${}_ZX^A = {}_2X^4 = {}_2He^4 \text{ or } \alpha\text{-particle} \quad ...(iv)$$

Example 13. *When four hydrogen atoms combine, they from a helium atom along with two positrons, each of mass 0·000549 amu. Calculate the energy released.* (*Mass of ${}_2He^4$ atom = 4·002604 amu*).

Solution. Nuclear reaction can be written as

$$4({}_1H^1) \rightarrow {}_2He^4 + 2({}_{+1}e^0)$$

Now mass of 4 hydrogen atoms

$$= 4 \times 1{\cdot}007825 = 4{\cdot}031300 \text{ amu} \quad ...(i)$$

Mass of one helium atom $= 4{\cdot}002604$ amu

Mass of two positrons

$$= 2 \times 0{\cdot}000549 = 0{\cdot}001098 \text{ amu}$$

Total final mass $= 4{\cdot}003702$ amu ...(*ii*)

$\therefore$ Loss in mass $= (4{\cdot}031300 - 4{\cdot}003702)$

$= 0{\cdot}027598$ amu

$\therefore$ Energy released $= 931 \times 0{\cdot}027598$

$= 25{\cdot}694$ MeV.

Example 14. *The uniform magnetic field applied to a cyclotron to accelerate the deutron is 3 Wb m^{-2}. Calculate the frequency of the oscillating potential that must be applied to the dees of the cyclotron. Mass of deutron = $3{\cdot}3 \times 10^{-27}$ kg and its charge = $1{\cdot}6 \times 10^{-19}$ C*).

Solution. $B = 3$ Wb m^{-2} ; $m = 3{\cdot}3 \times 10^{-27}$ Kg ; $q = 1{\cdot}6 \times 10^{-19}$ C

Period of revolution of charged particle in cyclotron,

$$T=2\pi\frac{m}{qB}$$

$\therefore$ Frequency of oscillating potential,

$$\nu=\frac{1}{T}$$

$$=\frac{qB}{2\pi m}$$

$$=\frac{1.6\times10^{-19}\times3}{2\times3.14\times3.3\times10^{-27}}=2.316\times10^{7}\text{ Hz}$$

$$=23.16\text{ MHz}$$

Example 15. *How much energy must a gamma ray photon have if it is to materialize into a proton-antiproton pair with each particle having a kinetic energy of 5.0 MeV and rest mass 1.007276 amu.*

Solution.

Mass of a proton $=1.007276$ amu

Mass of antiproton $=1.007276$ amu

Total Mass $=2.014552$ amu

Equivalent energy $=931\times2.014552$

$=1875.548$ MeV ...(*i*)

K.E. of both the particle

$=2\times5.0=10.000$ MeV ...(*ii*)

Total energy $=1885.548$ MeV

This is the required energy of gamma ray photon.

Example 16. *Calculate the power generated in fission of 1 g of uranium $_{92}U^{235}$ per day. The nuclear reaction takes place according to following equation :*

$$_0n^1+{}_{92}U^{235}\rightarrow{}_{56}Ba^{141}+{}_{36}kr^{92}+3\,({}_0n^1)$$

(Given mass of $_{92}U^{235}=235.045733$ amu

mass of $_{56}Ba^{141}=140.917700$ amu

mass of $_{36}Kr^{92}=91.885400$ amu

1 amu $=1.66\times10^{-27}$ Kg).

Solution. Total mass of one neutron and $_{92}U^{235}$

$=1.008665+235.045733$

$=236.054398$ amu

Total mass of $_{56}Ba^{141}$, $_{36}Kr^{92}$ and 3 neutrons

$=140.917700+91.885400+3\times1.008665$

$=235.829095$ amu

$\therefore$ Loss in mass $=236.054398-235.829095$

$=0.225303$ amu

$\therefore$ The energy released in a fission of one ${}_{92}U^{235}$ nucleus

$$=931\times0{\cdot}225303=209{\cdot}757093 \text{ MeV}$$
$$=209{\cdot}757093\times1{\cdot}6\times10^{-13} \text{ J}$$
$$=335{\cdot}611346\times10^{-13} \text{ J}$$

Now mass of one ${}_{92}U^{235}$ nucleus

$$=235{\cdot}045733 \text{ amu}$$
$$=235{\cdot}045733\times1{\cdot}66\times10^{-24} \text{ g.}$$
$$=390{\cdot}176\times10^{-24} \text{ g}$$

$\therefore$ Energy released due to fission of 1 g. of ${}_{92}U^{235}$,

$$E=\frac{335{\cdot}611349\times10^{-13}}{390{\cdot}176\times10^{-24}} \text{ J}$$

or $$E=86{\cdot}015\times10^{9} \text{ J}$$

Now $$t=1 \text{ day}=24\times60\times60 \text{ s}$$

$\therefore$ $$P=\frac{E}{t}.$$

$$=\frac{86{\cdot}015\times10^{9}}{86400} \text{ W}$$
$$=0{\cdot}995\times10^{6} \text{ W}$$
$$=995 \text{ KW.}$$

Example 17. *An electron-position pair is produced by a gamma ray of 2·02 meV. Calculate the amount of kinetic energy that is imparted to each of the charged particle (Mass of electron=0·000549 amu).*

Solution. Total mass of electron-positron pair

$$=0{\cdot}000549+0{\cdot}000549$$
$$=0{\cdot}001098 \text{ amu}$$

Equivalent energy$=931\times0{\cdot}001098=1{\cdot}02$ MeV.

$\therefore$ Total kinetic energy of electron and positron,

$$E=2{\cdot}02-1{\cdot}02=1{\cdot}00 \text{ MeV.}$$

$\therefore$ K.E. imparted to each,

$$\text{K.E.}=\frac{E}{2}$$
$$=\frac{1{\cdot}00}{2}=0{\cdot}50 \text{ MeV.}$$

Example 18. *An electron with kinetic energy 2 MeV collides head on with a position of the same kinetic energy. If they are annihilated to give gamma rays, calculate the total energy of the gamma rays. (Mass of an electron=0 000549 amu).*

Solution. Total K.E. of electron and positron,

$$=2+2=4 \text{ MeV} \qquad ...(i)$$

Total mass of electron and positron,

$$=0{\cdot}000549+0{\cdot}000549$$
$$=0{\cdot}001098 \text{ amu}$$

$\therefore$ Equivalent energy $=931\times0{\cdot}001098=1{\cdot}02$ MeV ...(*ii*)

$\therefore$ Total energy of resulting gamma rays $=5{\cdot}02$ MeV

Example 19. *The half life of a radioactive substance is one hour. Calculate how long will it take for 60% of the substance to decay.* [*A.I.S.S.E. 1981*]

Solution. $m_f=100-60=40$; $m_i=100$, T=1 hr.

Now $$\left(\frac{m_f}{m_i}\right)=\left(\frac{1}{2}\right)^{t/T}$$

$$\frac{40}{100}=\left(\frac{1}{2}\right)^{t/1}$$

$$\therefore \quad \log\left(\frac{2}{5}\right)=t\left(\log\frac{1}{2}\right)$$

or $$t=\frac{\log 2-\log 5}{-\log 2}$$

$$=\frac{0{\cdot}3010-0{\cdot}6990}{-{\cdot}3010}$$

$$=\frac{398}{301}\text{ hr.}\doteq 1\text{ hr. } 19\text{ mt. } 21\text{ sec.}$$

Example 20. *4 g of a radiactive substance disintegrates at the rate of $1{\cdot}38\times10^{20}$ disintegrations per second. The atomic mass of the substance is 220. Calculate its (a) disintegration constant (b) half life and (c) mean life.*

Solution.

(*a*) Number of atoms disintegrated in 1s,

$$=1{\cdot}38\times10^{20}$$

$\therefore$ The mass of the substance disintegrated in 1s

i.e. $$-\frac{dN}{dt}=\frac{1{\cdot}38\times10^{20}\times220}{6{\cdot}023\times10^{23}}$$

$$\therefore \quad \frac{dN}{dt}=-5{\cdot}04\times10^{-2}\text{ g}$$

and $$N=4\text{ g}$$

Now $$N=N_0\, e^{-\lambda t}$$

$$\therefore \quad \frac{dN}{dt}=N_0\, e^{-\lambda t}(-\lambda)$$

or $$\frac{dN}{dt}=-\lambda N$$

$$\therefore \quad \lambda = \frac{dN/dt}{-N}$$

$$= \frac{-5.04 \times 10^{-2}}{-4} = 1.26 \times 10^{-2}\ s^{-1}$$

∴ Disintegration const

$$= 1.26 \times 10^{-2}\ s^{-1}$$

∴ (*b*) Half life time

$$= \frac{0.693}{\lambda}$$

$$= \frac{0.693}{1.26 \times 10^{-2}} = 55\ s$$

(*c*) Mean life $= \frac{1}{\lambda}$

$$= \frac{1}{1.26 \times 10^{-2}} = 79.6\ s$$

EXERCISE 13

[Where necessary the following data may be used ; Mass of ${}_0n^1 = 1.008665$ amu ; mass of ${}_1H^1 = 1.007825$ amu : mass of ${}_2He^4 = 4.002604$ amu ; 1 amu=931 MeV.].

1. Calculate number of electrons, protons and neutrons in each of the following atoms (*a*) ${}_1H^3$ (*b*) ${}_2He^4$ (*c*) ${}_{20}Ca^{44}$ (*d*) ${}_{47}Ag^{107}$ (*e*) ${}_{90}Th^{232}$.
2. What is the number of protons and number of neutrons in each of the following nuclei : (*a*) ${}_6C^{12}$ (*b*) ${}_8O^{16}$ (*c*) ${}_{16}S^{34}$ (*d*) ${}_{27}Co^{59}$ (*e*) ${}_{83}Bi^{209}$ (*f*) ${}_6C^{13}$ (*g*) ${}_{92}U^{239}$. [*A.I.S.S.E. 1980*]
3. The mass of ${}_2He^3$ is 3.016030 amu. Calculate its binding energy.
4. Calculate the nuclear radius of (*a*) ${}_{47}Ag^{107}$, (*b*) ${}_{20}Ca^{44}$, (*c*) ${}_6C^{12}$.
5. The nuclear radius of ${}_8O^{16}$ is 3×10^{-15} m. Calculate the radius of ${}_{82}Pb^{205}$ nucleus.
6. The nuclear radius of ${}_8O^{16}$ is 3×10^{-15} m Calculate the density of nuclear matter. (1 amu$=1.66 \times 10^{-27}$ Kg).
7. The mass of ${}_{16}S^{34}$ is 33.967865 amu. Calculate its binding energy and binding energy per nucleon.
8. Calculate the binding energy and binding energy per nucleon for the following nuclei :
   (*a*) ${}_{20}Ca^{42}$ of atomic mass 41.9586 amu.
   (*b*) ${}_{48}Cd^{108}$ of atomic mass 107.904187 amu.
   (*c*) ${}_5B^{11}$ of atomic mass 11.009305 amu.
9. Calculate the mass of the following nuclei :
   (*a*) ${}_3Li^{17}$ having binding energy 37.69 MeV.
   (*b*) ${}_{92}U^{238}$ having binding energy per nucleon 7.368 MeV.

10. Calculate the energy released in the reaction

$$_1H^2+_1H^2 \rightarrow _1H^1+_1H^3$$

(Mass of $_1H^2$=2·014102 amu and that of $_1H^3$=3·016050 amu)

11. Calculate the kinetic energy of alpha particles in the following nuclear reaction, supposing the K.E. of other nuclei zero.

$$_2He^4+_7N^{14} \rightarrow _8O^{17}+_1H^1+6·508 \text{ MeV}.$$

Mass of $_7N^{14}$=14·003074 amu and that of $_8O^{17}$=15·999133 amu).

12. Calculate the mass of $_6C^{14}$ in the following nuclear reaction,

$$_0n^1+_7N^{14} \rightarrow _6C^{14}+_1H^1+0·55 \text{ MeV}.$$

[Mass of $_7N^{14}$=14·003074 amu]

13. Complete the following nuclear reactions :

(*a*) $_5B^{11}+_1H^1 \rightarrow _6C^{11}+......$

(*b*) $_3Li^7+...... \rightarrow _3Li^8+_1H^1$

(*c*) $_{17}Cl^{35}+_1H^1 \rightarrow _{16}S^{32}+......$

(*d*) $_7N^{13} \rightarrow _6C^{13}+......$

(*e*) α-particle$+_4Be^9 \rightarrow _0n^1+......$

14. An α-paticle strikes a $_{13}Al^{27}$ nucleus with the subsequent emission of a neutron. What is the atomic number, mass number and chemical name of the product ?

15. A neutron strikes a $_{11}Na^{23}$ nucleus with a subsequent emission of an electron What is the atomic number, mass number and chemical name of the product ?

16. The fission of single $_{92}U^{235}$ nucleus releases 200 MeV energy. How many fissions occur per second to produce a power of 10 KW.

17. Calculated the power generated in fission of 1 mg of uranium per minute. The nuclear reaction takes place as follows :

$$_0n^1+_{92}U^{235} \rightarrow _{56}Ba^{141}+_{36}Kr^{92}+3\ _0n^1$$

[Mass of $_{92}U^{235}$=235·045733 amu, mass of $_{56}Ba^{141}$=140·917700 amu, mass of $_{36}Kr^{92}$=91·985400 amu and 1 amu=$1·66\times10^{-27}$ Kg.]

18. A nuclear reactor using $_{92}U^{235}$ as fuel has an output of 235 MW How much uranium is consumed per day ? The energy released in a fission of single $_{92}U^{235}$ is 200 MeV.

19. The isotope $_{94}Pu^{241}$ decays successively to form $_{95}Am^{241}$, $_{93}Np^{237}$, $_{92}Pa^{233}$, $_{92}U^{233}$ and $_{90}Th^{229}$. What are the radiations in each decay ?

20. The uniform magnetic field applied to a cyclotron to accelerate deutron is 3·675 Wbm^{-2}. How rapidly the electric field between the plates be reversed ? Hence calculate the frequency of oscillating potential that must be applied.

(Mass of the deutron=$3·3\times10^{-27}$ Kg. and its charge=$1·6\times10^{-19}$ C).

21. How much of energy will be released when an electron-positron pair annihilates ? Calculate the frequency of the corresponding photon so emitted. (Mass of $_1e^0$=0·000549 amu and $h=6{\cdot}63\times10^{-34}$ Js).

22. A gamma ray of energy 2100 MeV materialises into a proton-antiproton pair. Calculate the energy released in the reaction and the energy shared by each of the particles.

23. An electron-positron pair is produced by a gamma ray of 6 00 MeV. How much of kinetic energy is imparted to each of the particle ?

24. Deuterium was bombarded with the gamma rays with the subsequent emission of a proton and a neutron. Calculate the energy and frequency of the γ-rays photons. [Mass of $_1H^2$= 2·014103 amu and $h=6{\cdot}63\times10^{-34}$ Js]

25. The Isotope uranium $_{92}U^{238}$ decays successively to form $_{90}Th^{234}$, $_{91}Pa^{234}$, $_{92}U^{234}$, $_{90}Th^{230}$, and $_{88}Ra^{226}$. Name the radiations emitted in these five steps. [*A.I.S.S.E. 1979* ; *D.S.S.E. 1981*]

26. The wavelength of a spectral line is 4000 A°. Calculate its frequency and energy. (Given $c=3\times10^8$ ms^{-1} and $h=6{\cdot}6\times10^{-34}$ Js). [*A.I.S.S.E. 1979*]

27. The atomic mass of $_8O^{16}$ is 16·000000 amu. Calculate the binding energy of $_8O^{16}$ in MeV per nucleon (Mass of proton =1·007825 amu. and mass of neutron =1·008665 amu). [*A.I.S.S.E. 1982*]

28. Calculate the energy released in the reaction

$$_3Li^6+{_0n^1}\rightarrow{_2He^4}+{_1H^3}$$

The respective masses are :

$_3Li^6\rightarrow$6·015226 amu

$_1H^3\rightarrow$3·016049 amu

$_2He^4\rightarrow$4·002604 amu

$_0n^1\rightarrow$1·008665 amu

1 amu=231 MeV [*D.S.S.E. 1985*]

29. If 1 kg of a substance is fully converted into energy, how much energy is relased. [*A.I.S.S.E. (Compt.) E. 1985*]

30. The half life of radium is 1600 years. What would be the fraction of a sample of radium that remains after 6400 years. [*C.P.M.T. 1980*]

31. An archaeologist analyses the wood in a prehistoric structure and finds that the ratio of C^{14} (half life=5700 years) to ordinary carbon is only one-fourth of that found in the cells of living plants. What is the age of the wood ? [*N.C.E.R.T. 1982*]

32. Half-life of polonium is 140 days. In how many days 15 gm out of 16 gm of this element will dacay ?

33. As a result of radio active decay a $_{92}U^{238}$ nucleus is changed to $_{91}Pa^{234}$. What particles are decayed during the decay ? [*N.C.E.R.T. 1982*]

34. What is the maximum distance at which two protons may attract each other ? [*A.I.I.M.S. 1982*]

35. How many electrons, protons and neutrons are there in a atom of atomic number 11 and mass number 24. [*I.I.T. 1983*]

36. A uranium nucleus (atomic number 92, mass number 238) emits an α-particle and then β-particle. What are the atomic and mass number of the final nucleus ? [*I.I.T. 1983*]

37. 1 gram of material is reduced by 2·1 mg in 4 years. Calculate the half life time of material.

38. The half life of the radioactive Radon is 3·8 days. What will be the time at the end of which $\frac{1}{20}$th of the sample will remain undecayed ? [*I.I.T. 1981*]

39. Find the longest wavelength that a singly ionised helium atom in its ground state will absorb.
(Rydberg constant$=1{\cdot}097\times10^{-7}$ m^{-1})

40. 200 MeV energy is released in the fission of one nucleus of $_{92}H^{235}$. Calculate the number of fissions per second that should occur for producing a power of 1 MW.
(1eV$=1{\cdot}6\times10^{-19}$ J) [*D.S.S.E. (Compt.) 1985*]

41. The fission of one nucleus of $_{92}U^{235}$ releases 250 MeV energy. How many fission should occur per second for producing a power 1 MW. (1eV$=1{\cdot}6\times10^{-19}$ J) ? [*A.I S.S E. 1986*]

42. Calculate the (*a*) mass defect (*b*) binding energy and (*c*) binding energy per nucleon for $_6C^{12}$ nucleus of atomic mass 12·000000 a.m.u. [*D S.S.E. 1986*]

43. The half life of a radioactive substance is 4 hours. In how much time 7/8 of the material would decay ? [*A.I.S S.E. 1987*]

44. A proton and antiproton annihilate into two photons of the same frequency. What is the wavelength of the photons produced ? (Mass of proton$=1{\cdot}67\times10^{-27}$ kg) [*D.S.S.E. 1987*]

45. Calculate the total binding energy in MeV of $_3Li^7$ nucleus of atomic weight of 7·01601 a.m.u. [*D.S.S.E. (Compt.) 1987*]

46. Calculate the binding energy for a $_1H^2$ nucleus of atomic mass 2·014103 a.m.u.

47. The normal activity of living carbon containing matter is found to be about 15 decays per minute for every gram of carbon C^{14}. A specimen from Mohanjodaro gives an activity of 9 decays perminute per gram of carbon. Estimate the approximate age of the Indus Valley civilisation. (Half life time of C^{14} is 5730 years).

48. A radioactive substance has a half life time of 30 days. Calculate (*a*) the disintegration constant (*b*) the time taken for 7/8 of the original number of atoms to disintegrate.

49. 1 g of a radioactive substance distintegrates at the rate of 3.7×10^{10} atoms per second. The atomic mass of the substance is 226. Calculate (*a*) distintegration constant (*b*) mean life time (0) half life time.

50. A certain radioactive substance has a distintegration constant $\lambda = 1.44 \times 10^{-3}$ per hour. In what time will 75% of the initial number of atoms disintegrates ?

OBJECTIVE TYPE QUESTIONS

51. From the following equations pick out the possible nuclear fusion reactions :

(*a*) ${}_6C^{13} + {}_1H^1 \rightarrow {}_6C^{14} + {}_1e^0 + 4.3$ MeV

(*b*) ${}_6C^{12} + {}_1H^1 \rightarrow {}_7N^{13} + 2$ MeV

(*c*) ${}_7N^{14} + {}_1H^1 \rightarrow {}_8O^{15} + 7.3$ MeV

(*d*) ${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + {}_0n^1 + {}_0n^1 + r + 200$ MeV.

[*I.I.T., J.E.E. 1984*]

52. The mass number of a nucleus is

(*a*) always less than its atomic number

(*b*) always more than its atomic number

(*c*) sometimes equal to its atomic number

(*d*) sometimes more than and sometimes equal to its atomic number. [*I.I.T., J.E.E. 1986*]

53. In the Uranium radioactive series the initial nucleus is ${}_{92}U^{238}$ and the final nucleus is ${}_{80}Pb^{206}$. When the Uranium decays to lead, the number of α-particles emitted is...............and the number of β-particles emitted is.................

[*I.I.T., J E.E. 1986*]

54. When Boron nucleus (${}_5B^{10}$) is bombarded by neutrons, α-particles are emitted. The resulting nucleus is of the element............and has the mass number...............

[*I.I.T., J.E.E. 1986*]

55. Atoms having the same...............but different............are called isotopes. [*I.I.T., J.E.E. 1986*]

56. During a negative beta decay

(*a*) an atomic electron is ejected

(*b*) an electron which is already present within the nucleus is ejected

(*c*) a neutron in the nucleus decays emitting an electron.

(*d*) a part of the binding energy of the nucleus is converted into an electron. [*I.I.T., J.E.E. 1987*]

57. During a nuclear fusion reaction :
(a) a heavy nucleus breaks into two fragments by itself.
(b) a light nucleus bombarded by thermal neutrons breaks up.
(c) a heavy nucleus bombarded by thermal neutrons breaks up.
(d) two light nuclei combine to give a heavier nucleus and possibly other products. [*I.I.T., J.E.E. 1986*]

58. If in above Question No. 57. instead of nuclear fusion reaction, there is a nuclear fission reaction, which of the answers from *a* to *d* is correct ?

59. A freshly prepared radioactive source of half life 2 hr. emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with source is
(a) 6 hr. (b) 12 hr.
(c) 24 hr. (d) 128 hr. [*I.I.T., J.E.E. 1988*]

60. The binding energies per nucleon for deutron ($_1H^2$) and helium ($_2He^4$) are 1·1 MeV and 7·0 MeV respectively. The total energy released when two deutrons fuse to form a helium nucleus ($_2He^4$) is............ [*I.I.T., J.E.E. 1988*]

61. In the following two reactions

$$_5B^{11} + _1H^1 \rightarrow _4Be^8 + _2He^4$$
$$_{13}Al^{27} + _2He^4 \rightarrow _{15}P^{30} + _1H^2$$

which is incorrect and which law governing nuclear reaction is violated. [*A.I.S.S.E., (Compt.) 1986*]

62. Complete the following nuclear reaction

$$_2He^4 + _4Be^9 \rightarrow _6C^{12} + ...$$ [*A.I.S.S.E., 1989*]

63. The binding energy of a nucleus is equivalent to the
(a) mass of the nucleus
(b) mass of the neutron
(c) mass of a proton
(d) mass of deficit of the nucleus [*D.P.M.T., 1985*]

64. The existance of fixed energy levels within the nucleus is shown by the emission of
(a) gamma rays (b) neutrons
(c) alpha particles (d) positrons [*D.P.M.T., 1985*]

65. In fission-fussion-fission bomb, Uranium 238 nuclei are split by
(a) high temperature (b) alpha particles
(c) fast neutrons (d) Uranium 235 nuclei [*D.P.M.T., 1985*]

66. Neutrino is
(a) chargeless and has no spin

(*b*) chargeless and has spin
(*c*) charged like electron and has spin
(*d*) uncharged but has mass nearly that of proton.
[*D.P.M.T., 1986, 88*]

67. Control rods used in a nuclear reactor are made of
(*a*) cadmium (*b*) steel
(*c*) copper (*d*) beryllium [*D.P.M.T., 1986*]

68. When a proton is moved from rest (or accelerated) by a potential difference of 1 volt. The kinetic energy associated with it is roughly equal to
(*a*) 1 eV (*b*) 1840 eV
(*c*) $\frac{1}{1840}$ eV (*d*) 1840 mc^2 eV
[*D.P M.T., 1986*]

69. After one hour, $\frac{1}{8}$ of the initial mass of a certain radioactive isotope remain undecayed. The half life of the isotope is
(*a*) 10 minute (*b*) 20 minute
(*c*) 30 minute (*d*) 45 minute. [*D.P.M.T., 1986*]

70. Which of the following elementary particles is unstable
(*a*) neutron (*b*) electron
(*c*) positron (*d*) proton [*D.P.M.T., 1987*]

71. A charged particle is moving in a uniform magnetic field in a circular path. The energy of the particle is doubled. If the initial radius of the circular path was R, the radius of the new circular bath after the energy is doubled will be
(*a*) $\frac{R}{2}$ (*b*) $\sqrt{2}$ R
(*c*) 2R (*d*) $\frac{R}{\sqrt{2}}$ [*D.P.M.T., 1987*]

72. A proton and an α-particle are accelerated through the same voltage. The ratio of their de Broglie wavelength will be
(*a*) 1 : 2 (*b*) $2\sqrt{2} : 1$
(*c*) $\sqrt{2} : 1$ (*d*) 2 : 1 [*D.P.M.T., 1987*]

73. The half life of a substance is $7\frac{1}{2}$ minutes. The fraction of substance left after one hour will be
(*a*) $\frac{1}{256}$ (*b*) $\frac{1}{16}$
(*c*) $\frac{1}{64}$ (*d*) $\frac{1}{4}$ [*D.P.M.T., 1984*]

74. Which of the following will penetrate minimum and ionize maximum

(a) α-rays (b) β-rays
(c) γ-rays (d) X-rays [*D.P.M.T., 1987*]

75. The ratio of the mass of an α-particle to the mass a β-particle is nearly

(a) 1840 (b) 7360
(c) 460 (d) 1920 [*D.P.M.T., 1987*]

76. The energy is an atom bomb is produced by the process of :

(a) nuclear fusion.
(b) nuclear fission.
(c) combination of hydrogen atom.
(d) combination of electrons and protons. [*D.P.M.T., 1987*]

77. The energy of hydrogen bomb is produced by the process of :

(a) uncontrolled fusion (b) controlled fission
(c) uncontrolled fission (d) controlled fusion [*D.P.M.T., 1989*]

78. The moderator used in a reactor is

(a) platinum (b) graphite
(c) thorium (d) cadmium [*C.P.M.T., 1988*]

79. An arrangement where the process of nuclear fission and release of energy goes in a controlled fashion is called

(a) thermopile (b) reactor
(c) thermostat (d) cloud chamber [*D.P.M.T., 1988*]

80. The half life of the radioactive Radon is 3·8 days. The time at the end of which $\frac{1}{20}$th of the Radon sample will remain undecayed is

(a) 3·8 days (b) 16·5 days
(c) 33 days (d) 76 days [*D.P.M.T., 1989*]

81. In the nuclear reaction

$$_5B^{10} + {}_2He^4 \rightarrow {}_7N^{13} + ?$$

82. β-rays emitted by a radioactive material are—

(a) electromagnetic radiations
(b) the electrons orbiting around the nucleus
(c) changed particles emitted by the atom
(d) neutral particles.

83. If elements with principal quantum number $n > 4$ were not allowed in nature, the number of possible elements would be :

(*a*) 60 (*b*) 32

(*c*) 4 (*d*) 64.

84. The scientist who discovered neutrons was

(*a*) Chadwich (*b*) Bohr

(*c*) Rutherford (*d*) Millikan [*D.P.M.T., 1989*]

85. Which is more energetic :

(*a*) Alpha rays (*b*) Beta rays

(*c*) Gamma rays (*d*) X-rays [*D.P.M.T., 1989*]

86. If the radiation from a radioactive material is passed through an electric field :

(*a*) only the gamma rays are deflected

(*b*) only the α-rays are deflected

(*c*) the alpha and beta rays are deflected

(*d*) all three kinds of rays will be deflected. [*D.P.M.T., 1985*]

87. A device that counts individual particles but cannot measure their energy is

(*a*) electroscope (*b*) cloud chamber

(*c*) Geiger counter (*d*) ionization chamber.

[*D.P.M.T., 1985*]

88. The ultimate source of solar energy is

(*a*) combustion (*b*) fusion

(*c*) thermionic emission (*d*) emission of electrons.

[*D.P.M.T., 1989*]

89. The one which does not effect the nature of the nucleus after its emission from the nucleus is

(*a*) neutron (*b*) electron

(*c*) positron (*d*) gamma rays.

[*D.P.M.T., 1989*]

□□

UNIT 14

Solids

IMPORTANT FORMULAE

1. Volume of unit cubic cell,

$$V=a^3$$

2. No. of atoms per unit cell,

$$N=\frac{N_c}{8}+\frac{N_f}{2}+N_i$$

3. Density of a substance,

$$\rho=\frac{nA}{NV}$$

4. Characteristics of a Unit Cubic Cell :

Characteristics	*sc.*	*bcc.*	*fcc.*
1. Unit cell volume (V)	a^3	a^3	a^3
2. Atoms per unit cell (N)	1	2	4
3. Coordination number (CN)	6	8	12
4. Atomic radius (r)	$\sqrt{4}.a/4$	$\sqrt{3}.a/4$	$\sqrt{2}.a/4$
5. Packing factor (PF)	$\pi/6$	$\sqrt{2}.\pi/8$	$\sqrt{2}.\pi/6$

5. For *hcp* structure,

$$\frac{c}{a}=1{\cdot}633$$

SOLVED EXAMPLES

Example 1. *How many atoms per unit cell are three in sc, bcc and fcc structure.*

Solution.

$$N=\frac{N_c}{8}+\frac{N_f}{2}+N_i$$

For sc :

$$N_e=8, N_f=4, N_i$$

$$\therefore \quad N=\frac{8}{8}+0+0=1$$

For bcc,

$$N_e=8, N_f=0, N_i=1$$

$$\therefore \quad N=\frac{8}{8}+0+1=2$$

For fcc,

$$N_e=8, N_f=6, N_i=0$$

$$\therefore \quad N=\frac{8}{8}+\frac{6}{2}+0=4.$$

Example 2. *Silver has fcc structure with lattice parameter 4·08 Å Calculate the atomic radius of silver.*

Solution. For fcc structure,

$$r=\frac{\sqrt{2}}{4}\,a$$

$$=\frac{\sqrt{2}}{4}\times 4{\cdot}08=1{\cdot}442\ \text{Å}.$$

Example 3. *Copper has fcc structure. Its interatomic spacing is 2·54 Å. Calculate (a) the atomic radius and (b) lattice constant for Cu.*

Solution. Interatomic spacing,

$$2r=2{\cdot}54\ \text{Å}$$

$$\therefore \quad r=\frac{2{\cdot}54}{2}=1{\cdot}27\ \text{Å}$$

$$\text{Now} \quad r=\frac{\sqrt{2}}{4}\,a$$

$$\therefore \quad a=\frac{4}{\sqrt{2}}\,r$$

$$=\frac{4\times 1{\cdot}27}{1{\cdot}414}$$

$$=3{\cdot}59\ \text{Å}.$$

Example 4. *The unit cell of aluminium is a cube having lattice constant 4·05 Å. Calculate the number of unit cells in an aluminium sheet of size 20 cm × 10 cm × 0·04 cm.*

Solution. Volume of one unit cell,

$$V=a^3$$

$$=(4{\cdot}05\times 10^{-10})^3=66{\cdot}43\times 10^{-30}\ \text{m}^3$$

Volume of aluminium sheet,

$$V_1 = 20 \times 10 \times 0.04 \text{ cm}^3$$
$$= 8 \text{ cm}^3 = 8 \times 10^{-6} \text{ m}^3$$

$$\therefore \text{ No. of unit cells} = \frac{V_1}{V}$$
$$= \frac{8 \times 10^{-6}}{66.43 \times 10^{-30}}$$
$$= 12.04 \times 10^{22}.$$

Example 5. *Copper has fcc structure with atomic radius as 1·278 Å. Calculate its density. (Atomic wt. of Cu=63·5).*

Solution. For fcc structure,

$$r = \frac{\sqrt{2}}{4} a$$

$$\therefore \quad a = \frac{4}{\sqrt{2}} r$$

$$= \frac{4}{\sqrt{2}} \times 1.278 = 3.615 \text{ Å} = 3.615 \times 10^{-10} \text{ m}$$

Now for fcc structure,

$$n = 4$$
$$N = 6.023 \times 10^{26} \text{ Kg mole}^{-1}.$$
$$A = 63.5$$

$$\therefore \quad \rho = \frac{nA}{Na^3}$$

$$= \frac{4 \times 63.5}{6.023 \times 10^{26} \times (3.615 \times 10^{-10})^3}$$

$$= 8.926 \times 10^3 \text{ kg m}^{-3}.$$

Example 6. *Aluminium has a fcc structure having density as 2700 Kg m^{-3} and atomic weight as 27. Calculate its radius.*

Solution. $\rho = 2700$ Kg m^{-3} ; $A = 27$; $N = 6.023 \times 10^{26}$ Kg mole^{-1} $n = 4$ for fcc structure.

$$\therefore \quad \rho = \frac{nA}{Na^3}$$

$$\therefore \quad a = \left(\frac{nA}{N\rho}\right)^{\frac{1}{3}}$$

$$= \left(\frac{4 \times 27}{6.023 \times 10^{26} \times 2700}\right)^{\frac{1}{3}}$$

$$= 4.051 \times 10^{-10} \text{ m}$$

$$= 4.051 \text{ Å}$$ (Solved by log method)

$$\therefore \qquad r=\frac{\sqrt{2}}{4}\times a=\frac{\sqrt{2}}{4}\times 4{\cdot}051\ \text{Å}$$

$$=1{\cdot}432\ \text{Å}.$$

Example 7. *The bond length of carbon in diamond is 1·545 Å Calculate the lattice constant.*

Solution. Carbon in diamond has bcc structure with bond length 1·545 Å.

$$\therefore \qquad 2r=1{\cdot}545\ \text{Å}$$

or $$r=\frac{1{\cdot}545}{2}=0{\cdot}7725\ \text{A}$$

Now $$r=\frac{\sqrt{3}}{4}\,a$$

$$\therefore \qquad a=\frac{4}{\sqrt{3}}\times 0{\cdot}7725\ \text{A}$$

$$=1{\cdot}784\ \text{A}.$$

Example 8. *The density of iron in bcc structure is 7870 Kg m^{-3} and its atomic weight is 55·8. Calculate (a) lattice parameter (b) the bond length. (Avogadro's number=$6{\cdot}02\times10^{26}$ Kg $mole^{-1}$ and atomic wt. of iron is 55·8).*

Solution. For iron in bcc structure,

$$\rho=7870\ \text{kg m}^{-3}$$

$$n=2$$

$$N=6{\cdot}02\times10^{26}\ \text{Kg mole}^{-1}$$

$$A=55{\cdot}8$$

$$\rho=\frac{nA}{Na^3}$$

$$\therefore \qquad a=\left(\frac{nA}{N\rho}\right)^{1/3}$$

$$=\left(\frac{2\times55{\cdot}8}{6{\cdot}02\times10^{26}\times7870}\right)^{1/3}$$

$$\therefore \qquad a=2{\cdot}866\times10^{-10}\ \text{m}=2{\cdot}866\ \text{A}$$

For bcc structure,

$$r=\frac{\sqrt{3}}{4}\,a$$

$$=\frac{1{\cdot}732}{4}\times2{\cdot}866=1{\cdot}241\ \text{A}$$

The bond length $=2r=2{\cdot}482$ A.

Example 9. *Calculate $\frac{c}{a}$ ratio for the hexagonal close packing of spheres.*

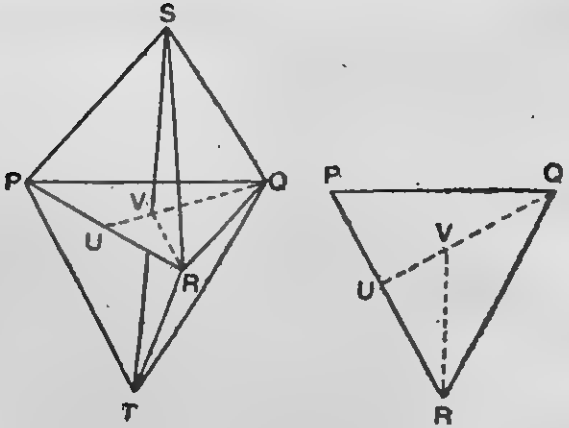


Fig. 14·1.

Solution. On joining the centres of the three adjacent atoms of the middle layer of the hexagonal close packing to the centres of the atoms at the top and the bottom layers, we get two tetrahedrals with a common base PQR in the form of equilateral triangle. Since the centres of the atoms at top and bottom layers S and T respectively are one above and other below the unit cell along c-axis, therefore

$$ST = c = 2\,SV$$

Now $\quad PQ = QR = RP = a$; also $QS = RS = PS = a$

In *rt.* angled triangle QRU,

$$QU = \sqrt{QR^2 - RU^2} = \sqrt{a^2 - (a/2)^2} = \frac{\sqrt{3}}{2}\,a$$

$$\therefore \quad QV = \frac{2}{3}\,QU = \frac{2}{3} \times \frac{\sqrt{3}}{2}\,a = \frac{a}{\sqrt{3}}$$

Now consider *rt.* angle triangle QVS,

$$SV = \sqrt{QS^2 - QV^2} = \sqrt{a^2 - \left(\frac{a}{\sqrt{3}}\right)^2}$$

or $$SV = \frac{\sqrt{2}}{\sqrt{3}}\,a$$

$$\therefore \quad \frac{c}{2} = \frac{\sqrt{2}}{\sqrt{3}}\,a$$

or $$\frac{c}{a} = \frac{2\sqrt{2}}{\sqrt{3}} = 1{\cdot}633$$

$$= \sqrt{\frac{8}{3}}$$

$$= \left(\frac{8}{3}\right)^{1/2}$$

Example 10. *Show that the fraction of total volume filled by spheres for hcp structure is 0 74.*

Solution. One *hcp* unit is formed by three such hexagonal unit cells as shown in the figure.

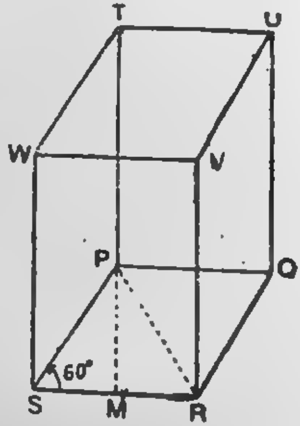


Fig. 14·2.

Now for a haxagonal unit cell,

PQ=PS=a, PT=c, and $\angle$QPS=120°

No. of atoms per unit cell,

$$n=\frac{8}{8}+1=2$$

and atomic radius $(r)=\frac{a}{2}$

$\therefore$ The volume of unit cell,

$$=\text{base area}\times\text{height}$$
$$=(SR\times PM)PT$$
$$=(a\times a\sin 60°)\ c$$
$$=\left(a\times a\frac{\sqrt{3}}{2}\right)\frac{2\sqrt{2}}{\sqrt{3}}\ a$$
$$=\sqrt{2}a^3$$

$$\therefore \text{ Density of packing}=\frac{\begin{pmatrix}\text{No. of atoms per}\\ \text{unit cell}\end{pmatrix}\times\begin{matrix}\text{Volume of an}\\ \text{atom}\end{matrix}}{\text{Volume of unit cell}}$$

$$=\frac{2\times\left[\frac{4}{3}\pi.\left(\frac{a}{2}\right)^3\right]}{\sqrt{2}a^3}$$

$$=0{\cdot}74.$$

Example 11. *Calculate the packing fraction for a simple cubic lattice of a crystal.* [*A.I.S.S. (Compt.) E. 1984*]

Solution.

$$P.F.=\frac{\begin{pmatrix}\text{No. of atoms per}\\ \text{unit cell}\end{pmatrix}\times\begin{pmatrix}\text{Volume of an}\\ \text{atom}\end{pmatrix}}{\text{Volume of unit cell}}$$

$$=\frac{1\times 4/3\pi r^3}{a^3}$$

But for *sc* structure,

$$r=\frac{a}{2}$$

$$P.F.=\frac{4}{3}\pi\frac{(a/2)^3}{a^3}$$

$$= \frac{\pi}{6}$$

$$= \frac{22}{7 \times 6} \times 100\%$$

$$= 52.38\%.$$

EXERCISE 14

1. Draw a unit cell for simple cubic structure. Find the number of atoms per unit cell.
2. How many lattice points are there in the cubic unit cell of the (*a*) body centred cubic (*bcc*) structure, (*b*) face centered cubic (*fcc*) structure.
3. Sodium has bcc structure with atomic radius of 1 857 Å. Calculate the lattice constant.
4. Silver crystallizes in fcc structure. If interatomic distance is 2·88 Å, calculate the lattice parameter.
5. Copper has fcc stucture with lattice parameter 3·6 Å. Calculate its atomic radius.
6. The iron has bcc structure with atomic radius 1·24 Å. Calculate (*a*) bond length, (*b*) lattice parameter.
7. The density of aluminium is 2770 kg m^{-3}. How many atoms are there per unit cubic centimetre. Also calculate the mass of an Al-atom. (At. wt. of Al=26·98).
8. The unit cell of copper is a cube having lattice parameter 3·6 Å. Find the number of unit cells in a copper foil of dimensions 36 cm × 18 cm × 0·002 cm.
9. Aluminium has fcc structure with lattice parameter 4·05 Å, Calculate its density (At. wt. of Al=27).
10. Gold has fcc structure Its density is 18983 Kg m^{-3}. Calculate (*a*) lattice constant, (*b*) atomic radius (At. wt. of gold= ·97).
11. I.on changes from fcc to bcc form. The atomic radii of iron atoms in the two structures are 1·270 Å and 1·240 Å respectively. Calculate the percentage change in density.
12. The unit cell of aluminium is a cube with lattice parameter 4·05 Å. How many unit cells are there in an aluminium foil of 15 cm square of thickness 0·004 cm ?
13. Show that *c*/*a* ratio of hcp structure is given by $(8/3)^{1/2}$.
14. Magnesium has hcp structure and has nearly spherical atoms with radius 1·61 Å. Calculate its density. (At. wt. of magnesium=26).
15. Calculate the density of packing for the hcp and fcc structures and show that in each case it is 74%.
16. Show that the percentage of the space occupied by the atoms to the volume available for a simple cubic structure is 52·4%.

17. How many atoms are there in each unit cell of copper. [*N.C.R.E.T. 1982*]

18. Calculate the lattice parameter of fcc aluminium if its density is 2700 kg m^{-3} and atomic weight is 26·98.

19. How many atoms per unit cell does a fcc lattic have ? [*A.I.S.S.E. 1986*]

20. Calculate coordination number for fcc lattice. [*A.I.S.S.E. 1987*]

21. Derive the relation between atomic radius and lattice parameter for bcc lattice.

22. Calculate packing factor for simple cubic lattice. [*D.S.S.E.* (*Compt.*) *1988*]

23. Show that the atomic packing fraction for fcc structure is nearly 74%. [*D.S.S.E.* (*Compt.*) *1989*]

24. Copper crystal has a fcc stucture and its atomic radius is 1·273 Å. Calculate (*a*) the lattice parameter, and (*b*) the density of copper. (Atomic weight of copper is 63·5). [*D.S.S.E. 1989*]

OBJECTIVE TYPE QUESTIONS

25. In a *n*-type semi-conductor, silicon is doped with :
(*a*) boron (*b*) carbon
(*c*) indium (*d*) phosphorus

26. In a *p*-type semi-conductor, silicon is doped with :
(*a*) Aluminium (*b*) Arsenic
(*c*) Carbon (*d*) Phosphorus

27. Match the following :
(*a*) Ionic binding (*e*) Water
(*b*) Covalent binding (*f*) Gite
(*c*) Metallic binding (*g*) Quartz
(*d*) Vander wall binding (*h*) Mg O.

Which of the following matching is correct :
(A) *a—e, b—g, c—f, d—h*
(B) *a—h, b—g, c—f, d—e*
(C) *a—f, b—e, c—h, d—g*
(D) *a—h, b—f, c—g, d—e*

28. The forbidden energy gap in semi-conductors :
(*a*) lies just below the valence band
(*b*) lies just above the conduction band
(*c*) is the same as the valence band
(*d*) lies between the valence band and the conduction band. [*D.P.M.T. 1985*]

29. Which type of crystals are generally good optical reflectors ?
(*a*) metals (*b*) covalent crystals
(*c*) ionic crystals (*d*) all of them [*C.P.M.T. 1986*]

30. Which one of the following is the best conductor of electricity ?
(*a*) Silver (*b*) Copper
(*c*) Gold (*d*) Zinc [*D.P.M.T. 1989*]

31. What is the coordination number (number of nearest neighbours) of sodium ions in the case of sodium chloride structure
(*a*) 4 (*b*) 6
(*c*) 8 (*d*) 12

32. A semi-conductor like Se or Ge doped with impurities like Boron or indium is called a............semi-conductor. [*D.S.S.E. 1988*]

33. The probability of promotion of electrons from valence band to conduction band increase with............temperature. [*D.S.S.E. 1988*]

34. The gap between valence band and............is known as............
35. Outermost energy levels in solids arrange themselves in............
36. Energy gap in insulators is of the order of............
37. Semiconductor are perfect............at............
38. Energy gap in semiconductor is of the order of............
39. In metals the valance band and............band............
40. The strongest of atomic bond is............bond.
41. The lustrous effect is the characteristic of............solid
42. The Vander Wall bond is the............
43. Solids bond having high............and............conductivity have ....bonds.
44. The number of atoms per unit call in *sc*, *bcc* and *fcc* structure are............,............and............respectively.
45. The number of atoms per unit cell for *hcp* structure is............
46. The coordination numbers of *sc*, *bcc* and *fcc* structures are......, ...and............ respectively.
47. The atomic packing factor of *sc*, *bcc* and *fcc* unit cell is............, ...and............respectivelly.
48. The *hcp* structure and *fcc* structure have the same value of...... and............
49. The ratio of radius of the atoms to the lattice parameter for *fcc* unit cell is
(*a*) 1 : 2 (*b*) 2 : 1
(*c*) $1 : 2\sqrt{2}$ (*d*) 1 : 4

50. For *hcp* structure, $\frac{c}{a}$ ratio is :
(*a*) 8 : 3 (*b*) $\sqrt{8} : \sqrt{3}$
(*c*) 8 : 5 (*d*) $\sqrt{8} : \sqrt{5}$

□ □

Semi-conductor Devices

IMPORTANT FORMULAE

1. The number density of a pure semi-conductor (n_i) is given by

$$n_i = \sqrt{n_e \times n_h}\ ;$$

n_e = no. density of conduction band electrons in a doped semi-conductor

and n_h = no. density of the valence band holes in a doped semi-conductor.

2. For *n*-type semi-conductor, density of donor atoms (N_d) is given by

$$N_d \simeq n_e >> n_h$$

3. For a *p*-type semi-conductor, density of acceptor atoms (N_a) is given by

$$N_a \simeq n_h >> n_e$$

4. The total current I in a semi-conductor is the sum of electron and hole currents

$$I = I_e + I_h$$
$$= n_e A e v_e + n_h A e v_h$$

where e = magnitude of the electron charge

A = Area of cross-section of the semi-conductor

v_h = electron drift velocity

v = hole drift velocity

$\therefore \quad I = eA\,(n_e v_h + n_h h)$

5. Mobility (μ) is defined as drift velocity per unit electric field :

$$\mu = \frac{v}{E}$$

6. The conductivity (σ) which is reciprocal of resistivity (ρ) is given by

$$\sigma = \frac{1}{\rho} = e(n_e \mu_e + n_h \mu_h)$$

7. (*a*) The current in forward biased diode,

$$I = I_0 \left[e^{\left(\frac{eV}{K_B T}\right)} - 1 \right]$$

(*b*) The current in a reverse biased semi conductor diode,

$$I = I_0 e^{\frac{qV}{KT}}$$

I_0 = Saturation current

$\frac{KT}{q} = 26\ mV$ at room temp.

(*c*) The incremental resistance of a semi-conductor diode,

$$r_e = \frac{KT}{q} \frac{1}{I}$$

8. (*a*) For a full wave rectifier semi-conductor diode, across the load resistance,

d.c. voltage, $V_{d.c.} = 0{\cdot}637\ V_0$

a.c. voltage, $V_{a.c.} = 0{\cdot}305\ V_0$

(*b*) For a half wave rectifier,

$V_{d.c.} = 0{\cdot}3185\ V_0$

$V_{a.c.} = 0{\cdot}385\ V_0$

9. For a transistor,

(*a*) $I_E = I_B + I_C$

and $\Delta_E = \Delta I_B + \Delta I_C$

(*b*) Common base current amplification factor,

$$\alpha = \frac{\Delta I_C}{\Delta I_E}$$

(*c*) Common emitter current amplification factor,

$$\beta = \frac{\Delta I_C}{\Delta I_B}$$

(*d*) $$\frac{1}{\alpha} - \frac{1}{\beta} = 1 \Rightarrow \alpha = \frac{\beta}{(1+\beta)} \Rightarrow \beta = \frac{\alpha}{(1-\alpha)}$$

(*e*) Figure of merit (transconductance),

$$g_m = \frac{\Delta I_C}{\Delta V_{BE}} = \frac{\beta}{R_i}$$

(*f*) The voltage gain of the amplifier is

$$A = \beta \frac{R_L}{R_i};$$

R_L = Load resistance

R_i = Input resistance of the transistor

(*g*) Power gain = current gain × voltage gain

$$= \beta \times \left(\beta \frac{R_L}{R_i} \right)$$

$$= \beta^2 \frac{R_L}{R_i}$$

10. In adding binary numbers the rules are
   (*a*) addition of binary 0 with 0 gives 0
   (*b*) addition of binary 0 with 1 gives 1.
   (*c*) addition of binary 1 with 1 gives 0 with carry 1.
   e.g.

$$\begin{array}{r} 10110 \\ +11011 \\ \hline 100001 \\ \hline \end{array}$$

SOLVED EXAMPLES

Example 1. *The r_p of a triode is 15 kΩ and the gain A of an amplifier using the triode is 40. The load resistance is 100 kΩ. Calculate the μ of the triode.*

Solution. $r_p=15\ \text{k}\Omega$; $A=40$; $R_L=100\ k\Omega$

Now $$A=\frac{\mu R_L}{r_p+R_L}$$

∴ $$40=\frac{\mu\times 100}{(15+100)}$$

or $$\mu=46$$

Example 2. *For a common base transistor circuit, $I_E=1$ mA and $I_C=0.96$ mA. Calculate α and I_B.*

Solution. $$\alpha=\frac{I_C}{I_E}$$

$$=\frac{0.96}{1}=0.96$$

and $$I_B=I_E-I_C$$

$$=1-0.97=0.04\ \text{mA}.$$

Example 3. *The α of a transistor is known as 0.97. Calculate its current gain in common emitter circuit.*

Solution. $$\alpha=0.97$$

$$\beta=\frac{\alpha}{1-\alpha}$$

$$=\frac{0.97}{1.0-0.97}=32.3$$

Example 4. *A transistor has β=65. Calculate α for this transistor.*

Solution. $$\beta=\frac{\alpha}{1-\alpha}$$

$$65\ (1-\alpha)=\alpha$$

$$\alpha=\frac{65}{66}=0{\cdot}985$$

Example 5. *In a common emitter circuit if V_{ce} is changed by 0·2 V, collector current changes by 0·004 mA. Calculate the output resistance.*

Solution. $\Delta V_{ce}=0{\cdot}2$ V, $\Delta I_{ce}=0{\cdot}004\times10^{-3}$ A

$$R_{out}=\frac{\Delta V_{ce}}{\Delta I_{ce}}$$

$$=\frac{0{\cdot}2}{0{\cdot}004\times10^{-3}}=50\ \text{K}\Omega$$

Example 6. *Find the gain of an amplifier using a transistor (BF 115) in common emitter circuit when $\beta=66$ and input and output resistances of the transistor being 0·5 KΩ aud 50 KΩ.*

Solution. Voltage gain = (Current amplification factor) × (Resistance gain)

$$=\beta\ \frac{R_{out}}{R_{in}}$$

$$=66\times\frac{50}{0{\cdot}5}$$

$$=6600.$$

Example 7. *Find the gain of an amplifier using a PNP junction transistor (OC 71) in common base circuit at $V_{cb}=-3V$ such that $\alpha=0{\cdot}972$. The input and output resistance of the transistor being 720 Ω and 3·00 MΩ.*

Solution. Voltage gain = (Current amplification factor) × (Resistance gain)

$$=\alpha\times\ \frac{R_{out}}{R_{in}}$$

$$=0\ 972\times\ \frac{3{\cdot}00\times10^{6}}{720}$$

$$=4050$$

Example 8. *Find the effect on electrical conductivity and resistivity of a silicon crystal at room temperature if every millionth silicon atom is replaced by an atom of Boron. Given that for silicon,*

Electron mobility $(\mu_e)=0{\cdot}135\ m^2V^{-1}s^{-1}$

Hole mobility $(\mu_h)=0{\cdot}048\ m^2V^{-1}s^{-1}$

Intrinsic carrier concentration $(n_i)=1{\cdot}5\times10^{16}\ m^{-3}$

Intrinsic conductivity $(\sigma)=4{\cdot}4\times10^{-4}\ sm^{-1}$

Intrinsic resistivity $(\rho)=2300\ \Omega m$

Concentration of atoms $=5\times10^{28}\ m^{-3}$

Solution. Concentration of silicon atoms$=5\times10^{28}$ m^{-3}

$\therefore$ Concentration of acceptor atoms (boron) *i.e.*

$$n_h=10^{-6}\times5\times10^{28}$$
$$=5\times10^{22}\text{ m}^{-3}$$

$\therefore$ New electron concentration,

$$n_e=\frac{n_i^2}{n_h}$$
$$=\frac{(1{\cdot}5\times10^{16})^2}{5\times10^{22}}$$

or $$n_e=0{\cdot}45\times10^{10}\text{ m}^{-3}$$

$\therefore$ The electrical conductivity of silicon doped with boron,

$$\sigma=e(n_e\mu_e+n_h\mu_h)$$
$$=1{\cdot}6\times10^{-19}\,(0{\cdot}45\times10^{10}\times0{\cdot}135+5\times10^{22}\times0{\cdot}048)$$
$$=(0{\cdot}9\times10^{-10}+384)$$
$$=384\text{ sm}^{-1},$$

which is much greater than the conductivity of pure silicon $0{\cdot}00044$ sm^{-1}.

The electrical resistivity of silicon doped with boron,

$$\rho=\frac{1}{\sigma}$$
$$=\frac{1}{384}$$
$$=0{\cdot}0026\ \Omega m,$$

which is much smaller as compared to the resistivity of pure silicon 2300 Ωm.

Example 9. *A p-n junction diode when for ard biased has a drop of 0·8 V which is assumed to be independent of current. The*

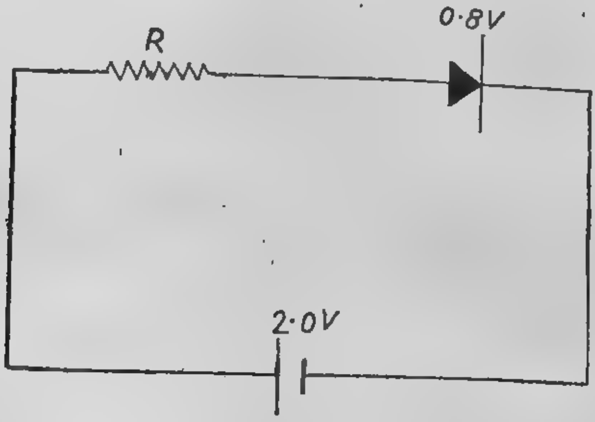


Fig. 15·1.

current is excess of 5 mA damages the diode. What should be the value of resistance used in series with diode to use it safely when it is forward biased with a battery of 2·0 volt.

Solution. Applying ohm's law,

$$V = IR$$

$$(2{\cdot}0 - 0{\cdot}8) = (5 \times 10^{-3})R$$

$$R = \frac{1{\cdot}2}{5} \times 10^{3}$$

$$= 240\ \Omega$$

Example 10. *An amplifier is represented by the following circuit, calculate its voltage gain. The voltage gain of the amplifier without load is 80.*

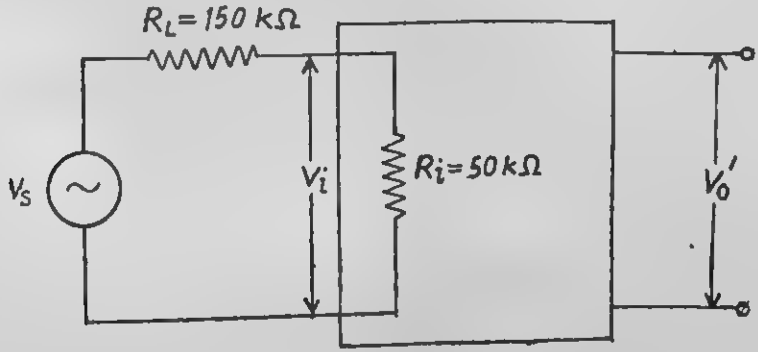


Fig. 15·2.

Solution. Without load R_L in the circuit,

$$A_0 = \frac{V_0}{V_s} = 80$$

With load R_L in the circuit,

$$A = \frac{V_0'}{V_s} \quad \text{...(1)}$$

Now $$V_i = \frac{V_s}{(R_i + R_L)} R_i \quad \text{[Ohm's law, } V = IR]$$

$\therefore$ $$V_0' = A_0 V_i$$

$$= A_0 \frac{V_s}{(R_i + R_L)} R_i$$

or $$\frac{V_0'}{V_s} = A_0 \frac{R_i}{(R_i + R_L)}$$

$\therefore$ From eq^n (1),

$$A = A_0 \frac{R_i}{(R_i + R_L)}$$

$$=80 \frac{50}{(50+150)}$$
$$=20$$

Example 11. *Assume that the silicon diode in the following circuit requires a minimum current of 2mA to be above the knee point (0 8 V) of its I-V characteristics. Also assume that the voltage across the diode is independent of current above the knee point.*

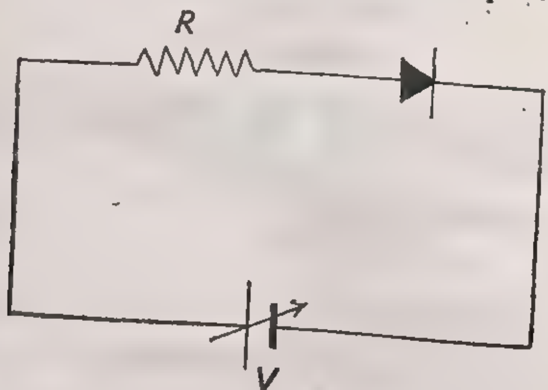


Fig. 15·3.

(a) *If V_B=6V, what should be the maximum value of R so that the voltage is above the knee point.*

(b) *If V_B=6V, what should be the value of R to establish the current of 5mA in the circuit.*

(c) *What is the power dissipated in the resistance R and in the diode, when the current of 5 mA flows in the circuit at V_B =8V.*

(d) *If R=2K Ω, what is the minimum voltage V_B reguired to keep the diode above the knee point ?*

Solution. (a) Applying Ohm's law,

$$(6-0{\cdot}8)=2\times10^{-3}\,R$$
$$R=\frac{5{\cdot}2}{2}\times10^{3}$$
$$R=2{\cdot}6\ k\Omega.$$

(b)
$$(6-0{\cdot}8)=5\times10^{-3}\,R$$
$$R=\frac{5{\cdot}2}{5}\times10^{-3}$$
$$=1040\ \Omega.$$

(c) Power dissipated in the resistance

$$=(8-0{\cdot}8)\times(5\times10^{-3})$$
$$=36\ m\ W$$

Power dissipated in the diode,

$$=0{\cdot}8\times(5\times10^{-3})$$

$$=4{\cdot}0 \text{ m W}$$

(*d*)
$$V_B=(I\times R+0{\cdot}8)$$
$$=(2\times10^{-3}\times2\times10^{3}+0{\cdot}8)$$
$$=4{\cdot}8 \text{ Volt.}$$

Example 12. *In a transistor base current is changed by 10 μA. This results in a change of 0 01 V in base to emitter voltage and change of 2·0 mA in the collector current.*

(a) Find β, input resistance, and transconductance of the transistor.

(b) What is the voltage gain of the transistor amplifier circuit when load resistance in the circuit is 8 kΩ.

Solution.

$$\beta=\frac{\Delta I_C}{\Delta I_B}$$

$$=\frac{2{\cdot}0\times10^{-3}}{10\times10^{-6}}$$

$$=200$$

$$R_i=\frac{\Delta V_{BE}}{\Delta I_B}$$

$$=\frac{0{\cdot}01}{10\times10^{-6}}$$

$$=1000\ \Omega=1\ \text{K}\ \Omega$$

$$g_m=\frac{\Delta I_C}{\Delta V_{BE}}=\frac{2\ 0\times10^{-3}}{0{\cdot}01}$$

$$=0{\cdot}2\ \Omega^{-1}$$

(*b*)
$$A=\beta\,\frac{R_L}{R_i}$$

$$=200\times\frac{8}{1}$$

$$=1600.$$

Example 13. *In Fig. 15·4 below the characteristic curve of crystal diode is shown. Calculate the d.c. and a c. resistance of diode around the point M.*

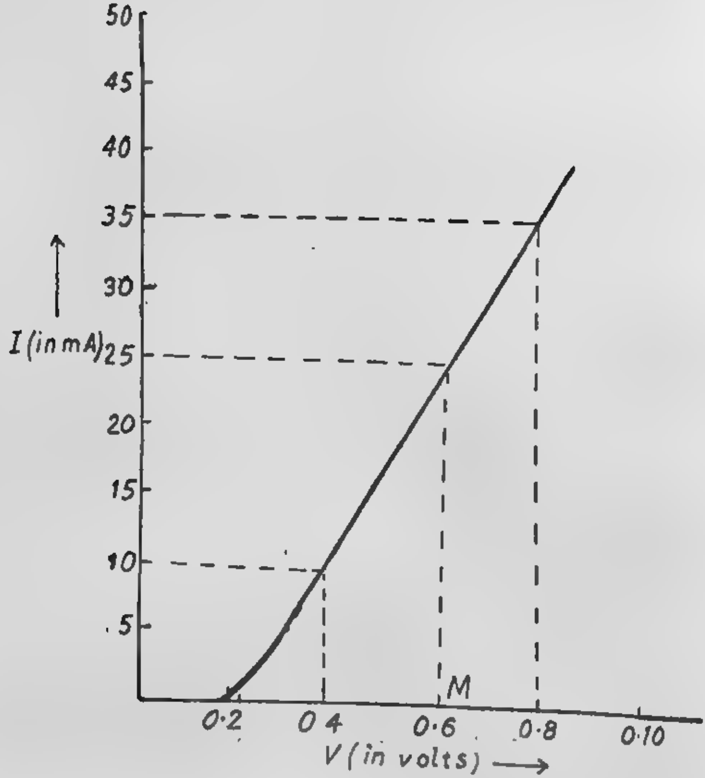


Fig. 15·4.

Solution. The value of d.c. resistance around the point M,

$$R_{d.c.} = \frac{V}{I}$$

$$= \frac{0{\cdot}6}{25 \times 10^{-3}}$$

$$= 24\ \Omega$$

The value of *a.c.* resistance around the point M,

$$R_{a.c.} = \frac{\Delta V}{\Delta I}$$

$$= \frac{(0{\cdot}8 - 0{\cdot}4)}{(35 - 10) \times 10^{-3}}$$

$$= 16\ \Omega.$$

Example 14. *In the following circuit, the value of β is 200. Find I_B, V_{CE}, V_{BE} and V_{BC}, when $I_C = 2{\cdot}5$ mA. The transistor is in active cutoff or saturation state.*

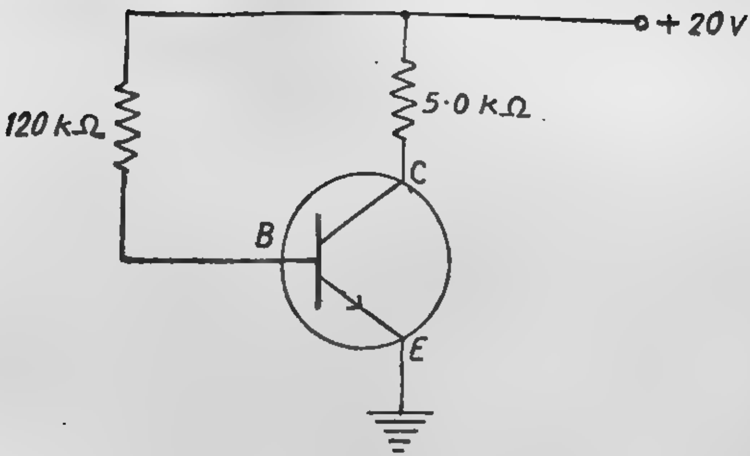


Fig. 15·5.

Solution.

$$\beta = \frac{I_C}{I_B}$$

$$\therefore \qquad I_B = \frac{I_C}{\beta}$$

$$= \frac{2{\cdot}5}{200}$$

$$= 0{\cdot}0125 \text{ mA}$$

Applying Kirchoff's law to Base emitter loop.

$$V_{BE} = V_C - I_B R_B$$

$$= 20 - 0{\cdot}0125 \times 120$$

$$= 20 - 1{\cdot}5$$

$$= 18{\cdot}5 \text{ V}$$

Applying Kirchoff's law to collector-emitter loop,

$$V_{CE} = V_C - I_C R_C$$

$$= 20 - 2{\cdot}5 \times 5$$

$$= 7{\cdot}5 \text{ V}$$

Now $\qquad V_{BC} = (V_{BE} - V_{CE})$

$$= (18{\cdot}5 - 7{\cdot}5)$$

$$= 11 \text{ V.}$$

Example 15. *In the following circuit if we assume that when the input voltage at the base resistance is +5V, V_{CE} is zero. Calculate the value of I_B, I_C and β when the barrier potential at base emitter junction is 0·5 V.*

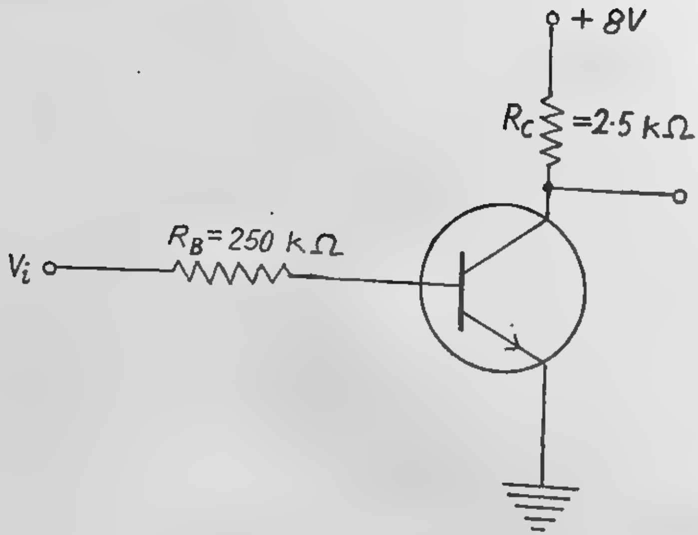


Fig. 15·6.

Solution. Since

$$V_{CE}=0$$

$$I_C=\frac{V_C}{R_C}$$

$$=\frac{8}{2{\cdot}5\times10^3}$$

$$=3{\cdot}2\times10^{-3}\text{ A}$$

$$=3{\cdot}2\text{ mA}$$

Since the input voltage is +5V, the voltage drop across R_B $=(5-0{\cdot}5)$ V$=4{\cdot}5$ V

$$\therefore \quad I_B=\frac{4{\cdot}5}{R_B}$$

$$=\frac{4{\cdot}5}{250\times10^3}$$

$$=18\ \mu\text{A}$$

$$\therefore \quad \beta=\frac{I_C}{I_B}$$

$$=\frac{(3{\cdot}2\times10^{-3})}{(18\times10^{-6})}$$

$$=177{\cdot}78.$$

Example 16. *In the following circuit, $V_s=0{\cdot}4$ V, $V_o=-8$ volt. Find V_i and gain*

$$A=\frac{V_o}{V_i} \text{ and } A'=\frac{V_o}{V_s}.$$

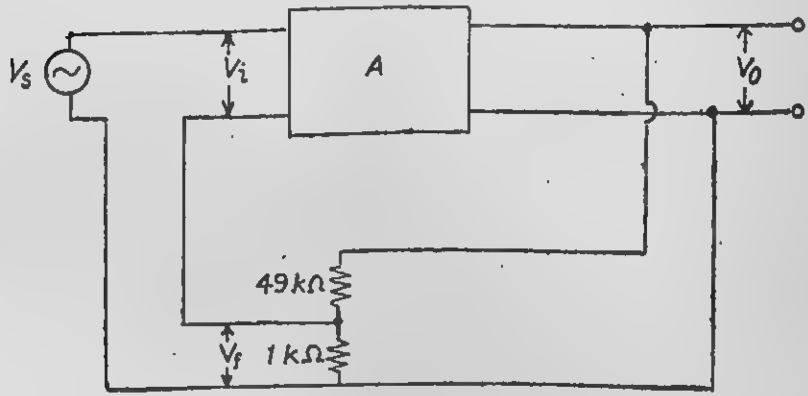


Fig. 15·7.

Solution.

$$\begin{aligned}
A' &= \frac{V_o}{V_s} \\
&= \frac{-8}{0{\cdot}4} \\
&= -20 \\
V_f &= \frac{V_o}{(49+1)} \times 1 \\
&= \frac{-8 \times 1}{50} \\
&= -0{\cdot}16 \text{ V} \\
V_i &= V_s + V_f \\
&= 0{\cdot}4 + (-0{\cdot}16) \\
&= 0{\cdot}24 \text{ V} \\
\therefore \quad A &= \frac{V_o}{V_i} \\
&= \frac{-8}{0{\cdot}24} \\
&= -\frac{100}{3} = 33{\cdot}3
\end{aligned}$$

Example 17. *The reverse saturation current of a diode at a room temperature is 25 μA. Calculate the current through the diode*

when the applied voltage is 130 mV. Also calculate the incremental diode resistance at the current.

Solution.

$$I = I_0\, e^{\left(\frac{qV}{KT}\right)}$$

At room temperature, we know

$$\frac{KT}{q} = 26 \text{ mV}$$

$\therefore$
$$I = 25\, e^{\left(\frac{130}{26}\right)} \mu A$$
$$= 25\, e^5$$

Let
$$e^5 = x$$
$$5 \log e = \log x$$

$\therefore$
$$\log x = 5 \times {\cdot}4343$$
$$\log x = 2{\cdot}1715$$
$$x = \text{Antilog } (2{\cdot}1715)$$
$$x = 148{\cdot}5$$
$$I = 25 \times 148{\cdot}5 \ \mu A$$
$$= 3712{\cdot}5 \ \mu A$$
$$= 3{\cdot}7125 \text{ mA}$$

The diode incremental resistance is given by

$$V_e = \frac{KT}{qI}$$
$$= \frac{26}{3{\cdot}7125}$$
$$= 7\Omega.$$

Example 18. *A transistor having an a.c. input resistance of 5·5 KΩ and current gain 180 is used as an amplifier. The signal source has a resistance of 10 KΩ and the load resistance has a value of 5 KΩ. Calculate the current, voltage and power gains assuming that the bias network offers a resistance of 5 KΩ.*

Solution. The current gain,

$$A_i = \beta \frac{i}{i_b} = \frac{\beta R_b}{R_b + R_i}$$

R_b = biasing resistance
R_i = input resistance

$$= \frac{180 \times 5}{(5 + 5{\cdot}5)}$$
$$= 85{\cdot}7$$

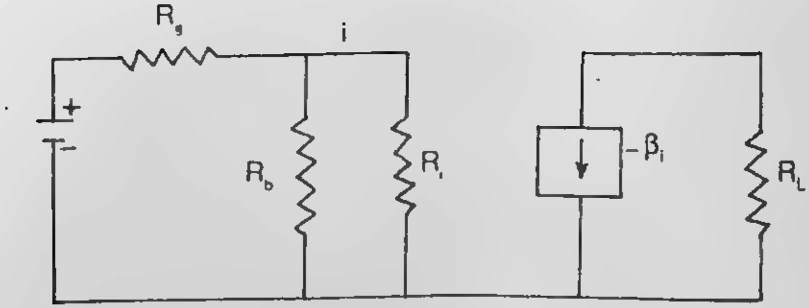


Fig. 15·8.

The voltage gain,

$$A_V = A_i \frac{R_L}{R_s + \frac{R_b R_i}{R_b + R_i}}$$

R_L = Load resistance

R_s = Signal source resistance

$$= 85{\cdot}7 \frac{5}{10 + \frac{5 \times 5{\cdot}5}{5 + 5{\cdot}5}}$$

$$= \frac{85{\cdot}7 \times 5}{(10 + 2{\cdot}62)}$$

$$= 33{\cdot}9$$

∴ Power gain,

$$A_p = A_i \times A_v$$
$$= 85{\cdot}7 \times 33{\cdot}9$$
$$= 2905{\cdot}23$$

EXERCISE 15

1. A transistor is connected in common base circuit. The biasing voltages are adjusted such that $I_E = 4$ m A and $I_C = 3.94$ mA. Calculate the base current I_B.
2. An NPN transistor (BF 115) is operated in common emitter circuit at $V_{ce} = 2$ Volt such that $I_C = 13$ 5 mA and $I_B = 250\ \mu$ A. Calculate the emitter current I_E.
3. A junction transistor has a common base current amplification factor as 0·985. Calculate the common emitter current amplification factor.

4. Find the valve of α for a transistor for which $\beta=50$.

5. A transistor (BF 125) is operating in a common base circuit. Calculate the current amplification factor such that a change in emitter current from 12·45 mA to 19·05 mA produces a change in the collector current from 12·30 mA to 17·70 mA.

6. A transistor AC 125 is connected in common base circut. When V_c is increased by 0·4 Volt, the I_c increases by 0·020 mA. Calculate the output resistance of the transistor.

7. For common emitter circuit of a transistor, when the base current changes by 80 μ A, the collector current changes by 4·8 mA. Calculate the current amplification factor.

8. In a common base circuit of a transistor, current amplification factor is 0·95. Calculate the base current when emitter current is 2 mA.

9. Find the voltage gain of an amplifier using a junction transistor (BF 115) in common emitter circuit such that $\beta=75$ and input and output resistance of the transistor are 500 ohm and 50 KΩ.

10. Find the gain of an amplifier using a transistor BF 115) in common base circuit such that $\alpha=0\ 987$ and input and output resistances of the transistor being 600 Ohm and 3200 KΩ.

11. Find the electrical conductivity and resistivity of germanium doped with phosphorous atoms at room temperature if the following datas are given for germanium—$n_e=4{\cdot}41\times10^{28}\ m^{-3}$, $\mu_e=0{\cdot}39\ m^2\ V^{-1}s^{-1}$, $n_h=1{\cdot}3\times10^{16}\ m^{-3}$ and $\mu_h=0{\cdot}19\ m^2\ V^{-1}s^{-1}$.

12. Find the effect on electrical conductivity and resistivity of a germanium crystal at room temperature if one arsenic atom is added in each 10^6 atom of germanium. Given that for germanium $\mu_e=0{\cdot}39\ m^2\ v^{-1}\ s^{-1}$, $\mu_h=0{\cdot}19\ m^2\ v^{-1}\ s^{-1}$, $n_i=2{\cdot}4\times10^{19}\ m^{-3}$, $\sigma=2{\cdot}18\ sm^{-1}$, $\rho=0{\cdot}46\ \Omega$ m and concentration of atoms$=4{\cdot}41\times10^{28}\ m^{-3}$.

13. A *p-n* junction diode when forward biased has a drop 0·6 V which is assumed to be independent of current. The current in excess of 8 mA damages the diode. What should be the value of resistance used in series with diode to use it safely when it is forward biased with a battery of 2·5 volt.

14. If in Question Number 13 above drop of potential on diode =0·4 V., I=1 mA and battery voltage= Volt, calculate 'R'.

15. If in the figure below R=200Ω and on connecting a 3 Volt battery in the circuit, the barrier potential developed in diode is 0·5 volt, calculate the value of current in the circuit

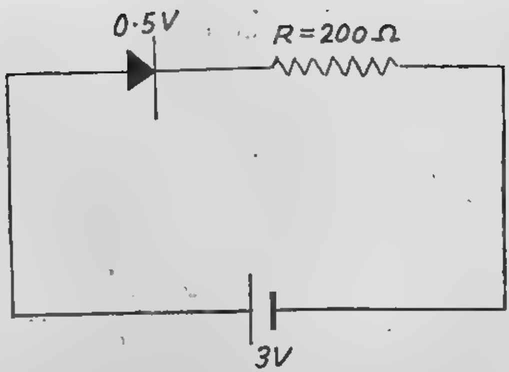


Fig. 15·9.

16. If in Fig. 15·2, on page 309, $R_L = 120$ k Ω & $R_i = 40$ k Ω, what will be the voltage gain of the amplifier circuit. The voltage gain of the amplifier without load is 100.

17. Determine the number density of donor atoms which have to be added in an intrinsic silicon to produce a *n* type semi-conductor with conductivity 300 $\Omega\ m^{-1}$. Given that the mobility of electrons in *n*-type silicon is 1350 $cm^2\ V^{-1}\ s^{-1}$. Neglect the contribution of holes to conductivity.

18. If the circuit in Fig. 15·3 on page 310 requires a minimum current of 1·5 mA to be above the knee point 1 V. (*a*) what is the power dissipated in the resistance R and in the diode when a current of 4 mA flows in the circuit at V=5 volt. (*b*) If R=2.5 k Ω what is the minimum voltage V required to keep the diode above knee point.

19. In a transistor base current is changed by 15 μ A. This results in a change of 0·03 V in base to emitter voltage and change of 1·5 mA in the collector current. (*a*) Find β, input resistance and transconductance of the transistor (*b*) If this transistor is used as an amplifier with the load resistance 6 k Ω, what is the voltage gain of the amplifier ?

20. Find the voltage gain of a transistor with a load resistance of 1 kΩ and internal resistance 200 Ω (β=120)

21. A transistor has base collector current gain β=80. The circuit is connected with the base grounded. Calculate the a.c. current for a change of 2·5 mA of the emitter current at constant collector potential.

22. A *n-p-n* transistor is operated in a common emitter circuit at a constant voltage $V_e = 3$V. The change in base current from 10·5 μA to 18.5 μA produces a change in the collector current from 12·4 mA to 13·6 mA. Find the value of α and β.

23. The common base current gain in an *n-p-n* transistor is 0·92. The reverse satuation current $I=8\mu$ A. Calculate the base collection currents for an emitter current of 2 mA.

24. In Fig. 15·10 the charcteristic curve of crystal diode in shown. Calculate the d.c. and a.c. resistance of diode around the point P.

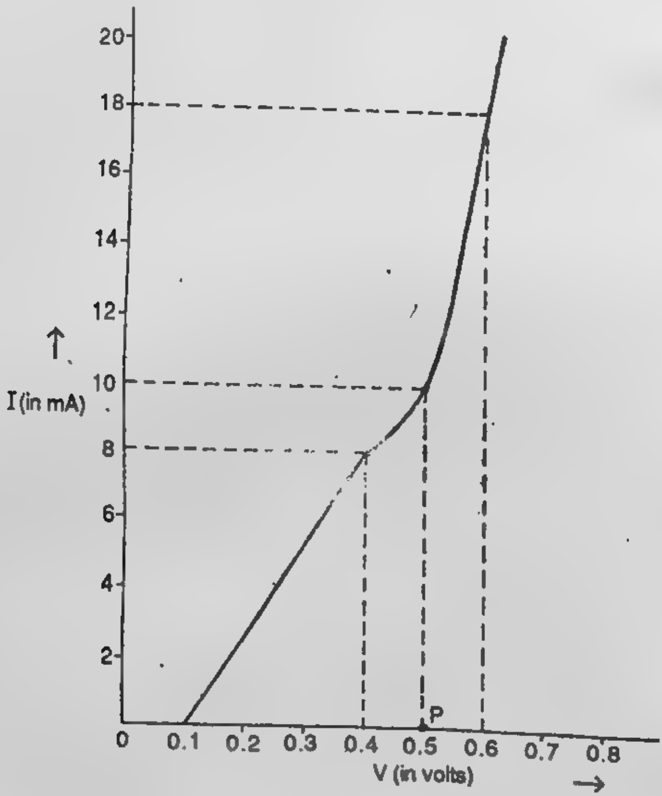


Fig. 15·10.

25. The current through a junction diode is 2 mA when a forward voltage of 104 mV is applied across the terminals. Calculate the reverse satuation current.

26. The revere saturation current of a transistor is 10 μA. Find the saturation current in the common emitter configuration if the current gain is 0·98.

27. Calculate R and R_L in the circuit given in Fig. 15·11 below. The circuit has a silicon NPN transistor and is operating at $I_C=10$ mA, $V_{CE}=3$ V and $V_{BE}=0·7$V. The d.c. current gdin of the circuit is 100 and battery voltage is 9 V.

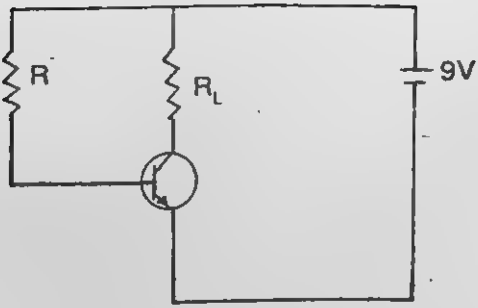


Fig. 15·11.

28. In the following circuit if we assume that when the input voltage at the base resistance is +6 V, V_{CE} is zero. Calculate the value of I_B, I_C and β when the barrier potential at base-emitter junction is 1 volt.

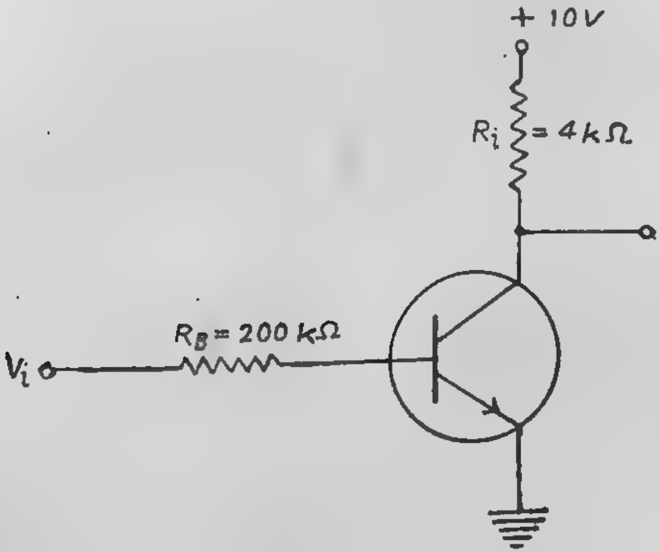


Fig. 15·12.

29. A half wave rectifier is fed with an input of 220 Volt 50 Hz. The output voltage is across a load resistance of 2 KΩ. What is the value of (*a*) a.c. voltage and (*b*) d.c. voltage, across the load.

30. For a full wave rectifier circuit using two semi-conductor diode, the input a.c. voltage is 20 Volt. Calculate the d.c. and a.c.

value of the rectified voltage across the load resistance R_L of 1 K Ω

31. Find the value of β for a transistor whose output characteristic curves are given in the Fig. 15·13 below, at $V_C=-3$ volt and $I_B=25\ \mu$ A, 75 μ A, 150 μ A and 200 μ A.

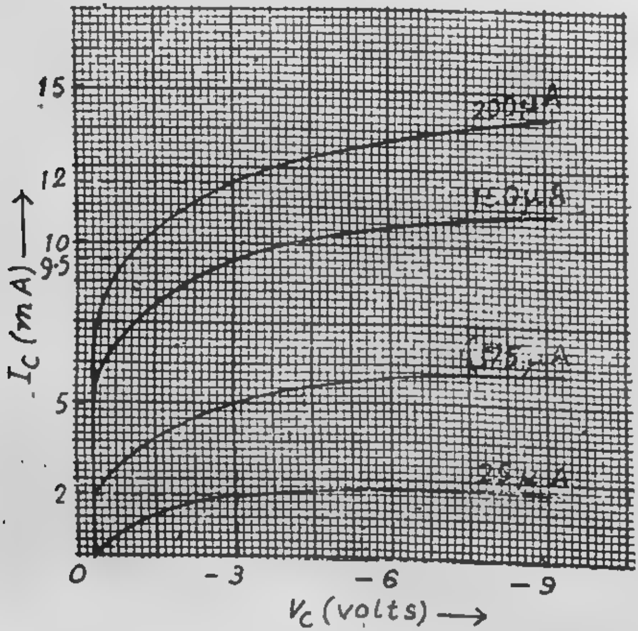


Fig. 15 13.

32. In a common base transistor amplifier, the input resistance is 400 ohms and the output resistance is 25 KΩ. If $\alpha=0{\cdot}98$, find the voltage gain of the amplifier circuit.

33. In the *n-p-n* transistor circuit, Fig. 15·14 below what is the potential difference between base and collector. What is the

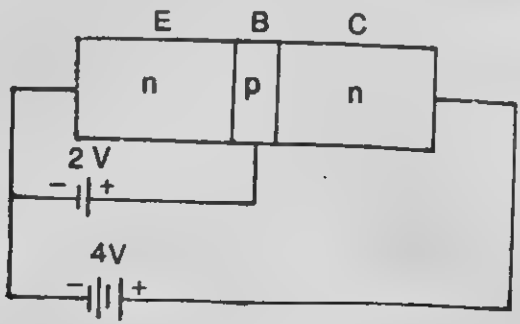


Fig. 15·14.

biasing (reverse and forward) between emitter-base and collector-base junction. [*A.I.S.SE. 1983*]

34. A transistor connected in common emitter configuration has input resistance $R_{CE}=2$ KΩ and load resistance of 5 KΩ. If $\beta=60$ and an input signal 12 mV is applied, calculate the resistance gain, voltage gain, the power gain and the value of output signal.

OBJECTIVE TYPE QUESTIONS

35. In a correctly biased NPN transistor,
(*a*) the base is positive with respect to emitter.
(*b*) the base is negative with respect to emitter.
(*c*) the base is positive with respect to collector.
(*d*) the base is positive with respect to both emitter and collector.

36. In the symbols of PNP transistors and NPN transistors, the arrow on the emitter shows the direction of flow of
(*a*) holes, electrons (*b*) holes, holes
(*c*) electrons, holes (*d*) electrons, electrons

37. Which of the following configuration has low thermal stability
(*a*) common collector (*b*) common base
(*c*) common emitter (*d*) All of these

38. The majority and minority carriers in a *n*-type semiconductors are
(*a*) electron and electron (*b*) holes and electrons
(*c*) holes and holes (*d*) electrons and holes

39. The order of magnitude of the barrier potential in a *p n* junction is
(*a*) 0·01 V (*b*) 0.05 V
(*c*) 0·1 V (*d*) 0·5 V

40. A *p-n* junction is forward biased, its resistance is
(*a*) ∞ (*b*) few kilo ohm
(*c*) few ohm (*d*) 0

41. A *p-n* junction is reverse biased, its resistance is
(*a*) ∞ (*b*) few kilo ohms
(*c*) few ohm (*d*) 0

42. A diode can act as—
(*a*) an amplifier (*b*) a detector
(*c*) a rectifier (*d*) an oscillator
[*D.P.M.T. 1989*]

43. A NPN or PNP transistor can act as
(*a*) an amplifier (*b*) a detector
(*c*) a rectifier (*d*) an oscillator

44. If the forward bias in a diode is increased, the length of depletion region will
(*a*) increase (*b*) decrease
(*c*) not change (*d*) can not be decided

45. For detecting light intensity, we use
(*a*) LED in forward bias (*b*) LED in reverse bias
(*c*) photodiode in reverse bias (*d*) photodiode in forward bias

46. When a *p-n* junction diode is forward biased, the flow of current across the function is mainly due to
(*a*) drift of electrons (*b*) diffusion of electrons
(*c*) both drift and diffusion of electrons
(*d*) none of them

47. A semi-conductor is known to have an electron concentration of 10^{19} m^{-3} and hole concentration of 5×10^{18} m^{-3}. The semi-conductor is
(*a*) *p*-type (*b*) *n*-type
(*c*) intrinsic (*d*) conductor

48. The following truth table represents
(*a*) NAND (*b*) AND
(*c*) NOR (*d*) NOT

Truth table

A	B	Y
0	0	1
1	0	0
0	1	0
1	1	0

49. To obtain a high output in an AND gate, the inputs A and B must be respectively
(*a*) 0, 1 (*b*) 1, 0
(*c*) 1, 1 (*d*) 0, 0

50. The input signal in a NAND gate are A=0 and B=0. The output is
(*a*) 0 (*b*) 1
(*c*) may be 0 or 1 (*d*) undecided

Universe

IMPORTANT FORMULAE

1. Newton's universal gravitational law,

$$F=G\frac{m_1 m_2}{x^2}$$

where F=Force of attraction
m_1 and m_2=Mass of heavenly bodies
x=Distance between their centres of mass
G=Gravitational constant
$=6{\cdot}67\times10^{-11}$ Nm^2/kg^2

2. Acceleration due to gravity at the surface of any planet or galaxy sattelite,

$$g=\frac{GM}{R^2}$$

where M=Mass of planet (or satellite)
R=Radius of the planet (or satellite)

3. Kepler's law ;

$$\frac{T^2}{a^3}=a \text{ constant ;}$$

where T=Period of revolution of planet
a=mean distance of planet from the sun.

4. The relation between the magnitudes m_1 and m_2 of the two stars corresponding to their brightness values is given by

$$\frac{l_1}{l_2}=100^{(m_2-m_1)/5} \Rightarrow (m_2-m_1)=-2{\cdot}5 \log\left(\frac{l_2}{l_1}\right)$$

5. Mass of a heavier body (*e g.* sun), around which a lighter body (*e.g.* earth) is revolving, is given by

$$M=\frac{4\pi^2}{G}\times\frac{a^3}{T^2}$$

6. Mean density of a planet,

$$D=\frac{3M}{4\pi R^3}=\frac{3g}{4\pi RG}$$

7. (*a*) The luminosity (the amount of solar energy emitted by sun or any other star per second),

$$L = 4\pi x^2 S\,;$$ S = Solar constant
x = Distance from sun (or star)

$$L = 4\pi R^2 \sigma T^4\,;$$ σ = Stefan's constant
$= 5{\cdot}67 \times 10^{-8}$ w m^{-2} k^{-4}
R = Radius of sun

(*b*) Stefan's law,

$$E = \sigma T^4\,;$$

8. Size of planet,

$$D = x\alpha\,;$$ D = Diameter of the planet
α = Angle subtended by planet on eye
x = Distance of the planet from earth

9. Red Shift,

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c},$$ $\Delta\lambda$ = Change in wavelength
v = Speed of star
c = Speed of light

10. Wien's law,

$$\lambda_m T = b, \text{ a constant}$$
$$b = 2{\cdot}897 \times 10^{-3} \text{ mK}$$

11. (*a*) The distance between a planet and the earth,

$$x = \cos \varepsilon\,;$$ ε = Angle of maximum elongation

(*b*) The distance between a planet and the Sun,

$$x' = \sin \epsilon$$

12. Hubble's law,

$$V = HR\,;$$ $H = 30$ (km/s)/million light year

13. Popular units,

1 A.U. $= 1{\cdot}5 \times 10^{11}$ m, 1 light year $= 9{\cdot}5 \times 10^{15}$ m

1 par sec. $= 3{\cdot}26$ light year, 1 magnitude $= \dfrac{I_1}{I_2} = 2{\cdot}512$

Solar mass $= 2 \times 10^{30}$ kg

SOLVED EXAMPLES

Example 1. *In case of planet mercury, the angle of maximum elongation* ε *is found to be* $22°47'$. *Calculate (a) the distance between mercury and the earth and the distance between mercury and the sun. (b) the orbital period of mercury in days.*

Solution.

(*a*)

$$x_{me} = \cos \varepsilon$$
$$= \cos 22°47'$$

$$=0{\cdot}9221 \text{ A.U.}$$
$$=0{\cdot}9221\times(1{\cdot}5\times10^{11})\text{ m}$$
$$=1{\cdot}38\times10^{11}\text{ m}$$
$$x_{ms}=\sin\varepsilon$$
$$=\sin 22°47'$$
$$=0{\cdot}387 \text{ A.U.}$$
$$=0{\cdot}387\times(1{\cdot}5\times10^{11})\text{ m}$$
$$=5{\cdot}8\times10^{10}\text{ m}$$

(b) $$\left(\frac{T_m}{T_e}\right)^2=\left(\frac{x_{ms}}{x_{es}}\right)^3$$

$\therefore$ $$T_m=\left(\frac{0{\cdot}387\text{ A.U.}}{1{\cdot}000\text{ A.U.}}\right)^{3/2}\times365\text{ days}\simeq88\text{ days}$$

Example 2. *Calculate the mass of the earth from the following data :*

$$g=9{\cdot}8\ ms^{-2}$$
$$R=6{\cdot}38\times10^6\ m$$
$$G=6{\cdot}67\times10^{-11}\ Nm^2/kg^2$$

[A.I.S.S.E. 1982]

Solution. We have

$$g=\frac{GM}{R^2}$$

$\therefore$ $$M=\frac{gR^2}{G}$$
$$=\frac{9{\cdot}8\times(6{\cdot}38\times10^6)^2}{6{\cdot}67\times10^{-11}}$$
$$=5{\cdot}98\times10^{24}\text{ kg}$$

Example 3. *It is a well known fact that during a total solar eclipse the disc of the moon almost completely covers the disc of the sun. The distance of the moon from the earth is $3{\cdot}84\times10^8$ m. If the sun's angular diameter is measured to be 1920", calculate the diameter of the moon.*

Solution. Sun's angular diameter,

$$\alpha=1920\times\frac{1}{3600}\times\frac{\pi}{180}\text{ radian}$$
$$=\frac{16}{30\times180}\times\frac{22}{7}$$
$$=9{\cdot}312\times10^{-3}\text{ rad.}$$

It is also the angular diameter of the moon on solar eclipse day.

$\therefore$ Diameter of the moon,

$$D=x\alpha$$

$$=(3{\cdot}84\times10^{8})(9{\cdot}312\times10^{-3})\text{ m}$$
$$=35{\cdot}758\times10^{5}\text{ m}$$
$$=3575{\cdot}8\text{ km.}$$

Example 4. *Compare the period of rotation of planet Mars about the Sun with that of the earth around it. The mean distance of the Mars from the sun is 1·52 A.U.*

Solution. We know

$$\frac{a^3}{T^2}\text{ is same for all planets}$$

$$\frac{G_e^3}{T_e^2}=\frac{a_m^3}{T_m^2}$$

$$\left(\frac{T_m}{T_e}\right)^2=\left(\frac{a_m}{a_e}\right)^3$$

$$\therefore\quad \frac{T_m}{T_e}=(1{\cdot}52)^{3/2}$$

$$=1\ 874:1$$

Example 5. *One of the satellites Jupiter has an orbital period 1·769 days and the radius of the orbit is $4{\cdot}22\times10^8$ m. Show that the mass of Jupiter is about one thousandth that of sun.*

Solution. Mass of the Jupiter,

$$M_j=\frac{4\pi^2}{G}\frac{R^3}{T^2}$$

$$=\frac{4\times(3{\cdot}14)^2\times(4{\cdot}22\times10^8)^3}{(6{\cdot}67\times10^{-11})(1{\cdot}769\times24\times60\times60)^2}$$

$$=2\times10^{27}\text{ kg}$$

We also know mass of the sun,

$$M_s=2\times10^{30}\text{ kg}$$

$$\therefore\quad \frac{M_j}{M_s}=\frac{2\times10^{27}}{2\times10^{30}}$$

$$=\frac{1}{1000}$$

Example 6. *The value of solar constant for earth is to be determined as $1{\cdot}388\times10^3$ watt m^{-2}. (a) Supposing it remits all the radiation it receives from the sun, calculate its temperature. (b) Calculate the value of solar constant and temperature for Venus (distance from the sun=0·723 A.U). (Given Stefan's constant $\sigma=5{\cdot}6\times10^{-8}$ $w\ m^{-2}\ k^{-4}$)*

Solution. (*a*) We know since S=E in this question,

$$S=\sigma T^4 \qquad \text{where S=solar constant}$$

$$T=\left(\frac{S}{\sigma}\right)^{\frac{1}{4}}$$

$$=\left(\frac{1{\cdot}388\times10^{3}}{5{\cdot}67\times10^{-8}}\right)^{\frac{1}{4}}$$

$$=395{\cdot}5\text{ K}$$

(*b*) **Luminosity of the sun**

$$L=4\pi x_1^2 S_1=4\pi x_2^2 S_2$$

$$\therefore\quad S_2=\left(\frac{x_1}{x_2}\right)^2 S_1$$

$$=\left(\frac{1\text{ A.U.}}{0{\cdot}723\text{ A.U.}}\right)^2\times1{\cdot}338\times10^{3}$$

$$=2560\text{ w m}^{-2}$$

$$T=\left(\frac{2560}{5{\cdot}67\times10^{-8}}\right)^{\frac{1}{4}}$$

$$=461\text{ K}$$

Example 7. *The dimmest star visible to the naked eye has a magnitude of 6·5. Compare [redacted] brightness with that of planet Mars having magnitude −2.*

Solution. $m_1=6{\cdot}5$ and $m_2=-2$

$\therefore$ $(m_1-m_2)=6{\cdot}5-(-2)=8{\cdot}5$

$\therefore$ The brightness ratio,

$$\frac{l_1}{l_2}=100^{\frac{(m_2-m_1)}{5}}$$

$$=100^{\frac{8\cdot5}{5}}$$

$$\frac{l_1}{l_2}=100^{1\cdot7}$$

$$\therefore\quad \log\left(\frac{l_1}{l_2}\right)=1{\cdot}7\log100=1{\cdot}7\times2=3{\cdot}4$$

$$\therefore\quad \frac{l_1}{l_2}=\text{Antilog }(3{\cdot}4)=2512$$

Example 8. *Calculate the range of temperature corresponding to which a star will appear blue. It is known that λ_{max} for blue light is between 4500 and 4900 A°.*

Solution. We know from Wien's law,

$$\lambda_{max}\,T=2\,897\times10^{-3}$$

$\therefore$ When $\lambda_m=4500\text{A}^\circ=4500\times10^{-10}$ m

$$T=\frac{2\,897\times10^{-3}}{4500\times10^{-10}}$$

$$=6438\text{ K}$$

and when $$\lambda_m = 4900\ \text{A}^\circ$$

$$T = \frac{2.897 \times 10^{-3}}{4900 \times 10^{-10}}$$

$$= 5912\ \text{K}.$$

Hence for blue light the temperature ranges from 5912 K to 6438 K.

Example 9. *In the constellation orion there is a bright reddish star called Betegeuse. Its luminosity is 10,000 times that of the sun and its surface temperature about 3000 K. How much larger is the radius of Betegeuse compared to that of the sun. (Surface temperature of the sun=5800 K).*

Solution. We know

$$L = 4\pi R^2 \sigma T^4$$

$$\frac{L_1}{L_2} = \frac{4\pi\sigma R_1^2 T_1^4}{4\pi\sigma R_2^2 T_2^4}$$

$$\therefore \quad \frac{R_1}{R_2} = \left(\frac{L_1}{L_2}\right)^{\frac{1}{2}} \left(\frac{T_2}{T_1}\right)^2$$

$$= (1000)^{\frac{1}{2}} \left(\frac{5800}{3000}\right)^2$$

$$= 100 \times 3.738$$

$$= 373.8\ \text{K}.$$

Example 10. *Consider a binary star system consisting of two stars separated by a distance of 30 A.U. with a period of revolution equal to 30 years. If one of the two stars is 5 times further from the centre of mass than the other. What is the mass of the two stars in comparison to the mass of the sun.*

Solution.

$$(M_1 + M_2) = \frac{4\pi^2}{G} \frac{a^3}{T^2}$$

If the masses are measured in terms of Solar mass, T in years and 'a' in A.U., then $4\pi^2 = G$

$$\therefore \quad (M_1 + M_2) = \frac{30^3}{30^2}$$

$$\therefore \quad (M_1 + M_2) = 30 \qquad \text{...(1)}$$

From the figure,

$$a_1 + a_2 = a$$

$$a_1 + a_2 = 30$$

But $$a_2 = 5a_1$$

$$\therefore \quad a_1 + 5a_1 = 30$$

$$\therefore \quad a_1 = 5$$

and $$a_2 = 25$$

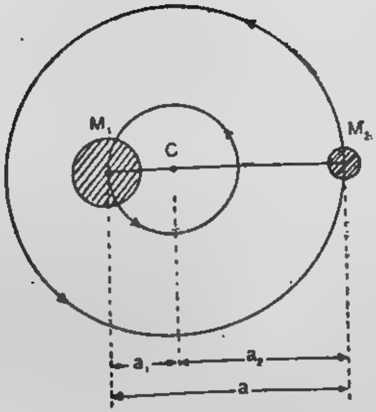


Fig. 16·1.

Now by principle of moments about the centre of mass,

$$M_1 a_1 = M_2 a_2$$

$$\therefore \quad \frac{M_1}{M_2} = \frac{a_2}{a_1} = \frac{25}{5} = 5 \qquad \text{[Equation (1)]}$$

But $\quad M_1 + M_2 = 30$

$\therefore \quad 5M_2 + M_2 = 30$

or $\quad M_2 = 5$

and $\quad M_1 = 25$

So the masses of the two stars are 5 and 25 times that of the sun.

Example 11. *Consider a white dwarf of one solar mass and its radius is that of earth (6400 km). Calculate the density of white dwarf.*

Solution. The density,

$$D = \frac{M}{V}$$

$$= \frac{M}{\frac{4}{3}\pi R^3}$$

$$= \frac{3M}{4\pi R^3}$$

$$= \frac{3 \times (2 \times 10)^{30}}{4 \times 3{\cdot}14 (6400 \times 10^3)^3}$$

$$= 1{\cdot}8 \times 10^9 \text{ kg/m}^3$$

Example 12. *A galaxy moving away from us with a speed 6500 km/s is at a distance of 430 million light years from us. Calculate Hubble's constant in (km/s)/million light year.*

Solution. By Hubble's law,

$$V = HR$$

$$H = \frac{V}{R}$$

$$= \frac{6500}{430} \text{ (km/s)/million light year}$$

$$= 15{\cdot}12 \text{ (km/)/million light year.}$$

Example 13. *A line in the spectrum from a galaxy in the constellation virgo is red shifted to a value of 3984 A°. In the laboratory experiments on earth this line has a wavelength 3968 A°. Find the speed of recession of the galaxy.*

Solution. Red shift

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{C}$$

$$\therefore \qquad v = \frac{\Delta\lambda}{\lambda} C$$

$$= \frac{(3984 - 3968)}{3968} \times 3 \times 10^8$$

$$= 1{\cdot}2 \times 10^6 \text{ m/s}$$

$$= 1200 \text{ km/s}$$

Example 14. *The lens of our eye has a diameter of 8 mm. How much fainter objects can be seen through a telescope of 120 cm aperture as compared to the faintest naked eye stars?*

Solution.

$$\frac{l_2}{l_1} = \frac{D_2^2}{D_1^2}$$

$$= \left(\frac{120 \times 10}{8}\right)^2$$

$$= 22500$$

Now

$$(m_2 - m_1) = -2{\cdot}5 \log\left(\frac{l_2}{l_1}\right)$$

$$= -2{\cdot}5 \log (22500)$$

$$= -2{\cdot}5 \times 4{\cdot}3522$$

$$= -10{\cdot}8$$

$$\simeq -11.$$

Therefore the telescope can make 11 magnitude smaller star visible compared to the one visible by naked eye.

Example 15. *The distance of the satellites of Mars are 25″ for phobos and 62″ for Deimos at mean opposition when mars is 0·524 A.U. from the earth calculate the distance of the two satellites from Mars in A.U. and in metres.*

Solution. (I) For phobos

$$\frac{S_1M}{EM}=25''$$

$$\therefore \quad S_1M=\left(\frac{25}{60\times 60}\times\frac{\pi}{180}\right)\times 0{\cdot}524 \text{ A.U.}$$

$$=6{\cdot}35\times 10^{-5} \text{ A.U.}$$

$$=6{\cdot}35\times 10^{-5}\times 1{\cdot}5\times 10^{11} \text{ m}$$

$$=9{\cdot}525\times 10^{6} \text{ m}$$

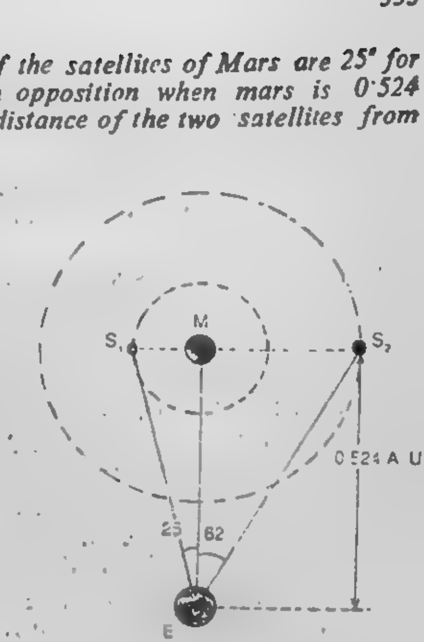


Fig. 16·2.

(II) For Deimos,

$$\frac{S_2M}{EM}=62''$$

$$\therefore \quad S_2M=\left(\frac{62}{60\times 60}\times\frac{3{\cdot}14}{180}\right)\times 0{\cdot}524 \text{ A.U.}$$

$$=1{\cdot}574\times 10^{-4} \text{ A.U.}$$

$$=1{\cdot}574\times 10^{-4}\times 1{\cdot}5\times 10^{11} \text{ m}$$

$$=2{\cdot}361\times 10^{7} \text{ m}$$

EXERCISE 16

1. The mass of the planet Mars is $6{\cdot}37\times 10^{23}$ kg and that of the sun is 2×10^{30} kg. The mean distance of Mars from the sun is $2{\cdot}28\times 10^{11}$ m. (*a*) Calculate the gravitational force which the sun exerts on Mars (*b*) Assuming the Mars moves in a circular orbit around the sun, calculate the speed of Mars.
2. Calculate the radius of the earth from the following data :
   Mass of the earth $=6\times 10^{24}$ kg,
   $G=6{\cdot}67\times 10^{-11}$ N m²/kg², $g=9{\cdot}8$ ms^{-2},
3. The radius of the moon is $1{\cdot}7\times 10^{6}$ m, and its mass is $7{\cdot}35\times 10^{22}$ kg. What is the acceleration due to gravity on the surface of the moon ?
4. The mass of Mars is 0·1065 times the mass of the earth. The diameter of Mars is $6\,880\times 10^{6}$ m and that of earth is $12{\cdot}800\times 10^{6}$ m. Calculate the value of acceleration due to gravity on Mars if g on earth is $9{\cdot}8$ m s^{-2}

5. A satellite revolves around a planet mercury in an orbit just above the surface of the planet. Taking $G=6.67\times10^{-11}$ Nm^2/kg^2 and mean density of planet$=5.4\times10^3$ kg m^{-3}, find the period of satellite. *[H.P.S.S.E. 1988]*
6. Calculate the mass of the moon, given that $G=6.67\times10^{-11}$ Nm^2/kg^2, the radius of moon$=1.7\times10^6$ m and g on moon $=1.7$ m s^{-2}.
7. The moon's radius is 27% of earth's radius and its mass is 1.2% of the earth's mass. What will be the weight of a man at the moon if his weight on the earth is 70 kg.
8. The mean distance of Mars from the sun is 1.524 A.U. Calculate the period of revolution of Mars.
9. Calculate the value of g on a hypothetical planet whose mass and radius both are one-third of that of the earth. (Value of g at earth$=9.8$ ms^{-2}).
10. Orbital period of moon is $27\frac{1}{3}$ days and its orbital radius is 3.85×10^8 m, calculate the mass of the earth.
11. Taking moon's period of revolution about the earth as 30 days and neglecting the effect of the sun and of the other planets on its motion, calculate its distance from the earth. (Given $G=6.67\times10^{-11}$ Nm^2/kg^2 and mass of the earth$=6\times10^{24}$ kg). *[Roorkee Engg. Ent. Exam. 1984]*
12. Calculate the period of revolution of Neptune around the Sun given that the diameter of its orbit is 30 times the orbit of the earth around the sun, both the orbits being assumed to be circular.
13. Calculate the mass of the sun, given that the distance between earth and the sun is 1.5×10^{11} m, $G=6.67\times10^{-11}$ Nm^2/kg^2 and one year$=365$ days.
14. How long will the earth take to rotate around the sun if its distance becomes half the present distance from the sun ?
15. The mean distance of Saturn from the sun is 9.54 A.U. Compare the period of rotation of saturn about sun with that of earth.
16. The mean distance of the earth from the sun is 1.5×10^{11} m and its period of revolution around the sun is 365.25 days. If period of Jupiter and Saturn around the sun is 4333 days and 10760 days respectively, calculate their mean distance from the sun.
17. The mean distance of the earth from the sun is 1.5×10^{11} m and its period of revolution around the sun is 365.25 days. If the distance of planents Mars and Venus from the sun is 2.28×10^{11} m and 1.08×10^{11} m respectively, calculate their period of revolution around the sun.

18. In case of Venus, the angle of maximum elongation is found to be 46° 18′. Determine the distance between venus and the sun and the distance between venus and the earth in A.U. and meters.

19. Assuming that the dimmest star visible to the naked eye has a magnitude of 6·0, compare its brightness with that of sun whose magnitude is −26·0.

20. What is known as the K line of singly ionized calcium has a wavelength of 3930A° as measured on earth in the spectrum of one of the observed galaxies, this spectral line is located at 5230 A°. Calculate (*a*) the speed with which this galvaxy is moving away from us, (*b*) the distance of galaxy from the earth in km.

21. The moon is known to revolve around the earth once in every 27·32 days in an orbit of average radius $3{\cdot}84 \times 10^8$ m. The centre of mass of the earth moon system lies at a distance of $4{\cdot}75 \times 10^6$ m from the centre of the earth. Calculate the mass of the earth and the moon.

22. The sun is known to give its maximum emission at 4750 A° and has a surface temperature of 1000 K. Calculate the temperature of a star giving its maximum emission at 2375A°.

23. The disc of the sun subtends an angle of 30′ to an observer on the earth. Find the diameter of the sun.

24. The parallex of sirius, the nearest star visible, is 0·375″. Find its distance from the sun in A.U.

25. The maximum and minimum distance of a comet from the sun are $1{\cdot}4 \times 10^{12}$ m and 7×10^{10} m. If its velocity nearest to the sun is 50 kms^{-1}, what is the velocity when farthest ?

26. Calculate the value of solar constant at Jupiter which is 5·3 A.U. away from the sun (*a*) if solar constant on the surface of the earth is 1388 Wm^{-2}. (*b*) if the temperature of the sun is 5800 K, radius of the sun is 7×10^8 m and Stefan's constant is $5{\cdot}7 \times 10^{-8}$ wm^{-2} k^{-4})

27. Solar constant on earth has a value of 1400 Wm^{-2}. If radius of the sun is 7×10^8 m, calculate the temperature of the sun. (Stefan's const.$=5{\cdot}7 \times 10^8$ $wm^{-2}k^{-4}$ and 1 A.U.$=1{\cdot}5 \times 10^{11}$ m).

28. Two stars have same absolute magnitudes. The temperature of one is 7000K and that of the other is 3500 K. Calculate the ratio of their diameters.

29. Luminosity of Rigel star in orion constellation is 17000 times that of our sun. If the surface temperature of the sun is 6000 K, what is the temperature of the star ?

30. Two stars have the same surface temperature. One is 10 magnitudes light than the other. Calculate the ratio of their surface areas and radii.

31. The period of a binary star system is 30 years and the semi-major axis of the orbit of the fainter star about the brighter is 20 A.U. Calculate the sum of the masses of the two stars.

32. If the rate of energy emitted by the sun is $3{\cdot}9 \times 10^{26}$ J/S, what is the value of solar constant at the earth ?

33. Calculate the surface temperature of the sun if the energy emitted per unit area per second by sun is $6{\cdot}41 \times 10^{7}$ w/m^2 ($\sigma = 5{\cdot}67 \times 10^{-8}$ wm^{-2} k^{-4}).

34. The period of sirius is 49·9 years. Its component stars revolve about a common centre of mass with a separation of 20 5 A.U. Calculate the total mass in solar units.

35. The moon subtends an angle of 1° at the base line equal to the radius of the earth. What is the distance of the moon from the earth ?

36. The period of revolution of a particular satellite of mars is 0·319 days. Calculate the mean distance of the satellite from the Mars. (Mass of Mars$=6{\cdot}73 \times 10^{23}$ kg).

37. If both components of Phxyon have same spectra while one component is 15000 times brighter than the other, calculate the ratio of their radii.

38. Calculate the distance in metre at which the velocity of a receding galaxy would approach the velocity of light. (Hubble's constant, H=30 (km/s)/million light year).

39. Two components of a binary star have a separation of 5″ and and a period of 50 years If the distance of binary is 10 light years, what is the total mass of the two stars ?

40. A binary system has a period of rotation of 36 years. The semi major axis of the orbit of fainter star is 20·25 A.U. Calculate the sum of masses of two stars.

41. Sun takes 24×10^{7} years to complete one rotation around the centre of its galaxy with a speed of 250 kms^{-1}. If the radius of galaxy is 30,000 light year, calculate the mass of the galaxy.

42. What must be the distance of a galaxy receding at a velocity 80% of the velocity of light ? (H=30 Km/s)/million light year).

43. When the spectrum emitted by a distant galaxy is analysed, it is found that the characteristic patterns of radiation emitted by similar elements at earth is present but their wavelengths shifted towards red by 12% (*a*) What is the speed of the galaxy ? (*b*) What is its distance from earth ? (H=30 (Km/s)/million light year).

44. When the light coming from a galaxy in the constellation Ursa Major was analysed, it was found to contain a line of wavelength 4170A° hile the wavelength of the same line observed on t th is 3970 A°. What is the velocity of recession of galaxy ?

OBJECTIVE TYPE QUESTIONS

45. According to Keplar's law of period, the following is same for all planets :

(a) $\frac{a^3}{T^2}$ (b) $\frac{a^3}{T^3}$

(c) $\frac{a^2}{T^3}$ (d) $\frac{a^2}{T^2}$.

46. The acceleration due to gravity at a distance 'x' from the centre of the earth ($x >$ radius of the earth) varies as :

(a) $g \propto \frac{1}{x^2}$ (b) $g \propto \frac{1}{\sqrt{x}}$.

(c) $g \propto \frac{1}{x}$ (d) $g \propto x$.

47. If the earth contracts to half of its radius, the duration of the day will be :

(a) 24 hrs. (b) 12 hrs.
(c) 6 hrs. (d) 48 hrs.

48. The maximum limiting size of the universe is

(a) 5 billion light years (bly)
(b) 10 bly
(c) 15 bly (d) 20 bly

49. From the stellar spectrum, we cannot conclude

(a) Recession velocity (b) mass
(c) composition (d) temperature.

50. The tail of the comet points

(a) away from the sun
(b) towards sun
(c) at some intermediate direction
(d) may be in any direction.

51. If R denotes the distance of the galaxy from the earth and V its recession velocity, then

(a) $V \propto \sqrt{R}$ (b) $V \propto R^2$

(c) $V \propto \frac{1}{R}$ (d) $V \propto R$.

52. Stars radiates light of their own by
(a) fusion reaction (b) chemical reaction
(c) fission reaction (d) all of these.

53. The hottest type of stars are called
(a) A– type (b) B—type
(c) G—type (d) M—type.

54. If a denotes the distance of planet from the sun and T the time period then
(a) $a \propto T^3$ (b) $a \propto T^2$
(c) $a \propto T^{1/2}$ (d) $a \propto T^{3/2}$.

55. The sun radiates energy of the rate of
(a) 4×10^{26} J/s (b) 2×10^{26} J/s
(c) 4×10^{30} J/s (d) 2×10^{30} J/s.

56. A Comet is moving faster when it is
(a) radiating maximum energy
(b) visible
(c) radiating no energy
(d) not visible.

57. The study of binary system is most helpful in finding
(a) their masses (b) their distance
(c) their temperature (d) their composition.

58. The galaxy to which our solar system belongs is
(a) radio galaxy (b) milky way
(c) normal galaxy (d) elliptical.

59. The number of stars in our galaxy is about
(a) 50 million (b) 250 million
(c) 100 billion (d) 150 billion.

60. The approximate temperature of the sun is
(a) 4800 K (b) 5800 K
(c) 6800 K (d) 7800 K.

61. Wien's displacement law states
(a) $\lambda_m T = a$ const. (b) $\frac{\lambda_m}{T} = a$ const.
(c) $\lambda_m T^4 = a$ const. (d) $\lambda_m^4 T = a$ const.

62. The solar constant on the surface of the earth is about
(a) 1000 W m^{-2} (b) 1200 W m^{-2}
(c) 1400 W m^{-2} (d) 1600 W m^{-2}.

63. 1 astronomical unit is equal to
(a) $1{\cdot}5 \times 10^8$ m (b) $1{\cdot}5 \times 10^{11}$ m
(c) $9{\cdot}5 \times 10^{12}$ m (d) $9{\cdot}5 \times 10^{15}$ m.

64. Heat energy falling on earth's surface per unit area per second is about
(*a*) $2{\cdot}4\times10^{2}$ J (*b*) $1{\cdot}4\times10^{3}$ J
(*c*) $2{\cdot}4\times10^{3}$ J (*d*) $1{\cdot}4\times10^{4}$ J.

65. 1 light year is the unit of
(*a*) time (*b*) distance
(*c*) frequency (*d*) none of these.

66. One light year is equal to
(*a*) 365 days (*b*) 3×10^{8} m
(*c*) $1{\cdot}5\times10^{11}$ m (*d*) $9{\cdot}5\times10^{15}$ m.

67. Luminosity of all the stars is closely related to their
(*a*) temperature (*b*) mass
(*c*) size (*d*) colour.

68. The pulsars are believed to be associated with
(*a*) Black holes (*b*) Andromeda Nebula
(*c*) Neutron stars (*d*) none of these.

69. The largest and heaviest member of the solar family is
(*a*) Sun (*b*) Saturn
(*c*) Jupiter (*d*) Neptune

70. Present evidence indicates that our universe is
(*a*) expanding (*b*) contracting
(*c*) static (*d*) none of these.

71. The brightest object other than moon ever seen in the night sky is
(*a*) Mars (*b*) Venus
(*c*) Pole star (*d*) Mercury.

72. The period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius 4 R=... [*B.H.U. 1982*]

73. If the amount of radiant energy received per unit area per second from the sun is measured at the earth, Mars and Jupiter it will be :
(*a*) the same at all the three
(*b*) in the decreasing order Jupiter, Mars, Earth
(*c*) in the increasing order Jupiter, Mars, Earth
(*d*) in the decreasing order Mars, Earth, Jupiter.
[*D.P.M.T. 1988*]

74. Moon has no atmosphere because
(*a*) r.m.s velocity of all gases is more than their escape velocity from moon's surface

(*b*) it does not have population as well as plants
(*c*) its surface is not smooth
(*d*) it is quite far off. [*D.P.M.T. 1989*]

75. If a star emitting yellow light stars accelerating towards the earth, its colours as seen from the earth will be
(*a*) turn gradually blue
(*b*) turn gardually red
(*c*) remains unchanged
(*d*) turn bright yellow. [*D.P.M.T. 1989*]

76. In accordance with Kepler's laws of planetry motion, the square of the periods of the planets are proportional to :
(*a*) their mean distance from the sun
(*b*) the square of the masses of the planets
(*c*) the square of their mean distance from the sun
(*d*) the cube of their mean distance from the sun.
[*M.A.M.C. 1983, D.P.M.T. 1989*]

77. The numerical value of the angular velocity of rotation of the earth should be.. ....rads^{-1} in order to make the effective acceleration due to gravity equal to zero at the equator.
[*I.I T. 1984*]

78. According to Kepler's second law, the radius vector to planet from the sun sweeps out equal areas in equal intervals of time. This law is a consequence of the conservation of..............

79. A geostationary satellite is orbiting the earth at a height of 6R above the surface of the earth, where R is the radius of the earth. The time period of another satellite at a height of 2·5 R from the surface of the earth is.........hours.

80. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K respectively. The energy radiated per second by the first sphere is grater than that by the second (True or False). [*I.I.T. J.E.E. 1988*]

□.□

ANSWERS

EXERCISE 1

1. (a) $\frac{1}{3}\times10^{-8}$ C, $\frac{2}{3}\times10^{-8}$ C (b) 3·314 cm from $\frac{1}{3}\times10^{-8}$C charge.
2. $3{\cdot}6\times10^{6}$ NC^{-1}. 3. 30 V. 4. 15·625 cm
5. $1{\cdot}2\times10^{26}$.
6. $1{\cdot}28\times10^{16}$ ms^{-2}, $2{\cdot}795\times10^{-9}$ S ; $6{\cdot}887\times10^{13}$ ms^{-2}, $1{\cdot}2\times10^{-7}$ S.
7. $1{\cdot}632\times10^{7}$ ms^{-1}
8. (a) $1{\cdot}6\times10^{-16}$ J (b) $1{\cdot}885\times10^{7}$ ms^{-1}
   (c) $1{\cdot}696\times10^{-23}$ kg ms^{-1}.
9. 720 V 10. $1{\cdot}602\times10^{-19}$ C 11. 9° 06′.
12. $2{\cdot}5\times10^{13}$ electrons from glass to silk ; there is a transfer of mass$=2{\cdot}25\times10^{-17}$ kg.
13. (a) $2{\cdot}88\times10^{-13}$ N (b) (i) $6{\cdot}48\times10^{-8}$ N, (ii) $3{\cdot}6\times10^{-6}$ N.
14. $1{\cdot}984\times10^{-6}$ N 15. $2{\cdot}388\times10^{-18}$ C.
16. $1{\cdot}214\times10^{-6}$ m 17. $1{\cdot}597\times10^{-19}$ C.
18. (a) 59 N at 21°46′ with the line AB
    (b) $4{\cdot}3\times10^{7}$ NC^{-1} at 69°46′ with the line joining centeroid and -3μC charge.
19. zero, zero.
20. (a) $\frac{100}{81}\times10^{5}$ NC^{-1} (b) $\frac{10}{3}\times10^{5}$.
21. 6 98 m from $+10\mu$C.
22. $1{\cdot}44\times10^{6}$ NC^{-1} parallel to dipole axis.
23. $+2{\cdot}3\times10^{-8}$ C, $-2{\cdot}3\times10^{-8}$ C.
24. -42 μC, $1{\cdot}028\times10^{9}$ NC^{-1}.
25. zero, $3\sqrt{3}\,K\,\frac{q}{a}$ 26. zero, 9×10^{5} V.
27. zero, 24×10^{5} V. 28. 7·2 N repulsion.
29. (a) 29·7 N repulsion (b) 5·4 N attraction.
30. (a) $6{\cdot}875\times10^{5}$ NC^{-1}, 303750 V.
    (b) $6{\cdot}65\times10^{5}$ NC^{-1} at 20°48′ with the line joining 4·5 μC and the point, $2{\cdot}43\times10^{5}$ V.
31. $-3{\cdot}47\times10^{-6}$ $\mu C\ m^{-2}$, $20{\cdot}8\times10^{-6}$ $\mu C\ m^{-2}$.
32. $2{\cdot}7\times10^{6}$ V, zero ; zero, No. 33. 7·05 J.

34. $-13{\cdot}6$ eV, $13{\cdot}6$ eV,
35. (*b*), 36. (*c*), 37. (*a*), 38. (*a*), 39. (*d*), 40. (*b*),
41. (*a*), 42. (*c*), 43. (*d*), 14. (*b*), 45. (*a*), 46. (*c*),
47. (*b*), 48. (*d*), 49. (*d*), 50. (*a*), 51. (*d*).

EXERCISE 2

1. $2320 \mu f$. 2. $10\ pf$, $2{\cdot}5 \times 10^6$ V, 0, 0. 3. $711\ \mu f$.
4. $\frac{4}{3} \times 10^{-8}$ C, $\frac{2}{3} \times 10^{-8}$ C, $133{\cdot}3$ V.
5. $3{\cdot}6 \times 10^6$ V, $14{\cdot}4\ \mu C$, $21{\cdot}6\ \mu C$ 6. $\frac{5}{6} \times 10^6$ V, $\frac{5}{6} \times 10^{-6}$ J.
7. 5 pt., $2{\cdot}5 \times 10^{-8}$ J, 140 V, $2{\cdot}4 \times 10^{-8}$ J.
8. 200 V, 1000 μC, 250 μC. 9. 530 *pf*. 10. 1452 *pf*.
11. 5/6 cm. 12. $2{\cdot}2\ \mu f$.
13. $\frac{20}{3}\ \mu f$, charge on first three capacitors$=2 \times 10^{-8}$ C ; charge on fourth capacitor$=1{\cdot}5 \times 10^{-8}$ C.
14. (*a*) 450 *pf*. 133·3 *pf*. and 160 *pf*. respectively
 (*b*) Fig. 8 (*a*) : 100×10^{-9} C on C_5 and $12{\cdot}5 \times 10^{-9}$ on C_1, C_2, C_3 and C_4 each ; 250 V on C_5 and 62·5 V on C_1, C_2, C_3 and C_4 each
 Fig. 8 (*b*) : $33{\cdot}3 \times 10^{-9}$ C on C_5 and $16{\cdot}65 \times 10^{-9}$ C on C_1, C_2, C_3 and C_4 each ; 83·3 V on C_5 and 83·35 V on C_1, C_2, C_3 and C_4 each
 Fig. 8 (*c*): 40×10^{-9} C on C_5, 30×10^{-9} C on C_1, 10×10^{-9} C on C_2, C_3 and C_4 each ; 100 V on C_5, 150 V on C_1, and 50 V on C_2, C_3 and C_4 each.
15. 10×10^{-6} J ; 5×10^{-6} J. 16. 30 μC.
17. 10 rows in parallel of 5 capacitors each.
 Total No. of capacitors$=50$.
18. $158{\cdot}4$ mm². 19. (*a*) 250 *pf*, 5×10^{-8} C (*b*) 5×10^{-6} J.
20. (*a*) 4000 *pf*. (*b*) 200 : 1 21. 2970 cm². 22. 50 cm.
23. (*d*), 24. (*b*), 25. (*c*), 26. (*d*), 27. (*a*). 28. (*d*),
29. (*b*), 30. (*b*), 31. (*c*), 32. (*c*), 33. (*a*), 34. (*b*),
35. (*b*).

EXERCISE 3

1. 0·25 A. 2. (*a*) 0·12 A (*b*) 0·42 A.
3. 1·5 Ω. 4. 6 Ω and 2 Ω.
5. 32 : 25 6. $3{\cdot}75 \times 10^{19}$
7. $1{\cdot}99 \times 10^{-4}$ ms^{-1} 8. 36·48 m
9. 1·3 V 10. 1 : 16 11. 20 Ω increase

12. 0·158 mm. 13. $26{\cdot}18\times10^{-9}\ \Omega$ m
14. 0·0199 mm 15. 2·5 Ω
16. 0·1 Ω, 2·15 V.
17. (*a*) $\frac{1}{101}\ \Omega$ in parallel (*b*) 9901 Ω in series
18. $\frac{1}{90}\ \Omega$ in parallel 19. 1990 Ω in series
20. (*a*) 0·12 Ω in parallel (*b*) 220 Ω in series
(*c*) 0·119 Ω for ammeter and 250 Ω for voltmeter.
21. (*a*) (*i*) $\frac{1}{49}\ \Omega$ in parallel (*ii*) 49 Ω in series
(*b*) (*i*) 0·02 Ω (*ii*) 50 Ω.
22. (*a*) (*i*) $\frac{5}{14}\ \Omega$ in parallel (*ii*) 10 Ω in series
(*b*) (*i*) $\frac{1}{3}\ \Omega$ (*ii*) 15 Ω
23. (*a*) (*i*) $\frac{5}{14}\ \Omega$ in parallel (*ii*) 10 Ω in series
(*b*) (*i*) $\frac{1}{3}\ \Omega$ (*ii*) 15 Ω
24. 0·012 Ω, 0·011 Ω
25. 1990 Ω in series and $\frac{10}{9}\ \Omega$ in parallel
26. (*a*) 3 Ω (*b*) 3 Ω (*c*) 1 Ω
(*d*) 10·3 Ω (*e*) 2 Ω (*f*) 10 Ω.
27. $1{\cdot}72\times10^{-4}\ ^\circ C^{-1}$ 28. 684°C
29. 0·2123 A, 0·0816 A, 0·294 A
30. $I=\frac{1}{6}$ A, $I_1=0{\cdot}1$ A, $I_2=\frac{1}{15}$ A
31. $I_1=\frac{7}{3}$ A, $I_2=-\frac{13}{3}$ A, $I_3=-\frac{23}{12}$ A
32. $I_1=1{\cdot}63$ A, $I_2=7{\cdot}1$ A
33. $I=2{\cdot}68$ A, $I_1=1{\cdot}26$ A, $I_2=0{\cdot}216$ A
34. (*a*) 6 Ω (*b*) 3 Ω
35. (*a*) $I=\frac{16}{3}$ A; $I_1=\frac{14}{3}$ A (*b*) $I_1=1$ A, $I_2=1$ A, $I_3=-0{\cdot}5$ A
36. 3 A 37. (*a*) 400 Ω (*b*) 40 V
38. $V=3\,(5+1)=1\times2=20$ V 39. $E=8\times10^{-4}$ J
40. (*a*) 20 Ω (*b*) 60 cm 41. 36 cm
42. $41\times10^{-4}\ ^\circ C^{-1}$ 43. 6·7 Ω 44. $1{\cdot}7\times10^{-8}\ \Omega$
45. (*d*), 46. (*c*), 47. (*a*), 48. (*c*), 49. (*a*), 50. (*d*),
51. (*b*), 52. (*b*), 53. (*b*), 54. (*d*), 55. (*c*), 56. (*d*),
57. (*b*), 58. (*d*), 59. (*a*), 60. (*a*) 61. (*a*), 62. (*c*).

63. (c), 64. (b), 65. (d), 66. (b), 67. (a), 68. (c)
69. (a).

EXERCISE 4

1. 8 A, 31·25 Ω, 3 min 9 S
2. 1 Kw.
3. (a) 625 Ω (b) Rs. 3·60
4. 31.5 k cal.
5. 48·4 Ω, 16 hr. 20 min.
6. 9·09 A, 24·2 Ω, Rs. 3·00
7. 14·78 A, Rs. 3·25
8. 14 min
9. 10°C
10. 151·2 S, 1 Paisa
11. (i) By putting 180 Ω resistance in series with a bulb
 (ii) Using 6 bulbs after connecting them in series.
12. Bulb L_1 will in fig. (a) ; Both the bulbs will fuse in fig. (b) Both the bulb lights up.
13. 1·05 Kw, 5 A
14. 2·297 cal s^{-1}, 3·445 cal s^{-1}
15. AB : 2 Js^{-1}, BC : 6 Js^{-1},
 AD : 3·2 Js^{-1}, DC : 9·6 Js^{-1}
16. (a) 0·5 Ω (b) 80 kwh. (c) 8 kwh.
17. 500 V, 6550 V
18. 17·94 cal s^{-1}
19. (a) 6·7% (b) 176·5 cal s^{-1}
20. (a) 72·98% (b) $4{\cdot}08\times10^4$ cal
21. (a) 31·2 w (b) 10·816 w
 (c) 20·384 w (d) 20·384 w.
22. (a) 71 w (b) 25·2 w
 (c) 45·8 w (d) 45·8 w.
23. 11 Ω, 1056 w 968 w, $6{\cdot}356\times10^4$ J
24. 8·41 Ω, 900 w, 874·8 w, $2{\cdot}268\times10^4$ J
25. 6Ω
26. 8Ω, 84 V
27. 2·8125 w
28. 50 V
29. $3{\cdot}5156\times10^4$ J
30. 4 : 9
31. 4·2 A
32. 1·3 A
33. 49 min. 1S.
34. 33·7 g
35. +0·05 A
36. 0 154 cm
37. (a) 7·34 Ω (b) 2 cells in series
38. 1·714 g, 0·0159 g, 0·1272 g
39. Cu : 58 85 g ; Ag : 200·19 g
40. 0·476 Ω
41. (a) 64 V (b) 16·67 w
42. 5°C
43. 590°C
44. 270°C
45. 60 μV
46. (a) 9 min. 18S (b) 9 min. 18S
47. Neutral temp.$=-\dfrac{a}{2\beta}$, Peltier coeff $=(\theta+273)\ (a+2\beta\theta)$ and Thomson coeff.$=2\ (\theta+273)\ \beta$

48. (*b*), 49 (*d*), 50. (*a*), 51. (*c*), 52. (*c*), 53. (*d*),
54. (*b*), 55. (*c*), 56. (*b*), 57. (*c*), 58. (*c*), 59. (*d*),
60. (*a*), 61. (*c*).

EXERCISE 5

1. 4×10^{-6} T, $2{\cdot}56\times10^{-20}$ N
2. $2{\cdot}4\times10^{-5}$ N, repulsive
3. 10^{-5} N, attractive
4. (*a*) 0·05 N (*b*) 0
5. $3{\cdot}2\times10^{-20}$ N
6. 0·875 N
7. 0·24 Tesla, horizontally and perpendicular to wire
8. $5{\cdot}1496\times10^{-3}$ T, perpendicular to the plane of the coil
9. $2{\cdot}512\times10^{-4}$ T
10. I=13 A, r=3 cm., n=1600 turns and length of the solenoid =75 cm.
11. 8×10^{-4} T, perpendicular to the plane of the arc.
12. (*a*) $4{\cdot}4\times10^{-4}$ T, perpendicular to plane of the arc
    (*b*) same magnitude but direction will get reversed.
13. 0·316 mm
14. $6{\cdot}787\times10^{-5}$ T
15. $58{\cdot}8\times10^{-5}$ T
16. (*a*) wire A (*b*) $1{\cdot}28\times10^{-4}$ T, $1{\cdot}115\times10^{-4}$ T
17. (*a*) $2{\cdot}32\times10^{-4}$ T (*b*) 0 (*c*) 0.
18. 0·8 T
19. 48
20. $1{\cdot}256\times10^{-4}$ T
21. (*a*) 0·024 Nm (*b*) 0
22. $1{\cdot}6\times10^{-3}$ T
23. $11{\cdot}264\times10^{-5}$ T
24. 2·4 MHz
25. $8{\cdot}25\times10^{-8}$ S
26. (*a*) $4{\cdot}175\times10^{-2}$ T (*b*) 2·53 MeV
27. 5 MeV
28. 47·9 MeV
29. 7·23 cm
30. $3{\cdot}67\times10^{7}$ ms^{-1}, 6·5 mm
31. Velocity = 8×10^{7} ms^{-1}
    (*a*) trajectory is a circle of radius 1·125 cm.
    (*b*) trajectory is a helical of radius 2·25 cm.
32. 0·32 A
33. 2·35 A
34. 10^{-5} N, repulsive
35. $2{\cdot}88\times10^{-6}$ N, attractive
36. 4×10^{-9} rad A^{-1}, $1{\cdot}6\times10^{-10}$ rad V^{-1}
37. (*a*) It is not easy to change A, B, R or K, so we will change increase number of turns 35 to 42.
    (*b*) original meter has more sensitivity.
38. 75 : 32
39. 45 : 32
40. (*a*) circular loop in north south direction
    (*b*) $200{\cdot}96\times10^{-5}$ Nm
41. 3×10^{-26} N
42. (*a*) $8{\cdot}64\times10^{-2}$ Nm (*b*) $4{\cdot}32\times10^{-2}$ Nm
43. 2 2 A
44. (*a*) 0·924 Nm (*b*) Unchanged

45. (*a*) 1.92×10^{-17} N (*b*) 1.92×10^{-17} N (*c*) 0
46. (*a*) 10.24×10^{-18} N (*b*) 10.24×10^{-18} N (*c*) 0
47. 46·77 A
48. 1·856 A
49. 7×10^{27}
50. 1·95 μV
51. 1.76×10^{-8} wb m^{-2}, 1.694×10^{-8} wb.
52. (*a*) 6.28×10^{-2} T (*b*) 5.62×10^{-4} T
53. (*a*) 1.57×10^{-2} T (*b*) 3.39×10^{-4} T
54. 7.73×10^{-4} T
55. 3.6×10^{-2} T
56. 3.0×10^{-5} Nm^{-1}, repulsive
57. 6A from right to left
58. 12·5 T
59. 4.8×10^{-4} N
60. (*c*), 61. (*a*), 62. (*a*), 63. (*b*), 64. (*c*), 65. (*b*),
66. (*a*), 67. (*b*), 68. (*a*), 69. (*c*), 70. (*d*), 71. (*c*).

EXERCISE 6

1. 63°26′
2. 0·174 Am^2
3. 5·3 s^{-1}
4. 22·5 Am^2
5. 16 : 25
6. 0·551 G
7. 25 : 7
8. 64 Am^2
9. 4×10^{5} Am^2
10. (*a*) $T'=2\pi\sqrt{\dfrac{I}{m(B-H)}}=5.67$ s

    (*b*) $T'=2\pi\sqrt{\dfrac{I}{m(B+H)}}=3$ s
11. (*a*) 2 Am^2 (*b*) (*i*) 0 (*ii*) 80° (*c*) (*i*) −0·5 J (*ii*) +0·5 J
12. (*a*) (*i*) 0·75 J (*ii*) 0·375 J (*a*) (*i*) 0 (*ii*) 0·375 Nm
13. (*a*) 50 JT^{-1} (*b*) 0·804 J
14. (*a*) 0·14 Nm
    (*b*) when solenoid is along the external field in one direction stable and in the reverse direction unstable.
    (*c*) 0·324 J
15. (*a*) 2Am^2 (*b*) 5×10^{-2} Nm
16. (*a*) 5°, 19°
    (*b*) 0·38 G in the magnetic median plane making an angle of 19° with the horizontal towards ground.
17. 0·275 G
18. 0·354 G
19. 0·346 G, 0·2 G
29. 0·5 G
21. 25° 46′
22. 30°
23. 0·32 G
24. (*a*) 12150 Am^2 (*b*) 18·84 cm
    (*c*) 1·08 G directed along *m*
25. 0·2419 Am^2
26. 4.73×10^{-4} kgm^2

27. 6.74×10^{-5} kgm^2 28. 2.34 Am^2, 58.5 Am
29. (*a*) 4.853 cm (*b*) 3.851 cm
30. (*a*) 10 cm (*b*) 7.938 cm
31. (*a*) 12°, 60° (*b*) 0.64 G
32. (*a*) 0.458 G, 70° 54′ (*b*) 0.557 G, 51° 9′
33. (*a*) 2.2 G (*b*) 1.9 G
34. The needle will reverse its original direction in each case (*a*) & (*b*)
35. (*a*) 1.0352×10^{-2} T (*b*) 0.732×10^{-2} T
36. (*a*) 3.75 mm (*b*) 15.15 cm
37. 3.125 T 38. 4.704 T
39. 6×10^{-4} TmA^{-1}, 477
40. 682, 41. 0.3 A, 42. (*d*), 43. (*b*), 44. (*a*), 45. (*c*),
46. (*b*), 47. (*a*), 48. (*d*), 49. (*b*), 50. (*c*), 51. (*a*)
52. (*d*), 53. (*b*), 54. (*a*), 55. (*b*).

EXERCISE 7

1. 8.4×10^{-4} *w* 1. 5.4×10^{-6} *wb*
3. 0.2475 *wb*, 8.25 V 4. 0.1 V
5. 0.216 V 6. 5 V
7. 3.75 ms^{-1} 8. 5.28 mV
9. 11.785 V, 8.33 V 10. 3.0 V
11. (*a*) 9.9 V (*b*) 18.57 V
(*c*) 4.95 V (*d*) 0
12. 4.5 V 13. 1 H
14. 0.667 H 15. 400 turns
16. 0.667 A 17. 2.5 A, 12.5 A
18. 0.005 A 19. 0.22 A, 0.088 A
20. (*a*) 4 (*b*) 10 A, 1.5 A
(*c*) 90% (*d*) 200 W (*e*) 19.22 W
21. (*a*) 1/20 (*b*) 2.5 A, 20 A
(*c*) 40% (*d*) 360 *w* (*e*) 7.5 W
22. 250 23. 400, 20 A
24. 25 A, 2.0 A
25. (*a*) 3.92 A (*b*) 8

26.
$$V = A\frac{d}{dt}(\mu_0 nI)$$
$$= A\mu_0 n\frac{dI}{dt}$$
$$= 14.3\ \mu V.$$

27. (*a*) 1.65×10^{-3} *wb* (*b*) 0·33 mH
28. (*a*) 1·92 mH (*b*) 32 mH
29. (*a*) 3·6 mH (*b*) 72 mV
30. 25·64 V
31. 0·1 T
32. 21 mA
33. 8·64 mA
34. 0·879 V
35. 2·94 V
36. (*a*), 37. (*b*), 38. (*c*), 39. (*a*), 40. (*d*), 41. (*c*),
42. (*b*), 43. (*a*), 44. (*a*), 45. (*c*).

EXERCISE 8

1. V=311·08 sin 314 *t*
2. (*a*) 7·07 A (*b*) (*i*) 6·06 A (*ii*) 3·535 A
3. 25Ω
4. 637 Hz
5. 112 V
6. 3·18 mΩ
7. 0·55 H
8. 100 Ω, 0·318 H
9. (*a*) 4·71 Ω (*b*) 3.77×10^{8} Ω
10. 20 kΩ
11. (*a*) 212·12 Ω (*b*) 2·65 Ω
12. 2·514 Ω, 15·154 Ω
13. 1·696 A
14. 120 Ω, 300 Ω, 274·95 Ω,
15. (*a*) 5·74 A (*b*) 180·52 V (*c*) 86·16 V
16. 0·445 A, 89° 5′ ahead of voltage
17. 2·13 A, 31°5′ logging behind the voltage, 113 V, 188·30 V
18. (*a*) 50 Hz (*b*) 302·45 Ω capacitive
    (*c*) 303·5 Ω (*d*) 0·659 A
    (*e*) 85° 14′ ahead of e.m.f. (*f*) I=0·93 sin (100 π*t*+1·486)
    (*g*) 16·475 V, 209·68 V, 10·355 V (*h*) 0·962 H
19. 1·687 pf
20. 53 pf
21. 0·688 H, 5 more lamps in series or 180 Ω resistance in series
22. 6 more lamps in series or an inductor in series to drop 180 V across it.
23. 28·675 kΩ
24. 800 Ω
25. 44·2 revolutions s^{-1}
26. 0·019 H
27. (*a*) 50 Hz (*b*) 3012·6 Ω
    (*c*) 3012·7 Ω (*d*) 0·727 A
    (*e*) 9·256 V.

28. (*a*) 254·96 Ω (*b*) 0·863 A

29. (*a*) 50 Ω (*b*) 4A
(*c*) current lags behind the voltage by 53·1°.

30. 5·652 V

31. (*a*) 0·1 H (*b*) 30 A

32. 0·2 A

33. 0·2 H

34. 20 mA

35. 50 Ω

36. 800 V

37. (*a*) 318 Hz (*b*) 8·77 A
(*c*) 175·4 V (*d*) 175·4 V

38. (*a*) 96 Ω (*b*) 98·9 Ω
(*c*) 2·21 A (*d*) 76°

39. 0·55 H

40. 48 Ω capacitive, 45°

41. (*a*) 136·5 Ω (*b*) 1·61 A

42. (*a*) 192 Ω
(*b*) 1·14 A, $I=1{\cdot}614\cos(200t-1{\cdot}013)$
(*c*) 0·807 A, 2·421 A, 1·614 A
(*d*) 0, 0, 160 W (*e*) 160 W (*f*) 0·64

43. (*a*) 250 Ω
(*b*) 0·85 A, $I=1{\cdot}2\sin(100\,t+0{\cdot}848)$
(*c*) 2·4 A, 1·2 A, 1·2 A (*d*) 0, 0, 108 W
(*e*) 108 W (*f*) 0·60

44. 0·975 V, 0·650 V

45. (*a*) 0·16 A (*b*) 750 : 1
(*c*) Circuit B (*d*) Same for both=1·28 W

46. (*a*) 64 J, yes it is conserved if R=0
(*b*) 636 Hz
(*c*) (*i*) 0, 0·785 mS, 1·57 mS, 2·35 mS...
(*ii*) 0·39 mS, 1·17 mS, 1·95 mS...
(*d*) 0·195 mS, 0·585 mS, 0·975 mS......
(*e*) whole of the energy.

47 (*a*) 143 Hz (*b*) 2000 W
(*c*) 27·5 Hz and 18·5 Hz, 10 A (*d*) 5

48. (*a*) 140 V (*b*) 2200 W, 1400 W 63·6 %

49. (*a*) 120 V (*b*) 3600 W, 1440 W, 40%

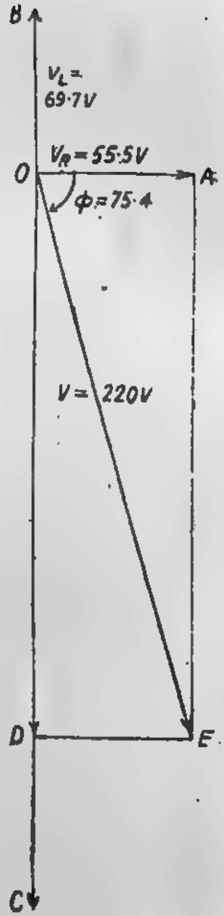
50. 205·8 MW

51. 85%

52. (*a*) 200 KW (*b*) 860 KW
(*c*) 1100 V—13600 V, step up transformer
(*d*) percentage power loss greatly roduced.

53. (*a*) 50 c/s capacitive (*b*) 96 Ω
(*c*) 99·2 Ω (*d*) 2 22 A.
(*e*) The current leads the voltage by 75·4° or 1·31 rad
(*f*) $I = 3{\cdot}13 \sin(314\,t + 1{\cdot}31)$
(*g*) $V_c = 286{\cdot}6$ V, $V_L = 69{\cdot}7$ V, $V_R = 55{\cdot}5$ V
(*h*) See diagram (*i*) 0·306 H.

54. (*a*) 55. (*c*) 56. (*b*) 57. (*d*) 58. (*c*) 59. (*d*)
60. (*a*) 61. (*a*) 62. (*a*) 63. (*c*) 64. (*a*) 65. (*a*)
66. (*b*) 67. (*b*) 68 (*a*) 69. (*d*) 70. (*b*).



(*h*)

EXERCISE 9

1. (*a*) 30 μA (*b*) 30 μA (*c*) $3{\cdot}39 \times 10^{-11}$ T
2. (*a*) 17·5 μA (*b*) 17·5 μA (*c*) $4{\cdot}95 \times 10^{-11}$ T
3. 700 μf 4. 42 42 μA
5. 10 MHz, 600 Vm^{-1}
6. (*a*) 1·2 m (*b*) 8×10^{-8} T
7. (*a*) 6000 A° (*b*) 7×10^{-8} T
8. 40 km 9. 500 m
10. 1, 40, 80, 000
11. (*a*) 48 km (*b*) 1,52,06,400 (*c*) 720 m
12. (*a*) 138·89 μf, $5\,76 \times 10^{8}$ Vs^{-1}
(*b*) 0·08 A
(*c*) (*i*) 0 (*ii*) $0{\cdot}64 \times 10^{-7}$ T (*iii*) 1×10^{-7} T
(*d*) (*i*) 2×10^{-7} (*ii*) $1{\cdot}6 \times 10^{-7}$ T (*iii*) 1×10^{-7} T
13. (*a*) 125 μf, $1{\cdot}6 \times 10^{9}$ Vs^{-1} (*b*) 0·2 A
(*c*) (*i*) 0 (*ii*) $0\,76 \times 10^{-7}$ T, (*iii*) $2{\cdot}5 \times 10^{-7}$
(*d*) (*i*) 5×10^{-7} T (*ii*) 4×10^{-7} T (*iii*) $2{\cdot}5 \times 10^{-7}$ T
14. 207 K 15. 5178 K, 5176 K
16. 3·625 micron 17. $3{\cdot}68 \times 10^{-8}$ T
18. $4{\cdot}76 \times 10^{-11}$ T 19. $4{\cdot}77 \times 10^{-18}$ 10 m^{-2}
20. 10^{-15} T, $1{\cdot}19 \times 10^{-10}$ Wm^{-2}
21. (*d*) 22. (*a*) 23. (*a*) 24. (*b*) 25. (*c*) 26. (*a*)
27. (*a*) 28. (*b*) 29. (*d*) 30. (*c*). 31. (*b*)

EXERCISE 10

1. (a) 3×10^{17} Hz, 3×10^{11} MHz
   (b) 3.33×10^{-18} s, 3.33×10^{12} μs
2. (a) 7.5×10^{-16} m, 7.5×10^{-6} A°
   (b) 2.5×10^{-24} s, 2.5×10^{-38} μs
3. 5000 KHz
4. 20 m
5. (a) 5×10^{14} Hz (b) 1.935×10^{8} ms^{-1}
   (c) 3870 A°
6. 6.89×10^{14} Hz, 3270 A°, 2602 A°
7. 9 : 4
8. 25 : 9
9. 0·5 mm
10. 6000 A°
11. 0·7 mm
12. 0·18 mm
13. 0·24 mm
14. 0·193 mm
15. (a) 0·2870 micron (b) 0·1435 micron
16. (a) 6933 A°, 6118 A°, 5474 A°, 4952 A°, 4522 A°, 4160 A°
    (b) 7429 A°, 6500 A°, 5778 A°, 5200 A°, 4727 A°, 4333 A°, 4000 A°.
17. 2·4 micron
18. 7·4 fringe
19. (a) 30° (b) 42° 21′, 19°3′
20. 4×10^{8} Wm^{-2}
21. 2.3×10^{-4} rad, 8.464×10^{9} m^{2}
22. (a) 0.8×10^{-10} s (b) 4×10^{4}
23. (a) 0·3 mm (b) 1·6 mm
24. 10^{-4} rad
25. 5460 A°
26. 9 : 1
27. 10^{-1} cm
28. 2×10^{8} ms^{-1}, 4×10^{-7} m
29. 5892 A°
30. 9 : 1
31. 5620 A°
32. 0·22 mm
33. (a) 3·5 m (b) 60 m
34. (a) 4×10^{6} (b) 8×10^{-11} s
35. 14°
36. (a) 4·4175 mm (b) 5·39 mm
37. (a) 6250 A° (b) 1 mm

38. (b)	39. (a)	40. (b)	41. (a)	42. (a)	43. (c)
44. (d)	45. (c)	46. (d)	47. (b)	48. (a)	49. (b)
50. (d)	51. (c)	52. (a)	53. (c)	54. (b)	

55. 2 : 1
56. 4000 A°, 5×10^{14} Hz
57. (c)
58. (c)

59. (d)	60. (a)	61. (c)	62. 3×10^{19}	63. 7·4 fringe	
64. (b)	65. (a)	66. (c)	67. (b)	68. (b)	69. (b)
70. (a)	71. (c)	72. (c)	73. (d)	74. (c)	75. (b)
76. (d).					

EXERCISE 11

1. 5 metre
2. 133·3 cm from 64 cd lamp
3. 36%
4. 3·23 m
5. 775 r p.m.
6. 2.976×10^8 ms^{-1}
7. 60°
8. 60°
9. (*a*) 60 cm, 6 cm, real image, 3 (*b*) 21 cm
10. 24 cm
11. 22·5 cm, 5 cm × 15 cm
12. (*a*) 27 cm (*b*) 27 cm
13. 6 mm
14. 37·5 cm
15. (*a*) 36 cm, 2·4 cm, real image
    (*b*) 0·8 (*c*) 6 cm towards mirror
16. 50 cm
17. 24 cm, 8 cm behind the mirror
19. (*a*) 26°18′ (*b*) 36°6′ (*c*) 38°42′
20. 2
21. 37°40′
22. 30°43′
23. 40 cm
24. $\mu = \frac{1}{\sin C}$, 1·414
25. 45°
26. 1·414
27. 1·556
28. 60°20′
29. 51°12′
30. 0·13%
31. 1·6384
32. 52°
33. 18·6°, 48·6°
34. 0·12%
35. 20 cm, virtual image $m=5$
36. 30 cm or 60 cm
37. 15 cm for real image, 5 cm for virtual image
38. Concave lens, 24 cm, 8 cm
39. 4 cm
40. 8·2 cm
41. 12 cm
42. 2·454 cm
43. 24 cm
44. 1·6
45. 5·60 m
46. The radius of curvature of the concave mirror in first case, $2 \times 25 = 50$ cm will be the focal length of the plano convex lens case. The radius of curvature of the concave mirror in second $2 \times 9 = 18$ cm will be the radius of curvature of the concave surface of the lens. So $\mu = 1.56$.
47. 1·5
48. 15 cm, 1·33
49. +4 diopter, 25 cm.
50. +1 diopter, 100 cm.
51. Convex lens, $f = +9.38$ cm, (P = +10·67 diopter.)
52. −1 diopter.
53. −2 diopter.
54. −1 diopter.
55. (*a*) +2·0 diopter. (*b*) −0·667 diopter.
56. +4 diopter
57. 10.

58. (*a*) 487·5 (*b*) 519·75
(*c*) 553·5.

59. 20, 42 cm.

60. 16 cm, 2 cm.

61. $m=\frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)=17\cdot5$

62. 15, 64 cm

63. $m=\frac{V}{v}=\frac{800/19}{200/33}=6\cdot95.$

$L=(V+v)=48\cdot16$ cm.

64. 0·019, 0·03.

65. 0·0289″.

66. 60 cm. 67. (*c*) 68. 15 cm. 69. (*c*) 70. (*d*) 71. (*c*) 72. (*a*)
73. (*d*) 74. (*c*) 75. (*a*) 76. (*b*) 77. (*a*) 78. (*d*) 79. (*b*)
80. (*c*) 81. (*b*) 82. (*c*) 83. (*a*) 84. (*d*) 85. (*d*) 86. (*c*) 87. (*d*)
88. (*a*) 89. (*c*) 90. (*a*) 91. (*d*) 92. (*b*) 93. (*a*) 94. (*d*)
95 (*b*) 96. (*a*) 97. (*d*) 98. (*d*) 99. (*a*) 100. (*d*) 101. (*c*)
102. (*c*) 103. (*c*) 104. (*b*) 105. (*b*) 106. (*d*) 107. (*a*)
108. (*b*) 109. (*a*) 110. (*b*) 111. (*a*) 112. (*a*) 113. (*b*)
114. (*d*) 115. (*b*) 116. (*a*) 117. (*b*) 118. (*a*) 119. (*d*)
120. (*a*) 121. (*d*) 122. (*b*) 123. (*a*)

EXERCISE 12

1. 2.5×10^7 ms^{-1}
2. 114Å, 3 spectral lines
3. $1\cdot47\times10^{-9}$ m
4. 8×10^{-18} J
5. $8\cdot433\times10^7$ ms^{-1}
6. $4\cdot14\times10^{-14}$ m
7. (*a*) $2\cdot18\times10^6$ ms^{-1} (*b*) $-13\cdot6$ eV (*c*) 13·6 eV
8. 0·53 Å
9. 4·77 Å
10. 182·8 eV
11. 12·09
12. (*a*) $9\cdot428\times10^7$ ms^{-1} (*b*) $6\cdot03\times10^{18}$ Hz (*c*) 0·4975 Å
13. 0·45 Å
14. 2·29 cm^{-1}
15. $6\cdot629\times10^{-34}$ Js
16. $8\cdot2875\times10^{-15}$ J
17. (*a*) $1\cdot2\times10^{16}$ Hz (*b*) $1\cdot8\times10^{16}$ Hz
(*c*) 6×10^{16} Hz
18. (*a*) $3\cdot315\times10^{-19}$ J (*b*) $7\cdot956\times10^{-19}$ J
(*c*) $9\cdot945\times10^{-19}$ J (*d*) $6\cdot63\times10^{-27}$ J
(*e*) $1\cdot326\times10^{-29}$ J
19. (*a*) 2·26 eV (*b*) 1·66 eV;
(*c*) 7·77 kV
20. (*a*) $4\cdot34\times10^{14}$ Hz (*b*) $5\cdot55\times10^{14}$ Hz
(*c*) $8\cdot2\times10^{14}$ Hz
21. 4·8 eV ; 2·48 eV

22. (*a*) 6.63×10^{-24} kg ms^{-1}, same for the both
(*b*) 7.36×10^{6} ms^{-1}, 3.96×10^{3} ms^{-1}
(*c*) 2.413×10^{-17} J, 1.312×10^{-20} J

23. 5.56×10^{14} Hz.
24. 122.7 Å, 0.066 Å
25. 2.388×10^{-18} *c*
26. 0.55 Å
27. 3603 Å
28. 2.22×10^{6} ms^{-1}
29. (*a*) 8×10^{14} Hz (*b*) 1.32×10^{-19} J
30. (*a*) 50.56 cm (*b*) 1.7×10^{11} *c* kg^{-1}
31. (*a*) 2.8×10^{-6} m (*b*) 4.15×10^{-18} *c*
32. (*a*) 1.61×10^{-6} m (*b*) 1.41×10^{-14} kg
(*c*) 0.23
33. (*a*) 1.46×10^{-10} m (*b*) 23.3
34. 3.06×10^{-12} m
35. 0.39 A°
36. (*a*) 3×10^{-20} J (*b*) 6.6×10^{-19} J
(*c*) $\simeq$2 eV
37. 3×10^{-19} J
38. 10^{22}
39. 1.1 A°
40. 6.517×10^{5} ms^{-1}
41. (*a*), (*c*)
42. False
43. False
44. A
45. (*b*) 46. (*c*) 47. (*a*), (*d*) 48. 10.31×10^{15} Hz
49. (*c*) 50. $ML^{2}T^{-1}$ 51. (*a*) 52. (*c*) 53. (*a*)
54. (*b*) 55. (*d*) 56. (*a*) 57. (*c*) 58. (*b*) 59. (*a*)
60. (*a*) 61. (*c*) 62. (*c*) 63. (*a*) 64. (*c*) 65. (*b*)
66. (*a*) 67. (*c*) 68. (*c*) 69. (*c*) 70. (*d*) 71. (*a*)
72. (*c*) 73. (*d*) 74. (*a*) 75. (*d*) 76. (*b*) 77. (*a*)

EXERCISE 13

1. (*a*) 1, 1, 2 (*b*) 2, 2, 2
(*c*) 20, 20, 24 (*d*) 47, 47, 60
(*e*) 90, 90, 142
2. (*a*) 6, 7 (*b*) 8, 10
(*c*) 16, 18 (*d*) 27, 32
(*e*) 83, 126 (*f*) 6, 6 (*g*) 92, 146
3. 7.7 MeV
4. (*a*) 5.696×10^{-15} m (*b*) 4.236×10^{-15} m
(*c*) 2.746×10^{-15} m
5. 7×10^{-15} m
6. 2.34×10^{17} kg m^{-3}
7. 291.7 MeV, 8.58 MeV
8. (*a*) 361.7 MeV, 8.61 MeV (*b*) 922.61 MeV, 8.545 MeV
(*c*) 76.165 MeV, 6.924 MeV
9. (*a*) 7.018 amu (*b*) 238.13 amu

10. $4{\cdot}0303$ MeV
11. $5{\cdot}5$ MeV
12. $14{\cdot}003324$ amu
13. (a) $_0n^1$ (b) $_1H^2$
(c) $_2He^4$ (d) $_1e^0$
(f) $_6C^{12}$
14. 15, 30, Phosphorous $_{15}P^{30}$
15. 12, 24, Magnesium ($_{12}Mg^{24}$)
16. 3125×10^{11}
17. 1433 kW
18. $247{\cdot}5$ g
19. β, α, α, β, α
20. $1{\cdot}76 \times 10^{-8}$ s, $2{\cdot}837 \times 10^7$ Hz
21. $1{\cdot}02$ MeV, $2{\cdot}46 \times 10^{20}$ Hz
22. 224 MeV, 112 MeV
23. $2{\cdot}49$ MeV
24. $2{\cdot}22$ MeV, $5{\cdot}36 \times 10^{20}$ Hz
25. α, β, β, α, α
26. $7{\cdot}5 \times 10^{14}$ Hz, $4{\cdot}97 \times 10^{-19}$ J
27. $7{\cdot}68$ MeV
28. $4{\cdot}783$ MeV
29. 9×10^{16} J
30 $\frac{1}{16}$
31. 11400 years
32. 560 days
33. one α-particle, one β-particle
34. 10^{-15} m
35. 11 electrons, 11 protons and 13 neutrons
36. 91 and 234
37. $1337{\cdot}6$ years
38. $16{\cdot}425$ days
39. $303{\cdot}9$Å
40. 3×10^{18} fissions/sec.
41. $2{\cdot}5 \times 10^8$ fissions/sec.
42. (a) $0{\cdot}098940$ amu. (b) $92{\cdot}1$ MeV (c) $7{\cdot}68$ MeV
43. 12 hours
44. $1{\cdot}3 \times 10^{-15}$ m
45. $39{\cdot}2$ MeV
46. $2{\cdot}22$ MeV
47. 4224 years
48. (a) $0{\cdot}023$ per day (b) 90 days.
49. (a) $1{\cdot}389 \times 10^{-11}$ per s (b) 2282 years
(c) $1581{\cdot}4$ years
50. $962{\cdot}9$ years.
51. (b), (c)
52. (d)
53. 8, 9
54. Lithium, 7
55. atomic number, mass number
56. (c) 57. (d) 58. (b) 59. (b) 60. $23{\cdot}6$ MeV
61. The reaction number two is incorrect because it violets the law of conservation of nucleons
62. $_0n^1$ 63. (d) 64. (a) 65. (c) 66. (b) 67. (a)
68. (a) 69. (b) 70. (a) 71. (b) 72. (d) 73. (a)
74. (a) 75. (b) 76. (b) 77. (a) 78. (b) 79. (b)
80. (b) 81. (c) 82. (c) 83. (b) 84. (a) 85. (c)
86. (c) 87. (c) 88. (b) 89. (d).

EXERCISE 14

1. 1
2. (*a*) 2, (*b*) 4
3. 4·293 Å
4. 4·07 Å
5. 1·27 Å
6. (*a*) 2·48Å (*b*) 2·86A°
7. $6·18 \times 10^{22}$, $4·48 \times 10^{-23}$ g
8. $2·77 \times 10^{22}$
9. 2700 kg m^{-3}
10. (*a*) 4·1A° (*b*) 1·45 A°
11. 1·594% decrease
12. $9·032 \times 10^{20}$
14. 2585·966 kg m^{-3}
17. 4
18. 4A°
19. 4
20. 12
21. $r = \sqrt{2}\frac{a}{4}$
22. $\frac{\pi}{6}$
24. 3·6 A°, 9038·8 kg m^{-3}
25. (*d*)
26. (*a*)
27. (B)
28. (*d*)
29. (*a*)
30. (*a*)
31. (*c*)
32. *p*-type
33. rise in
34. conduction band, forbidden gap
35. Energy bands
36. 7 eV
37. insulator, OK
38. 1 eV
39. conduction, overlap
40. ionic
41. metallic
42. weakest
43. electrical, thermal, metallic
44. 1, 2, 4
45. 2
46. 6, 8, 12
47. 52·4%, 68%, 74%
48. Coordination number and atomic packing factor
49. (*c*)
50. (*b*)

EXERCISE 15

1. 0·06 mA
2. 13·75 mA
3. 65·6
4. 0·98
5. 0·818
6. 20 kΩ
7. 60
8. 0·1 m A
9. 7500
10. 5264
11. 2751·84 sm^{-1}, 0·00036 Ωm
12. 275 sm^{-1} (>>2·18 sm^{-1} for pure Ge)
    0·0036Ωm(<<0·46 Ωm for pure Ge)
13. 237·5 Ω
14. 3·6 kΩ
15. 12·5 mA
16. 25
17. $1·38 \times 10^{22}$ m^{-3}
18. (*a*) 16 mW, 4 mW (*b*) 4·75 volt
19. (*a*) $\beta = 100$, $R_i = 2k\,\Omega$, $g_m = 0·05\Omega^{-1}$ (*b*) 300

20. 600 21. 2·469 mA
22. $\alpha=0·993$, $\beta=150$
23. $I_C=1·848$ mA, $I_B=0·152$ mA
24. $R_{a.c}=40\ \Omega$ $R_{d.c}=20\ \Omega$
25. 36·6 μA 26. 500 μA
27. $R_L=0·6$ k Ω, $R=83$ k Ω
28. $I_C=2·5$ mA, $I_B=25\ \mu$A and $B=100$
29. $V_{d.c.}=99$ V, $V_{a.c.}=119\ 75$ V 30. $V_{d.c}=18$ V, $VX_{a.c.}=8·6$ V
31. 80, 66·7, 63·3, 60 32. 30625
33. -2 V, forward biased, reverse biased
34. 2·5, 125, 9000, 1·8 V 35. (*a*)
36. (*b*) 37. (*b*) 38. (*d*) 39. (*d*) 40. (*c*) 41. (*a*)
42. (*b*), (*c*) 43. (*a*), (*d*) 44. (*b*) 45. (*c*) 46. (*b*)
47. (*b*) 48. (*c*) 49. (*c*) 50. (*b*).

EXERCISE 16

1. (*a*) $1·635\times10^{19}$ N (*b*) 24·2 kms^{-1}
2. $6·39\times10^{6}$ m 3. 1·7 ms^{-2}
4. 2·935 ms^{-2} 5. $T=\dfrac{2\pi R}{R\sqrt{\frac{4}{3}\pi\rho G}}=\sqrt{\dfrac{3\pi}{\rho G}}$ $=1$ hr. 25 min. 16 S
6. $7·35\times10^{22}$ kg
7. 11·52 kg 8. 688·7 days
9. 29·4 ms^{-2} 10. $6·06\times10^{24}$ kg
11. $4·48\times10^{8}$ m 12. 164·3 years
13. 2×10^{30} kg 14. 129 days
15. 29·46 years 16. $7·8\times10^{11}$ m, $14·3\times10^{11}$ m
17. 684·47 days, 223·15 days
18. 0·723 A.U. ($=1·0845\times10^{11}$ m) ;
0·6909 A.U.($=1·03635\times10^{11}$ m)
19. $1 : 6·31\times10^{12}$
20. (*a*) 10^{5} km/s (*b*) $3·17\times10^{22}$ km
21. $6·96\times10^{24}$ kg, $7·45\times10^{22}$ kg
22. 1200 K 23. $1·3\times10^{9}$ m
24. $x=\dfrac{1}{\phi}=5·5\times10^{5}$ A.U. 25. 2·5 km s^{-1}
26. (*a*) 49·4 W m^{-2} (*b*) 51·95 wm^{-2}
27. 5780 K 28. 4 : 1
29. 6820 K 30. 10,000 ; 100
31. 10^{31} kg. approx. 32. 1380 W m^{-2}

33. 5800 K

34. $(M_1+M_2)=\frac{a^3}{T^2}=3{\cdot}41$ of solar mass

35. $3{\cdot}844\times10^8$ m 36. $9{\cdot}52\times10^8$ m

37. $122{\cdot}5:1$

38. 10 billion light years$=8{\cdot}5\times10^{24}$ m

39. $2{\cdot}85\times10^{30}$ kg 40. $6{\cdot}407$ of Solar mass

41. $11{\cdot}75\times10^{10}$ of Solar mass$=2{\cdot}35\times10^{41}$ kg

42. 8×10^9 light year

43. (*a*) $3{\cdot}6\times10^4$ km s^{-1} (*b*) 1200 million light years

44. $1{\cdot}5\times10^4$ km/s 45. (*a*)

46. (*a*) 47. (*c*) 48. (*b*) 49. (*b*) 50. (*a*) 51. (*d*)

52. (*a*) 53. (*c*) 54. (*c*) 55. (*a*) 56. (*b*) 57. (*a*)

58. (*b*) 59. (*d*) 60. (*b*) 61. (*a*) 62. (*c*) 63. (*b*)

64. (*d*) 65. (*b*) 66. (*d*) 67. (*a*) 68. (*c*) 69. (*c*)

70. (*a*) 71. (*b*) 72. 8T 73. (*c*) 74. (*a*) 75. (*b*)

76. (*d*) 77. $1{\cdot}273\times10^{-8}$ rad s^{-1} 78. energy

79. $5\frac{1}{6}$ hrs.

80. False, energy radiated per Sec.$=\sigma A\,T^4$,

$$E_1=\sigma4\pi(1)^2\,(4000)^4$$
$$E_2=\sigma4\pi(4)^2\,(2000)^4$$

So both spheres will radiate same energy per second.

□ □

C.B.S.E. EXAMINATION PAPERS

PHYSICS, 1990

(Held in Kendriya Vidyalyas)

Paper I

1. A 900 pF capacitor is charged by a 100 V battery. How much electrostatic energy is stored by the capacitor ? 2

2. A series LCR circuit with $L=0{\cdot}12$ H, $C=4{\cdot}8\times10^{-7}$ F, $R=23\Omega$ is connected to a variable frequency supply. At what frequency the current is maximum ? 2

3. In a plane electromagnetic wave the electric field oscillates sinusoidally with a frequency of $2{\cdot}0\times10^{10}$ Hz and amplitude 48 Vm^{-1}. What is the wavelength of the wave ? 2

4. Red light of wavelength 6500 A from a distant source falls on a slit 0·5 mm wide. What is the distance between the two dark bands on each side of the central bright bands of the diffraction pattern observed on a screen placed 1·8 m from the slit ? 2

5. An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0·66 A. What is the maximum energy of a photon in the radiation ? ($h=6{\cdot}6\times10^{-34}$ JS) 2

6. An object placed 45 cm from a lens forms an image on a screen placed 90 cm. on the other side of the lens. Identify the type of the lens and find its focal length. 3

7. Calculate the binding energy of an alpha particle in MeV, given that

   m_p (mass of proton) $=1{\cdot}007825\ u$

   m_n (mass of neutron) $=1{\cdot}008665\ u$

   Mass of helium nucleus $=4{\cdot}002800\ u$ 3

   $(1u=931$ MeV)

8. A circuit with $R=70\ \Omega$ in series with a parellel combination of $T=1{\cdot}5$ H and $C=30\ \mu F$ is driven by 230 V supply of angular frequency 300 rad s^{-1}.

   (*i*) Find the impedance of the circuit.

   (*ii*) What is the rms value of the total current ?

   (*iii*) What are the current amplitudes in L and C arms of the circuit ? 4

Paper II

1. The magnetic flux threading a coil changes from 12×10^{-3} wb to 6×10^{-3} wb in 0·01 S. Calculate the induced e.m.f. 1
2. Calculate the frequency associated with a photon of energy $3{\cdot}3\times10^{-20}$ J ($h=6{\cdot}6\times10^{-34}$ JS) 1
3. The total energy of an electron in the first excited state of hydrogen atom is $-3{\cdot}4$ eV. What is its Kinetic energy ? 1
4. The e.m f. of a Cu—Fe thermocouple varies with the temperature θ of the hot junction (cold junction at 0°C) as
$$E\,(\mu V)=14\,\theta-0{\cdot}02\,\theta^2$$
Determine the neutral temperature. 2
5. Monochromatic X-rays, when reflected from a crystal with lattice spacing 2·0 A, produce first order diffraction maximum at $\theta=30°$. What is the wavelength of X-rays ? 2
6. The charging current for a capacitor is 0·25 A. What is the displacement current a cross its plates ? 2
7. Calculate the distance a beam of light of wavelength 500 nm can travel without significant broadening, if the diffracting aperture is 3 mm wide.
8. Pure Si at 300 K has equal electron (n_e) and hole (n_h) concentrations of $1{\cdot}5\times10^{16}$ m^{-3}. Doping by indium increases n_h to $4{\cdot}5\times10^{22}$ m^{-3}. Calculate n_e in the doped silicon. 3

ANSWERS

Paper I

1. $4{\cdot}5\times10^{-6}$ J
2. 663 Hz
3. 1·5 cm.
4. 4·68 mm.
5. 3×10^{-15} J
6. $f=30$ cm., convex lens.
7. 28·098 MeV
8. (*i*) 163·3 Ω (*ii*) 1·41 A (*iii*) $I°_L=0{\cdot}654$ A, $I°_C=2{\cdot}65$ A.

Paper II

1. 0·6 V
2. 5×10^{13} Hz
3. 1·7 eV
4. 700°C
5. 2 A
6. 0·25 A
7. 18 m
8. 5×10^{9} m^{-3}

Appendix

TABLES

1. PHYSICAL CONSTANTS

Symbol	*Name*	*Magnitude*
c	Speed of light	3×10^{8} ms^{-1}
h	Planck's constant	$6 \cdot 63 \times 10^{-34}$ Js
e	Electron charge	$1 \cdot 602 \times 10^{-19}$ coulomb
m_e	Electron mass	$9 \cdot 107 \times 10^{-31}$ Kg.
m_n	Neutron mass	$1 \cdot 67479 \times 10^{-27}$ Kg $= 1 \cdot 008665$ amu
m_p	Proton mass	$1 \cdot 67239 \times 10^{-27}$ Kg $= 1 \cdot 007276$ amu
m_H	Hydrogen atom mass	$1 \cdot 67330 \times 10^{-27}$ Kg $= 1 \cdot 007825$ amu.
N	Avogadro's number	$6 \cdot 023 \times 10^{26}$ Kg mol^{-1}
R	Gas constant	$8 \cdot 314$ J mol^{-1} K^{-1}
μ_0	Permeability of free space	$4\pi \times 10^{-7}$ Wb A^{-1} m^{-1}
ϵ_0	Permitivity of free space	$8 \cdot 854 \times 10^{-12}$ C^{2}N^{-1} m^{-2}
—	Hydrogen atom radius	$5 \cdot 29 \times 10^{-11}$ m
r_0	Hydrogen nucleus radius	$1 \cdot 2 \times 10^{-15}$ m
$\frac{1}{4\pi\epsilon_0} = K$	Coulomb's constant	9×10^{9} Nm^2 C^{-2}

2. PREFIXES FOR UNIT

Prefix	*Power of ten*	*Abbreviation*
Atto	10^{-18}	a
Femto	10^{-15}	f
Pico	10^{-12}	p
Nano	10^{-9}	n
Micro	10^{-6}	μ
Milli	10^{-3}	m
Centi	10^{-2}	c
Kilo	10^{3}	K
Mega	10^{6}	M
Giga	10^{9}	G
Tera	10^{12}	T

3. CONSTANTS

$\pi = 3{\cdot}1416$, $\log_{10} \pi = 0{\cdot}4971$

1 radian $= 57{\cdot}296$ degree

$e = 2{\cdot}7183$, $\log_{10} e = 0{\cdot}4343$

$\log_e N = 2{\cdot}3026 \log_{10} N$;

$\log_{10} N = 0{\cdot}4343 \log_e N$

$\sqrt{\pi} = 1{\cdot}7724$;

$\pi^2 = 9{\cdot}8696$

4. THE GREEK ALPHABETS

Name	*Symbol*	*Name*	*Symbol*
Alpha	α	Xi	ξ
Beta	β	Pi	π
Gamma	γ	Rho	ρ
Delta	δ or Δ	Sigma	σ, Σ
Epsilon	ϵ	Tau	τ
Zeta	ζ	Upsilon	υ
Eeta	η	Phi	ϕ
Theta	θ	Psi	ψ
Iota	ι	Omega	ω
Kappa	K	Omcrion	o
Lambda	λ	Chi	χ
Mu	μ		
Nu	ν		

5. DENSITIES OF COMMON SUBSTANCES (g./c.c.)

Substance	*Density*	*Substance*	*Density*	*Substance*	*Density*
Solids		*Liquids*		*Gases*	
Aluminium	2·7	Alcohol	0·80	Air	·00129
Brass	8·6	Benzene	0·88	Carbon dioxide	·00198
Copper	8·89	Ether	0·74	Helium	·000179
Glass (Crown)	2·6	Glycerine	1·26	Hydrogen	·000090
Glass (Flint)	4·0	Lubricating oil	0·91	Steam (100°C)	·00061
Gold	19·3	Mercury	13·60		
Iron (cast)	7·5	Methylated spirit	0·83		
Iron (wrought)	7·9	Turpentine	0·87		
Lead	11·34				
Platinum	21·45				
Silver	10·5				
Steel	7·8				
Zinc	7·1				

6. RESISTOR COLOUR CODE

Colour	*Number*	*Multiplier*	*Tolerance %*
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5
Silver		10^{-2}	10
No colour	–		20

7 ELECTRICAL RESISTIVITIES OF SOME SUBSTANCES

Material	*Resistivity* (Ωm) *at 0°C* ρ	*Temprature coefficient* $(°C)^{-1}$ $\frac{1}{\rho}\left(\frac{d\rho}{dT}\right)$	*Number of outer (valence) electrons per unit cell*
A. Conductors			
Silver	$1{\cdot}6\times10^{-8}$	0·0041	1
Copper	$1{\cdot}7\times10^{-8}$	0·0068	1
Aluminium	$2{\cdot}7\times10^{-8}$	0·0043	3
Tungsten	$5{\cdot}6\times10^{-8}$	0·0045	6
Iron	10×10^{-8}	0·0065	8
Platinum	11×10^{-8}	0·0039	10
Mercury	98×10^{-8}	0·0009	2
Nichrome (alloy of Ni, Fe, Cr)	100×10^{-8}	0·0004	
B. Semiconductors			
Carbon	$3{\cdot}5\times10^{-5}$	−0·0005	4
Germanium	0·46	−0·05	4
Silicon	2300	−0·07	4
C. Insulators			
Glass	$10^{10}-10^{14}$		?
Hard Rubber	$10^{13}-10^{16}$		?
NaCl			8

8. PROPERTIES OF Si AND Ge AT 300K

	Si	*Ge*
Energy gap E_g (eV)	1·1	0·7
Electron mobility μ_n ($m^2V^{-1}s^{-1}$)	0·135	0·39
Hole mobility μ_p ($m^2V^{-1}s^{-1}$)	0·048	0·19
Intrinsic carrier concentration $n_i (m^{-3})$	$1{\cdot}5\times10^{16}$	$2{\cdot}4\times10^{19}$
Intrinsic conductivity σ (S_m^{-1})	$4{\cdot}4\times10^{-4}$	2·18
Intrinsic resistivity (Ωm)	2300	0·46
Density (gm^{-3})	$2{\cdot}3\times10^{6}$	$5{\cdot}32\times10^{6}$
Concentration of atoms (m^{-3})	5×10^{28}	$4{\cdot}41\times10^{28}$

9. PHYSICAL PROPERTIES OF THE OBJECTS IN THE SOLAR SYSTEM

Object	*Period of revolution in years*	*'a' in A.U.*	*radius R (Earth)*	*Rotation period*	*Mass M(Earth)*	*Mean density (kg/m × 10^{-3})*	*g/g(Earth)*	*Temperature (Degree C)*	*Albedo*	*Surface pressure in atmospheres*	*Atmospheric chemical composition*	*No. of satellites etc.*
Moon	—	—	0.27	d 27.32	0.0123	3.34	0.170	+110 (day) −150 (night)	0.07	0	Vacuum	—
Mercury	0.241	0.387	0.38	d 58.6	0.056	5 4	0.367	+340 (day) −120 (night)	0.06	0	Vacuum	0
Venus	0.615	0.723	0.96	d 243.0 (retrograde)	0 815	5.1	0.886	+480 (surface) −40 (clouds)	0.85	100	Co_2 (95%)	0
Earth	1.000	1.000	1.10	23p56m.1	1.000	5.52	1.000	+45 (Equator) −50 (Poles)	0·40	1	N_2 (80%) O_2(20%)	1
Mars	1.881	1.524	0.53	24h27m.4	0.107	3.97	0.383	+30 (Eq.) −130 (Poles)	0.15	0.006	CO_2(97%) N_2(3%)	2
Cares (Largest Asteroid)	4.603	2.767	0.055	90h05m	0.0001	3.34	0.18	—	0.07	0	Nil	++
Jupiter	11.864	3.203	11.23	9h50m5	317.9	1.33	2.522	−140(Clouds)	0.45	—	H_2, He, CH_4, NH_3	14
Saturn	29.46	9.540	9.41	10h14m	95.2	0.70	1.074	−175(Clouds)	0.61	,,	H_2, He, CH_4, NH_3	10+ring
Uranus	84.01	19.18	3.98	10h49m (retrograde)	14.6	1.33	0.922	−220(Clouds)	0.35	,,	H_2, He, CH_4	5+ring
Neptune	164.6	30.01	3.88	15h	17.2	1.66	1.435	−230(Clouds)	0.35	,,	H_2, He, CH_4	2
Pluto	247	39.44	1.5	d 6.39	0.1		0.440	−240 ?	0.14	?	?	Nil

Notes : 1 year = 365.257 days ; 1 A.U. = 1.496×10^{8} Km ; R (Earth) = 6378 Km ; M (Earth) = 5.977×10^{27} g ; g (Earth) = 9.82 m/sec^2 ;
+Comet Nucleur diameter 10 Km, head 10000 Km ; ++Total number of asteroids > 1600.
d = day h = hour m = minute

10. PROPERTIES OF STARS OF VARIOUS SPECIAL TYPES

Spectral type	*Colour*	*Example*	*Description of Spectrum*	*Temperature*	$\frac{M}{M\,(Sun)}$	$\frac{R}{R\,(Sun)}$	*Absolute Magnitude MV*	$\frac{L}{L\,(Sun)}$
O	Very Blue	θ Orionis	Lines of ionised Helium	35000	40	20	−6·0	10^5
B	Blue	Spica (Chitra)	Lines of neutral Helium	20000	15	7	−3·1	10^4
A	White	Sirius A (Vyadha)	Balmer lines of Hydrogen	9500	2·3	1·8	+1·4	25
F	Green	Procyon	Lines of Hydrogen and ionised metals	7000	1·4	1·2	+2·7	3
G	Yellow	Sun (Surya)	Lines of ionised and natural metals	5800	1·0	1·0	+4·8	1
K	Orange	Σ Eridani	Lines of neutral metals and molecular bands	4500	0·7	0·8	+6·1	0·4
M	Red	Kruger 60	Molecular bands of Tio	3500	0·3	0·4	+11·8	1/40
M	Red	Betelgeuse (Ardra)	Tio band	3000	20	220	−6	5×10^4
A	White	Sirius B	Broad lines of hydrogen	9500	1·0	1/50	+11·5	1/200

APPENDIX
Logarithm Tables
LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						5	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	16	20	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	18	22	26	29	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	6	10	13	16	19	23	26	29
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	14	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	6	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	3	5	8	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	17	19	21
						2672	2695	2718	2742	2765	2	4	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	[illegible]	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4712	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	[illegible]	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	[illegible]	7	8	9
45	6832	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

ANTILOGARITHMS

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·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 2	2 2 2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1	1 2 2	2 3 3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1	1 2 2	2 3 3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1	1 2 2	2 3 3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1	1 2 2	3 3 3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1	2 2 2	3 3 3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1	2 2 2	3 3 3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0 1 1	2 2 3	3 4 4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1 1 2	2 3 3	4 4 5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2285	1 1 2	2 3 3	4 4 5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 3 4	5 6 6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 3 4	5 6 6

ANTILOGARITHMS

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.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

NATURAL SINES

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences 1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	·0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	·0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	·0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	·1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	·1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	·1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	·1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	·1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	·1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	·2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	·2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	·2419	2436	2452	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	·2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	·2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	·2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	·3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	·3256	3273	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	·3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	·3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	·3746	3762	3778	3795	3811	2827	3843	3859	3875	3891	3	5	8	11	14
23	·3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	·4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	·4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	·4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	·4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	·4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	·4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	·5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	·5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	·5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	·5446	5461	6476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	·5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	·5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	·5878	5892	5906	5920	5934	5848	5962	5976	5990	6004	2	5	7	9	12
37	·6018	6032	6046	6060	6074	6088	61·1	6115	6129	6143	2	5	7	9	12
38	·6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	·6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	·6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	·6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	·6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	·6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	·6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences 1	2	3	4	5
45	·7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	·7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	·7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	·7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	·7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	·7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	·7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	·7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	·7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	·8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	·8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	·8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	·8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	·8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	·8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	·8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	·8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	·8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	·8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	·8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	·9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	·9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	·9205	9212	9219	9225	9232	9239	9345	9252	9259	9265	1	2	3	4	6
68	·9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	·9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	·9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	·9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	·9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	·9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	·9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	·9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	·9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	·9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	·9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	·9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	·9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	·9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	·9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	·9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	·9945	9947	9949	9951	9952	8854	9956	9957	9959	9960	0	1	1	1	2
85	·9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	·9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	·9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	·9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	·9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0
90	1·000														

NATURAL COSINES

[Numbers in difference columns to be subtracted, not added.]

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences 1	2	3	4	5
0	1·000	1·000	1·000	1·000	1·000	1·000	9999	9999	9999	9999	0	0	0	0	0
1	·9998	9998	9998	9997	9997	9997	9996	[illegible]	9995	9995	0	0	0	0	0
2	·9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	·9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	·9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	·9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	·9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	·9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	·9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	·9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	·9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	·9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	·9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	·9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	·9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	·9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	·9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	·9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
18	·9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	·9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	·9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	·9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	·9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	·9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
24	·9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	·9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	·8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	·8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	·8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	·8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	·8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	·8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	·8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	·8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	·8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	·8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	·8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
37	·7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	·7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	·7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	·7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	·7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	·7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	·7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	·7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

NATURAL COSINES

[Numbers in difference columns to be subtracted, not added.]

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences 1	2	3	4	5
45	·7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	·6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	·6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	·6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	·6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50	·6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	·6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	·6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	·6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	·5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	·5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	·5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	·5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	·5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	·5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	·5000	4985	4970	4955	4939	4924	4900	4894	4879	4863	3	5	8	10	13
61	·4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	·4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	·4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	·4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	·4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	·4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
67	·3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	14
68	·3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	·3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	·3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	·3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	·3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	·2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	·2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	·2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	·1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
80	1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	·1564	1547	1530	151	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	·1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
83	·1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	·1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	·0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	15
86	·0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	·0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	·0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	·0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15
90	·0000														

NATURAL TANGENTS

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences 1	2	3	4	5
0	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	·0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences 1	2	3	4	5
45	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5830	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	2.0057	2.0145	2.0233	2.0323	2.0413	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2·9042	9208	9375	9544	9714	9887	3.0061	3.0237	3.0415	3.0595	29	58	87	116	145
72	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646	Mean differences cease to be sufficiently accurate.				
78	4·7046	7453	7867	8288	8716	9152	9594	5·0045	5·0504	5·0970					
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5·6713	7297	7894	8502	9124	9758	6·0405	6·1066	6·1742	6·2432					
81	6·3138	3859	4596	5350	6122	6912	7720	8548	9395	7·0264					
[illegible]	7·1154	2066	3002	3962	4947	5958	6996	8062	9158	8·0285					
83	8·1443	2636	3863	5126	6427	7769	9152	9·0579	9·2052	9·3572					
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16.83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23.86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40.92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·5	143.2	191·0	286·5	573·0					
90	∞														

LOGARITHMS OF SINES

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences 1	2	3	4	5
0	$-\infty$	$\bar{3}$·2419	$\bar{3}$·5429	7190	8439	9408	$\bar{2}$·0200	$\bar{2}$·0870	$\bar{2}$·1450	$\bar{2}$·1961					
1	$\bar{2}$·2419	2832	3210	3558	3880	4179	4459	4723	4971	5206					
2	$\bar{2}$·5428	5640	5842	6035	6220	6397	6567	6731	6889	7041					
3	$\bar{2}$·7188	7330	7468	7602	7731	7857	7979	8098	8213	8326					
4	$\bar{2}$·8436	8543	8647	8749	8849	8946	9042	9135	9226	9315	16	32	48	64	80
[illegible]	$\bar{2}$·9403	9489	9573	9655	9736	9816	9894	9970	$\bar{1}$·0046	$\bar{1}$·0120	13	26	39	52	65
[illegible]	$\bar{1}$·0192	0264	0334	0403	0472	0539	0605	0670	0734	0797	11	22	33	44	55
7	$\bar{1}$·0859	0900	0981	1040	1099	1157	1214	1271	1326	1381	10	19	29	38	48
8	$\bar{1}$·1436	1489	1542	1594	1646	1697	1747	1797	1847	1895	8	17	25	34	42
9	$\bar{1}$·1943	1991	2038	2085	2131	2176	2221	2266	2310	2353	8	15	23	30	38
10	$\bar{1}$·2397	2439	2482	2524	2565	2606	2647	2687	2727	2767	7	14	20	27	34
11	$\bar{1}$·2806	2845	2883	2921	2959	2997	3034	3070	3107	3143	6	12	19	25	31
12	$\bar{1}$·3179	3214	3250	3284	3319	3353	3387	3421	3455	3488	6	11	17	23	28
13	$\bar{1}$·3521	3554	3585	3618	3650	3682	3713	3745	3775	3806	5	11	16	21	26
14	$\bar{1}$·3837	3867	3897	3927	3957	3986	4015	4044	4073	4102	5	10	15	20	24
15	$\bar{1}$·4130	4158	4186	4214	4242	4269	4296	4323	4350	4377	5	9	14	18	23
16	$\bar{1}$·4403	4430	4456	4482	4508	4533	4559	4584	4609	4634	4	9	13	17	21
17	$\bar{1}$·4659	4684	4709	4733	4757	4781	4805	4829	4853	4876	4	8	12	16	20
18	$\bar{1}$·4900	4923	4946	4969	4992	5015	5037	5060	5082	5104	4	8	11	15	19
19	$\bar{1}$·5126	5148	5170	5192	5213	5235	5256	5278	5299	5320	4	7	11	14	18
20	$\bar{1}$·5341	5361	5382	5402	5423	5443	5463	5484	5504	5523	3	7	10	14	17
21	$\bar{1}$·5543	5563	5583	5602	5621	5641	5660	5679	5698	5717	3	6	10	13	16
22	$\bar{1}$·5736	5754	5773	5792	5810	5828	5847	5865	5883	5901	3	6	9	12	15
23	$\bar{1}$·5919	5937	5954	5972	5990	6007	6024	6042	6059	6076	3	6	9	12	15
24	$\bar{1}$·6093	6110	6127	6144	6161	6177	6194	6210	6227	6243	3	6	8	11	14
25	$\bar{1}$·6259	6276	6292	6308	6324	6340	6356	6371	6387	6403	3	5	8	11	13
26	$\bar{1}$·6418	6434	6449	6465	6480	6495	6510	6526	6541	6556	3	5	8	10	13
27	$\bar{1}$·6570	6585	6600	6615	6629	6644	6659	6673	6687	6702	2	5	7	10	12
28	$\bar{1}$·6716	6730	6744	6759	6773	6787	6801	6814	6828	6842	2	5	7	9	12
29	$\bar{1}$·6856	6869	6883	6896	6910	6923	6937	6950	6863	6977	2	4	7	9	11
30	$\bar{1}$·6990	7003	7016	7029	7042	7055	7068	7080	7093	7106	2	4	6	9	11
31	$\bar{1}$·7118	7131	7144	7156	7168	7181	7193	7205	7218	7230	2	4	6	8	10
32	$\bar{1}$·7242	7254	7266	7278	7290	7302	7314	7326	7338	7349	2	4	6	8	10
33	$\bar{1}$·7361	7373	7384	7396	7407	7419	7430	7442	7453	7464	2	4	6	8	10
34	$\bar{1}$·7476	7487	7498	7509	7520	7531	7542	7553	7564	7575	2	4	6	7	9
35	$\bar{1}$·7586	7597	7607	7618	7629	7640	7650	7661	7671	7682	2	4	5	7	9
36	$\bar{1}$·7692	7703	7713	7723	7734	7744	7754	7764	7774	7785	2	3	5	7	9
37	$\bar{1}$·7795	7805	7815	7825	7835	7844	7854	7864	7874	7884	2	3	5	7	8
38	$\bar{1}$·7893	7903	7913	7922	7932	7941	7951	7960	7970	7979	2	3	5	6	8
39	$\bar{1}$·7989	7998	8007	8017	8026	8035	8044	8053	8063	8072	2	3	5	6	8
40	$\bar{1}$·8081	8090	8099	8108	8117	8125	8134	8143	8152	8161	1	3	4	6	7
41	$\bar{1}$·8169	8178	8187	8195	8204	8213	8221	8230	8238	8247	1	3	4	6	7
42	$\bar{1}$·8255	8264	8272	8280	8289	8297	8305	8313	8322	8330	1	3	4	6	7
43	$\bar{1}$·8338	8346	8354	8362	8370	8378	8386	8394	8402	8410	1	3	4	5	7
44	$\bar{1}$·8418	8426	8433	8441	8449	8457	8464	8472	8480	8487	1	3	4	5	6

LOGARITHMS OF SINES

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences 1	2	3	4	5
45	$\bar{1}$·8495	8502	8510	8517	8525	8532	8540	8547	8555	8562	1	2	4	5	6
46	$\bar{1}$·8569	8577	8584	8591	8598	8606	8613	8620	8627	8634	1	2	4	5	6
47	$\bar{1}$·8641	8648	8655	8662	8669	8676	8683	8690	8697	8704	1	2	3	5	6
48	$\bar{1}$·8711	8718	8724	8731	8738	8745	8751	8758	8765	8771	1	2	3	4	6
49	$\bar{1}$·8778	8784	8791	8797	8804	8810	8817	8823	8830	8836	1	2	3	4	5
50	$\bar{1}$·8843	8849	8855	8862	8868	8874	8880	8887	8893	8899	1	2	3	4	5
51	$\bar{1}$·8905	8911	8917	8923	8929	8935	8941	8947	8953	8959	1	2	3	4	5
52	$\bar{1}$·8965	8971	8977	8983	8989	8995	9000	9006	9012	9018	1	2	3	4	5
53	$\bar{1}$·9023	9029	9035	9041	9046	9052	9057	9063	9069	9074	1	2	3	4	5
54	$\bar{1}$·9080	9085	9091	9096	9101	9107	9112	9118	9123	9128	1	2	3	4	5
55	$\bar{1}$·9134	9139	9144	9149	9155	9160	9165	9170	9175	9181	1	2	3	3	4
56	$\bar{1}$·9186	9191	9196	9201	9206	9211	9216	9221	9226	9231	1	2	3	3	4
57	$\bar{1}$·9236	9241	9246	9251	9255	9260	9265	9270	9275	9279	1	2	2	3	4
58	$\bar{1}$·9284	9289	9294	9298	9303	9308	9312	9317	9322	9326	1	2	2	3	4
59	$\bar{1}$·9331	9335	9340	9344	9349	9353	9358	9362	9367	9371	1	1	2	3	4
60	$\bar{1}$·9375	9380	9384	9388	9393	9397	9401	9406	9410	9414	1	1	2	3	4
61	$\bar{1}$·9418	9422	9427	9431	9435	9439	9443	9447	9451	9455	1	1	2	3	3
62	$\bar{1}$·9459	9463	9467	9471	9475	9479	9483	9487	9491	9495	1	1	2	3	3
63	$\bar{1}$·9499	9503	9506	9510	9514	9518	9522	9525	9529	9533	1	1	2	3	3
64	$\bar{1}$·9537	9540	9544	9548	9551	9555	9558	9562	9566	9569	1	1	2	2	3
65	$\bar{1}$·9573	9576	9580	9583	9587	9590	9594	9597	9601	9604	1	1	2	2	3
66	$\bar{1}$·9607	9610	9614	9617	9621	9624	9627	9631	9634	9637	1	1	2	2	3
67	$\bar{1}$·9640	9643	9647	9650	9653	9656	9659	9662	9666	9669	1	1	2	2	3
68	$\bar{1}$·9672	9674	9678	9681	9684	9687	9690	9693	9696	9699	0	1	1	2	2
69	$\bar{1}$·9702	9704	9707	9710	9713	9716	9719	9722	9724	9727	0	1	1	2	2
70	$\bar{1}$·9730	9733	9735	9738	9741	9743	9746	9749	9751	9754	0	1	1	2	2
71	$\bar{1}$·9757	9759	9762	9764	9767	9770	9772	9775	9777	9780	0	1	1	2	2
72	$\bar{1}$·9782	9785	9787	9789	9792	9794	9797	9799	9801	9804	0	1	1	2	2
73	$\bar{1}$·9806	9808	9811	9813	9815	9817	9820	9822	9824	9826	0	1	1	2	2
74	$\bar{1}$·9828	9831	9833	9835	9837	9839	9841	9843	9845	9847	0	1	1	1	2
75	$\bar{1}$·9849	9851	9853	9855	9857	9859	9861	9863	9865	9867	0	1	1	1	2
76	$\bar{1}$·9869	9871	9873	9875	9876	9878	9880	9882	9884	9885	0	1	1	1	2
77	$\bar{1}$·9887	9889	9891	9892	9894	9896	9897	9899	9901	9902	0	1	1	1	1
78	$\bar{1}$·9904	9906	9907	9909	9910	9912	9913	9915	9916	9918	0	1	1	1	1
79	$\bar{1}$·9919	9921	9922	9924	9925	9927	9928	9929	9931	9932	0	0	1	1	1
80	$\bar{1}$·9934	9935	9936	9937	9939	9940	9941	9943	9944	9945	0	0	1	1	1
81	$\bar{1}$·9946	9947	9949	9950	9951	9952	9953	9954	9955	9956	0	0	1	1	1
82	$\bar{1}$·9958	9959	9960	9961	9962	9963	9964	9965	9966	9967	0	0	1	1	1
83	$\bar{1}$·9968	9968	9969	9970	9971	9972	9973	9974	9975	9975	0	0	0	1	1
84	$\bar{1}$·9976	9977	9978	9978	9979	9980	9981	9981	9982	9983	0	0	0	0	1
85	$\bar{1}$·9983	9984	9985	9985	9986	9987	9987	9988	9988	9989	0	0	0	0	0
86	$\bar{1}$·9989	9990	9990	9991	9991	9992	9992	9993	9993	9994	0	0	0	0	0
87	$\bar{1}$·9994	9994	9995	9995	9996	9996	9996	9996	9997	9997	0	0	0	0	0
88	$\bar{1}$·9997	9998	9998	9998	9998	9999	9999	9999	9999	9999	0	0	0	0	0
89	$\bar{1}$·9999	9999	0.0000	0000	0000	0000	0000	0000	0000	0000					
90	0·0000														

LOGARITHMS OF COSINES

[Numbers in difference columns to be subtracted, not added.]

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences 1	2	3	4	5
0	0.0000	0000	0000	0000	0000	0000	0000	0000	0000	$\bar{1}$.9999	0	0	0	0	0
1	$\bar{1}$.9999	9999	9999	9999	9999	9999	9998	9998	9998	9998	0	0	0	0	0
2	$\bar{1}$.9997	9997	9997	9996	9996	9996	9996	9995	9995	9994	0	0	0	0	0
3	$\bar{1}$.9994	9994	9993	9993	9992	9992	9991	9991	9990	9990	0	0	0	0	0
4	$\bar{1}$.9989	9989	9988	9988	9987	9987	9986	9985	9985	9984	0	0	0	0	0
5	$\bar{1}$.9983	9983	9982	9981	9981	9980	9979	9978	9978	9977	0	0	0	0	1
6	$\bar{1}$.9976	9975	9975	9974	9973	9972	9971	9970	9969	9968	0	0	0	1	1
7	$\bar{1}$.9968	9967	9966	9965	9964	9963	9962	9961	9960	9959	0	0	1	1	1
8	$\bar{1}$.9958	9956	9955	9954	9953	9952	9951	9950	9949	9947	0	0	1	1	1
9	$\bar{1}$.9946	9945	9944	9943	9941	9940	9939	9937	9936	9935	0	0	1	1	1
10	$\bar{1}$.9934	9932	9931	9929	9928	9927	9925	9924	9922	9921	0	0	1	1	1
11	$\bar{1}$.9919	9918	9916	9915	9913	9912	9910	9909	9907	9906	0	1	1	1	1
12	$\bar{1}$.9904	9902	9901	9899	9897	9896	9894	9892	9891	9889	0	1	1	1	1
13	$\bar{1}$.9887	9885	9884	9882	9880	9878	9876	9875	9873	9871	0	1	1	1	2
14	$\bar{1}$.9869	9867	9865	9863	9861	9859	9857	9855	9853	9851	0	1	1	1	2
15	$\bar{1}$.9849	9847	9845	9843	9841	9839	9837	9835	9833	9831	0	1	1	1	2
16	$\bar{1}$.9828	9826	9824	9822	9820	9817	9815	9813	9811	9808	0	1	1	2	2
17	$\bar{1}$.9806	9804	9801	9799	9797	9794	9792	9789	9787	9785	0	1	1	2	2
18	$\bar{1}$.9782	9780	9777	9775	9772	9770	9767	9764	9762	9759	0	1	1	2	2
19	$\bar{1}$.9757	9754	9751	9749	9746	9743	9741	9738	9735	9733	0	1	1	2	2
20	$\bar{1}$.9730	9727	9724	9722	9719	9716	9713	9710	9707	9704	0	1	1	2	2
21	$\bar{1}$.9702	9699	9696	9693	9690	9687	9684	9681	9678	9675	0	1	1	2	2
22	$\bar{1}$.9672	9669	9666	9662	9659	9656	9653	9650	9647	9643	1	1	2	2	3
23	$\bar{1}$.9640	9637	9634	9631	9627	9624	9621	9617	9614	9611	1	1	2	2	3
24	$\bar{1}$.9607	9604	9601	9597	9594	9590	9587	9583	9580	9576	1	1	2	2	3
25	$\bar{1}$.9573	9569	9566	9562	9558	9555	9551	9548	9544	9540	1	1	2	2	3
26	$\bar{1}$.9537	9533	9529	9525	9522	9518	9514	9510	9506	9503	1	1	2	3	3
27	$\bar{1}$.9499	9495	9491	9487	9483	9479	9475	9471	9467	9463	1	1	2	3	3
28	$\bar{1}$.9459	9455	9451	9447	9443	9439	9435	9431	9427	9422	1	1	2	3	3
29	$\bar{1}$.9418	9414	9410	9406	9401	9397	9393	9388	9384	9380	1	1	2	3	4
30	$\bar{1}$.9375	9371	9367	9362	9358	9353	9349	9344	9340	9335	1	1	2	3	4
31	$\bar{1}$.9331	9326	9322	9317	9312	9308	9303	9298	9294	9289	1	2	2	3	4
32	$\bar{1}$.9284	9279	9275	9270	9265	9260	9255	9251	9246	9241	1	2	2	3	4
33	$\bar{1}$.9236	9231	9226	9221	9216	9211	9206	9201	9196	9191	1	2	3	3	4
34	$\bar{1}$.9186	9181	9175	9170	9165	9160	9155	9149	9144	9139	1	2	3	3	4
35	$\bar{1}$.9134	9128	9123	9118	9112	9107	9101	9096	9091	9085	1	2	3	4	5
36	$\bar{1}$.9080	9074	9069	9063	9057	9052	9046	9041	9035	9029	1	2	3	4	5
37	$\bar{1}$.9023	9018	9012	9006	9000	8995	8989	8983	8977	8971	1	2	3	4	5
38	$\bar{1}$.8965	8959	8953	8947	8941	8935	8929	8923	8917	8911	1	2	3	4	5
39	$\bar{1}$.8905	8899	8893	8887	8880	8874	8868	8862	8855	8840	1	2	3	4	5
40	$\bar{1}$.8843	8836	8830	8823	8817	8810	8804	8797	8791	8784	1	2	3	4	5
41	$\bar{1}$.8778	8771	8765	8758	8751	8745	8738	8731	8724	8718	1	2	3	5	6
42	$\bar{1}$.8711	8704	8697	8690	8683	8676	8669	8662	8655	8648	1	2	3	5	6
43	$\bar{1}$.8641	8634	8627	8620	8613	8606	8598	8591	8584	8577	1	2	4	5	6
44	$\bar{1}$.8569	8562	8555	8547	8540	8532	8525	8517	8510	8502	1	2	4	5	6

LOGARITHMS OF COSINES

[Numbers in difference columns to be subtracted, not added].

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences 1	2	3	4	5
45	$\bar{1}.8495$	8487	8480	8472	8464	8457	8449	8441	8433	8426	1	3	4	5	6
46	$\bar{1}.8418$	8410	8402	8394	8386	8378	8370	8362	8354	8346	1	3	4	5	7
47	$\bar{1}.8338$	8330	8322	8313	8305	8297	8289	8280	8272	8264	1	3	4	6	7
48	$\bar{1}.8255$	8247	8238	8230	8221	8213	8204	8195	8187	8178	1	3	4	6	7
49	$\bar{1}.8169$	8161	8152	8143	8134	8125	8117	8108	8099	8090	1	3	4	6	7
50	$\bar{1}.8081$	8072	8063	8053	8044	8035	8026	8017	8007	7998	2	3	5	6	8
51	$\bar{1}.7989$	7979	7970	7960	7951	7941	7932	7922	7913	7903	2	3	5	6	8
52	$\bar{1}.7893$	7884	7874	7864	7854	7844	7835	7825	7815	7805	2	3	5	7	8
53	$\bar{1}.7795$	7785	7774	7764	7754	7744	7734	7723	7713	7703	2	3	5	7	9
54	$\bar{1}.7692$	7682	7671	7661	7650	7640	7629	7618	7607	7597	2	4	5	7	9
55	$\bar{1}.7586$	7575	7564	7553	7542	7531	7520	7509	7498	7487	2	4	6	7	9
56	$\bar{1}.7476$	7464	7453	7442	7430	7419	7407	7396	7384	7373	2	4	6	8	10
57	$\bar{1}.7361$	7349	7338	7326	7314	7302	7290	7278	7266	7254	2	4	6	8	10
58	$\bar{1}.7242$	7230	7218	7205	7193	7181	7168	7156	7144	7131	2	4	6	8	10
59	$\bar{1}.7118$	7106	7093	7080	7068	7055	7042	7029	7016	7003	2	4	6	9	11
60	$\bar{1}.6990$	6977	6963	6950	6937	6923	6910	6896	6883	6869	2	4	7	9	11
61	$\bar{1}.6856$	6842	6828	6814	6801	6787	6773	6759	6744	6730	2	5	7	9	12
62	$\bar{1}.6716$	6702	6687	6673	6659	6644	6629	6615	6600	6585	2	5	7	10	12
63	$\bar{1}.6570$	6556	6541	6526	6510	6495	6480	6465	6449	6434	3	5	8	10	13
64	$\bar{1}.6418$	6403	6387	6371	6356	6340	6324	6308	6292	6276	3	5	8	11	13
65	$\bar{1}.6259$	6243	6227	6210	6194	6177	6161	6144	6127	6110	3	6	8	11	14
66	$\bar{1}.6093$	6076	6059	6042	6024	6007	5990	5972	5954	5937	3	6	9	12	15
67	$\bar{1}.5919$	5901	5883	5865	5847	5828	5810	5792	5773	5754	3	6	9	12	15
68	$\bar{1}.5736$	5717	5698	5679	5660	5641	5621	5602	5583	5563	3	6	10	13	16
69	$\bar{1}.5543$	5523	5504	5484	5463	5443	5423	5402	5382	5361	3	7	10	14	17
70	$\bar{1}.5341$	5320	5299	5278	5256	5235	5213	5192	5170	5148	4	7	11	14	18
71	$\bar{1}.5126$	5104	5082	5060	5037	5015	4992	4969	4946	4923	4	8	11	15	19
72	$\bar{1}.4900$	4876	4853	4829	4805	4781	4757	4733	4709	4684	4	8	12	16	20
73	$\bar{1}.4659$	4634	4609	4584	4559	4533	4508	4482	4456	4430	4	9	13	17	21
74	$\bar{1}.4403$	4377	4350	4323	4296	4269	4242	4214	4186	4158	5	9	14	18	23
75	$\bar{1}.4130$	4102	4073	4044	4015	3986	3957	3927	3897	3867	5	10	15	20	24
76	$\bar{1}.3837$	3806	3775	3745	3713	3682	3650	3618	3586	3554	5	11	16	21	26
77	$\bar{1}.3521$	3488	3455	3421	3387	3353	3319	3284	3250	3214	6	11	17	23	28
78	$\bar{1}.3179$	3143	3107	3070	3034	2997	2959	2921	2883	2845	6	12	19	25	31
79	$\bar{1}.2806$	2767	2727	2687	2647	2606	2565	2524	2482	2439	7	14	20	27	34
80	1.2397	2353	2310	2266	2221	2176	2131	2085	2038	1991	8	15	23	30	38
81	$\bar{1}.1943$	1895	1847	1797	1747	1697	1646	1594	1542	1489	8	17	25	34	42
82	$\bar{1}.1436$	1381	1326	1271	1214	1157	1099	1040	0981	0920	10	19	29	38	48
83	$\bar{1}.0859$	0797	0734	0670	0605	0539	0472	0403	0334	0264	11	22	33	44	55
84	$\bar{1}.0192$	0120	0046	$\bar{2}.9970$	$\bar{2}.9894$	$\bar{2}.9816$	$\bar{2}.9736$	$\bar{2}.9655$	$\bar{2}.9573$	$\bar{2}.9489$	13	26	39	52	65
85	$\bar{2}.9403$	9315	9226	9135	9042	8946	8849	8749	8647	8543	16	32	48	64	80
86	$\bar{2}.8436$	8326	8213	8098	7979	7857	7731	7602	7468	7330					
87	$\bar{2}.7188$	7041	6889	6731	6567	6397	6220	6035	5842	5640					
88	$\bar{2}.5428$	5206	4971	4723	4459	4179	3880	3558	3210	2832					
89	$\bar{2}.2419$	1961	1450	0870	0200	$\bar{3}.9408$	$\bar{3}.8439$	$\bar{3}.7190$	$\bar{3}.5429$	$\bar{3}.2419$					
90	∞														

LOGARITHMS OF TANGENTS

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences 1	2	3	4	5
0	$-\infty$	$\bar{3}.2419$	$\bar{3}.5429$	$\bar{3}.7190$	$\bar{3}.8439$	$\bar{3}.9409$	$\bar{2}.0200$	$\bar{2}.0870$	$\bar{2}.1450$	$\bar{2}.1962$					
1	$\bar{2}.2419$	2833	3211	3559	3881	4181	4461	4725	4973	5208					
2	$\bar{2}.5431$	5643	5845	6038	6223	6401	6571	6736	6894	7046					
3	$\bar{2}.7194$	7337	7475	7609	7739	7865	7988	8107	8223	8336					
4	$\bar{2}.8446$	8554	8659	8762	8862	8960	9056	9150	9241	9331	16	32	48	64	81
5	$\bar{2}.9420$	9506	9591	9674	9756	9836	9915	9992	$\bar{1}.0068$	1.0143	13	26	40	53	66
6	$\bar{1}.0216$	0289	0360	0430	0499	0567	0633	0699	0764	0828	11	22	34	45	56
7	$\bar{1}.0891$	0954	1015	1076	1135	1194	1252	1310	1367	1423	10	20	29	39	49
8	$\bar{1}.1478$	1533	1587	1640	1693	1745	1797	1848	1898	1948	9	17	26	35	43
9	$\bar{1}.1997$	2046	2094	2142	2189	2236	2282	2328	2374	2419	8	16	23	31	39
10	$\bar{1}.2463$	2507	2551	2594	2637	2680	2722	2764	2805	2846	7	14	21	28	35
11	$\bar{1}.2887$	2927	2967	3006	3046	3085	3123	3162	3200	3237	6	13	19	26	32
12	$\bar{1}.3275$	3312	3349	3385	3422	3458	3493	3529	3564	3599	6	12	18	24	30
13	$\bar{1}.3634$	3668	3702	3736	3770	3804	3837	3870	3903	3935	6	11	17	22	28
14	$\bar{1}.3968$	4000	4032	4064	4095	4127	4158	4189	4220	4250	5	10	16	21	26
15	$\bar{1}.4281$	4311	4341	4371	4400	4430	4459	4488	4517	4646	5	10	15	20	25
16	$\bar{1}.4575$	4603	4632	4660	4688	4716	4744	4771	4799	4826	5	9	14	19	23
17	$\bar{1}.4853$	4880	4907	4934	4961	4987	5014	5040	5066	5092	4	9	13	18	22
18	$\bar{1}.5118$	5143	5169	5195	5220	5245	5270	5295	5320	5345	4	8	13	17	21
19	$\bar{1}.5370$	5394	5419	5443	5467	5491	5516	5539	5563	5587	4	8	12	16	20
20	$\bar{1}.5611$	5634	5658	5681	5704	5727	5750	5773	5796	5819	4	8	12	15	19
21	$\bar{1}.5842$	5864	5887	5909	5932	5954	5976	5998	6020	6042	4	7	11	15	19
22	$\bar{1}.6064$	6086	6108	6129	6151	6172	6194	6215	6236	6257	4	7	11	14	18
23	$\bar{1}.6279$	6300	6321	6341	6362	6383	6404	6424	6445	6465	3	7	10	14	17
24	$\bar{1}.6486$	6506	6527	6547	6567	6587	6607	6627	6647	6667	3	7	10	13	17
25	$\bar{1}.6687$	6706	6726	6746	6765	6785	6804	6824	6843	6863	3	7	10	13	16
26	$\bar{1}.6882$	6901	6920	6939	6958	6977	6996	7015	7034	7053	3	6	9	13	16
27	$\bar{1}.7072$	7090	7109	7128	7146	7165	7183	7202	7220	7238	3	6	9	12	15
28	$\bar{1}.7257$	7275	7293	7311	7330	7348	7366	7384	7402	7420	3	6	9	12	15
29	$\bar{1}.7438$	7455	7473	7491	7509	7526	7544	7562	7579	7597	3	6	9	12	15
30	$\bar{1}.7614$	7632	7649	7667	7684	7701	7719	7736	7753	7771	3	6	9	12	14
31	$\bar{1}.7788$	7805	7822	7839	7856	7873	7890	7907	7924	7941	3	6	9	11	14
32	$\bar{1}.7958$	7975	7992	8008	8025	8042	8059	8075	8092	8109	3	6	8	11	14
33	$\bar{1}.8125$	8142	8158	8175	8191	8208	8224	8241	8257	8274	3	5	8	11	14
34	$\bar{1}.8290$	8306	8323	8339	8355	8371	8388	8404	8420	8436	3	5	8	11	14
35	$\bar{1}.8452$	8468	8484	8501	8517	8533	8549	8565	8581	8597	3	5	8	11	13
36	$\bar{1}.8613$	8629	8644	8660	8676	8692	8708	8724	8740	8755	3	5	8	11	13
37	$\bar{1}.8771$	8787	8803	8818	8834	8850	8865	8881	8897	8912	3	5	8	10	13
38	$\bar{1}.8928$	8944	8959	8975	8990	9006	9022	9037	9053	9068	3	5	8	10	13
39	$\bar{1}.9084$	9099	9115	9130	9146	9161	9176	9192	9207	9223	3	5	8	10	13
40	$\bar{1}.9238$	9254	9269	9284	9300	9315	9330	9346	9361	9376	3	5	8	10	13
41	$\bar{1}.9392$	9407	9422	9438	9453	9468	9483	9499	9514	9529	3	5	8	10	13
42	$\bar{1}.9544$	9560	9575	9590	9605	9621	9636	9651	9666	9681	3	5	8	10	13
43	$\bar{1}.9697$	9712	9727	9742	9757	9772	9788	9803	9818	9833	3	5	8	10	13
44	$\bar{1}.9848$	9864	9879	9894	9909	9924	9939	9955	9970	9985	3	5	8	10	13

LOGARITHMS OF TANGENTS

Degrees	0' 0°.0	6' 0°.1	12' 0°.2	18' 0°.3	24' 0°.4	30' 0°.5	36' 0°.6	42' 0°.7	48' 0°.8	54' 0°.9	Mean Differences 1	2	3	4	5
45	·0000	0015	0030	0045	0061	0076	0091	0106	0121	0136	3	5	8	10	13
46	·0152	0167	0182	0197	0212	0228	0243	0258	0273	0288	3	5	8	10	13
47	·0303	0319	0334	0349	0364	0379	0395	0410	0425	0440	3	5	8	10	13
48	·0456	0471	0486	0501	0517	0532	0547	0562	0578	0593	3	5	8	10	13
49	·0608	0624	0639	0654	0670	0685	0700	0716	0731	0746	3	5	8	10	13
50	·0762	0777	0793	0808	0824	0839	0854	0870	0885	0901	3	5	8	10	13
51	·0916	0932	0947	0963	0978	0994	1010	1025	1041	1056	3	5	8	10	13
52	·1072	1088	1103	1119	1135	1150	1166	1182	1197	1213	3	5	8	10	13
53	·1229	1245	1260	1276	1292	1308	1324	1340	1356	1371	3	5	8	11	13
54	·1387	1403	1419	1435	1451	1467	1483	1499	1516	1532	3	5	8	11	13
55	·1548	1564	1580	1596	1612	1629	1645	1661	1677	1694	3	5	8	11	14
56	·1710	1726	1743	1759	1776	1792	1809	1825	1842	1858	3	5	8	11	14
57	·1875	1891	1908	1925	1941	1958	1975	1992	2008	2025	3	6	8	11	14
58	·2042	2059	2076	2093	2110	2127	2144	2161	2178	2195	3	6	9	11	14
59	·2212	2229	2247	2264	2281	2299	2316	2333	2351	2368	3	6	9	12	14
60	·2386	2403	2421	2438	2456	2474	2491	2509	2527	2545	3	6	9	12	15
61	·2562	2580	2598	2616	2634	2652	2670	2689	2707	2725	3	6	9	12	15
62	·2743	2762	2780	2798	2817	2835	2854	2872	2891	2910	3	6	9	12	15
63	·2928	2947	2966	2985	3004	3023	3042	3061	3080	3099	3	6	9	13	16
64	·3118	3137	3157	3176	3196	3215	3235	3254	3274	3294	3	6	10	13	16
65	·3313	3333	3353	3373	3393	3413	3433	3453	3473	3494	3	7	10	13	17
66	·3514	3535	3555	3576	3596	3617	3638	3659	3679	3700	3	7	10	14	17
67	·3721	3743	3764	3785	3806	3828	3849	3871	3892	3914	4	7	11	14	18
68	·3936	3958	3980	4002	4024	4046	4068	4091	4113	4136	4	7	11	15	19
69	·4158	4181	4204	4227	4250	4273	4296	4319	4342	4366	4	8	12	15	19
70	·4389	4413	4437	4461	4484	4509	4533	4557	4581	4606	4	8	12	16	20
71	·4630	4655	4680	4705	4730	4755	4780	4805	4831	4857	4	8	13	17	21
72	·4882	4908	4934	4960	4986	5013	5039	5066	5093	5120	4	9	13	18	22
73	·5147	5174	5201	5229	5256	5284	5312	5340	5368	5397	5	9	14	19	23
74	·5425	5454	5483	5512	5541	5570	5600	5629	5659	5689	5	10	15	20	25
75	·5719	5750	5780	5811	5842	5873	5905	5936	5968	6000	5	10	16	21	26
76	·6032	6065	6097	6130	6163	6196	6230	6264	6298	6332	6	11	17	22	28
77	·6366	6401	6436	6471	6507	6542	6578	6615	6651	6688	6	12	18	24	30
78	·6725	6763	6800	6838	6877	6915	6954	6904	7033	7073	6	13	19	26	32
79	·7113	7154	7195	7236	7278	7320	7363	7406	7449	7493	7	14	21	28	35
80	·7537	7581	7626	7672	7718	7764	7811	7858	7906	7954	8	16	23	31	39
81	·8003	8052	8102	8152	8203	8255	8307	8360	8413	8467	9	17	26	35	43
82	·8522	8577	8633	8690	8748	8806	8865	8924	8985	9046	10	20	29	39	49
83	·9109	9172	9236	9301	9367	9433	9501	9570	9640	9711	11	22	34	45	56
84	·9784	9857	9932	1·0008	1·0085	1·0164	1·0244	1·0326	1·0409	1·0494	13	26	40	53	66
85	1·0580	0669	0759	0850	0944	1040	1138	1238	1341	1446	16	32	48	64	81
86	1·1554	1664	1777	1893	2012	2135	2261	2391	2525	2663					
87	1·2806	2954	3106	3264	3429	3599	3777	3962	4155	4357					
88	1·4569	4792	5027	5275	5539	5819	6119	6441	6789	7167					
89	1·7581	8038	8550	9130	9800	2·0591	2·1561	2·2810	2·4571	2·7581					